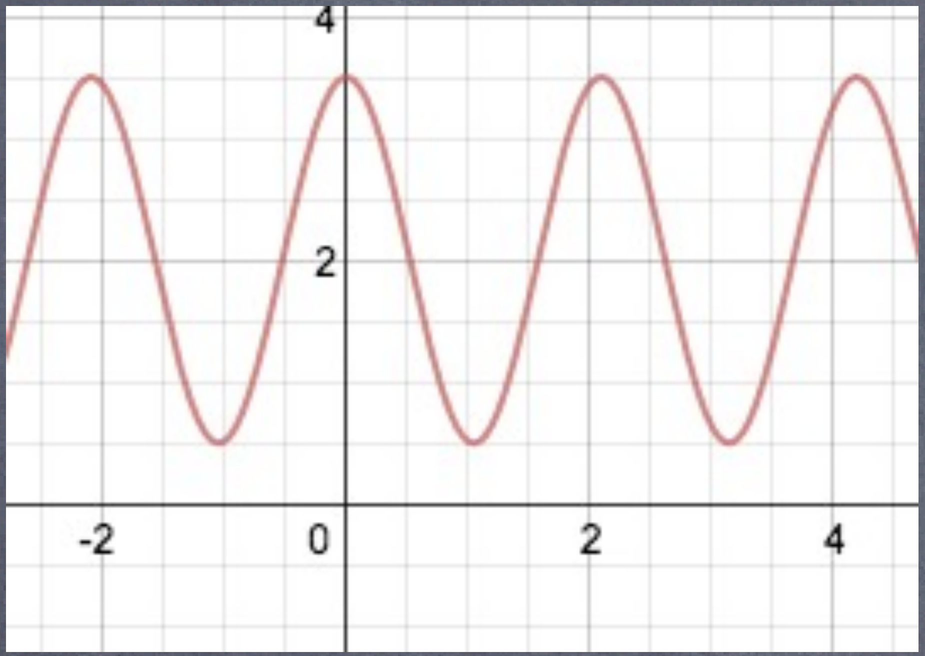
Today

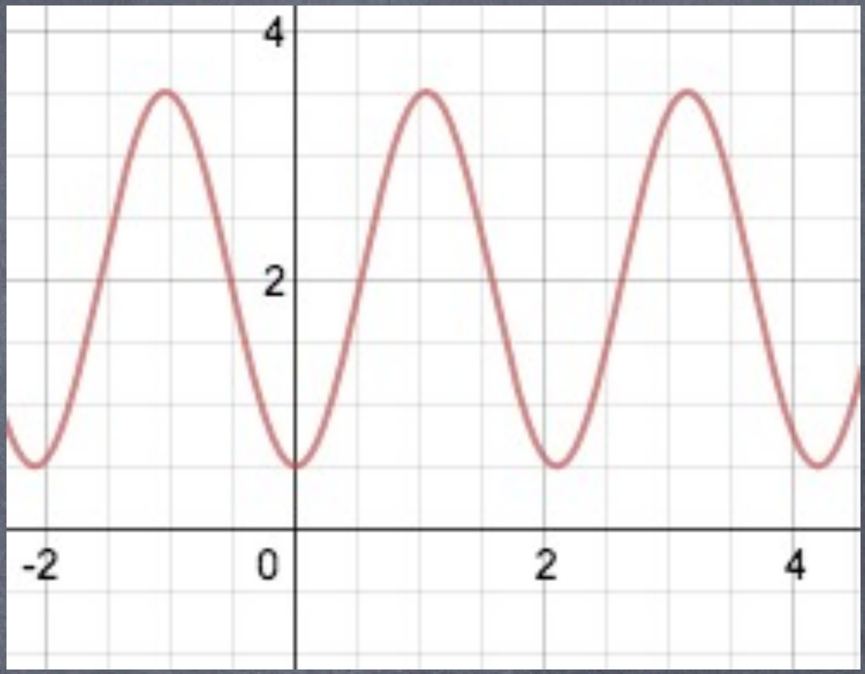
- Rhythmic processes
- Derivatives of trig functions
- Derivatives of inverse trig functions

Which is the graph of $y = 2 + 1.5\sin(3x - \pi/2)$?

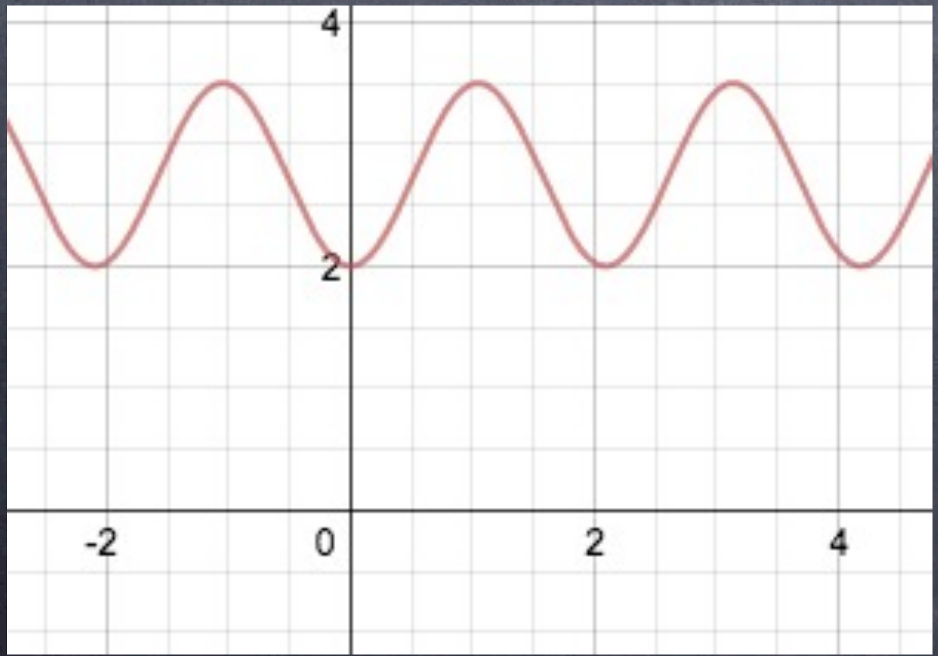
(A)



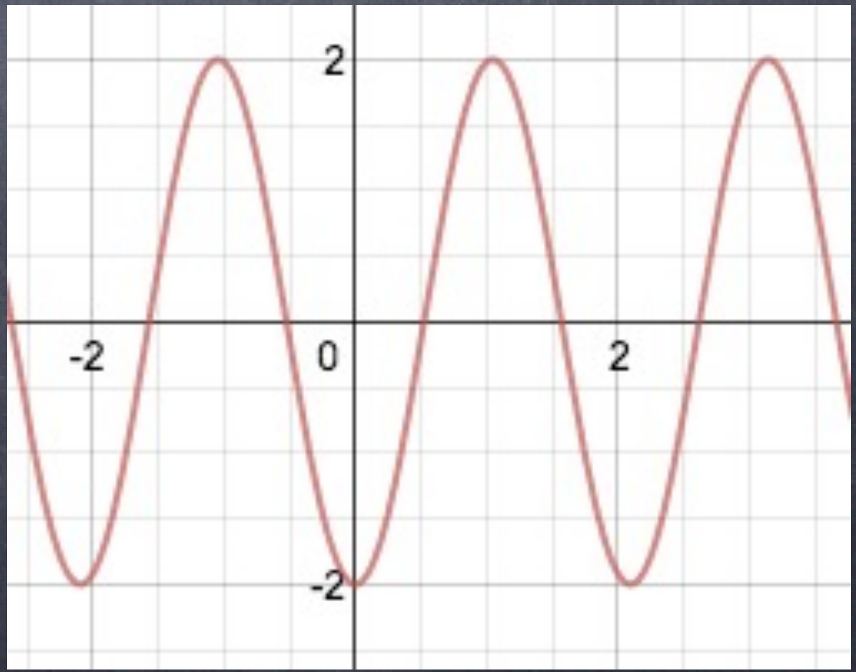
(B)



(C)

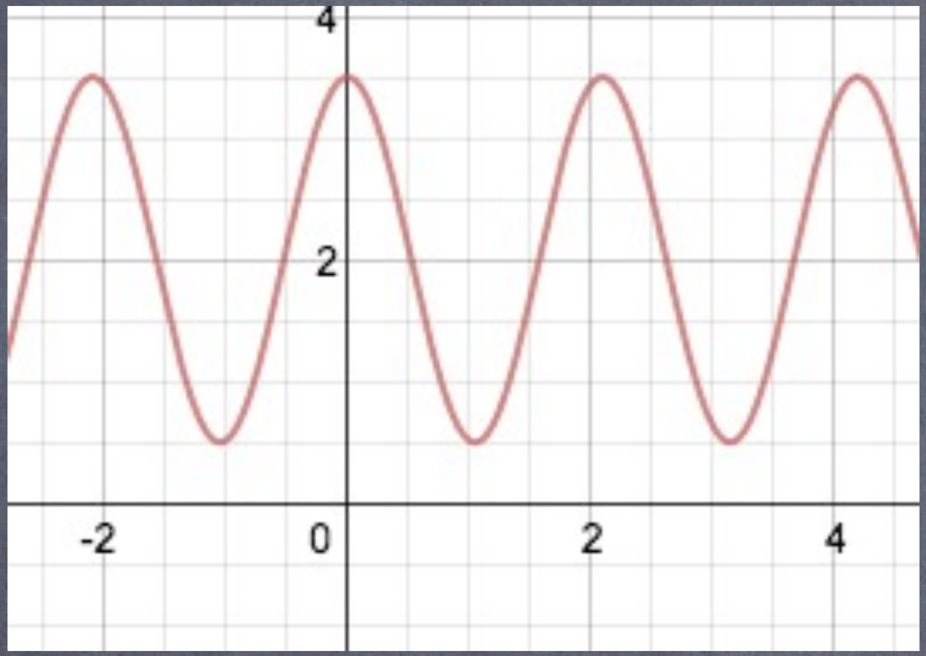


(D)

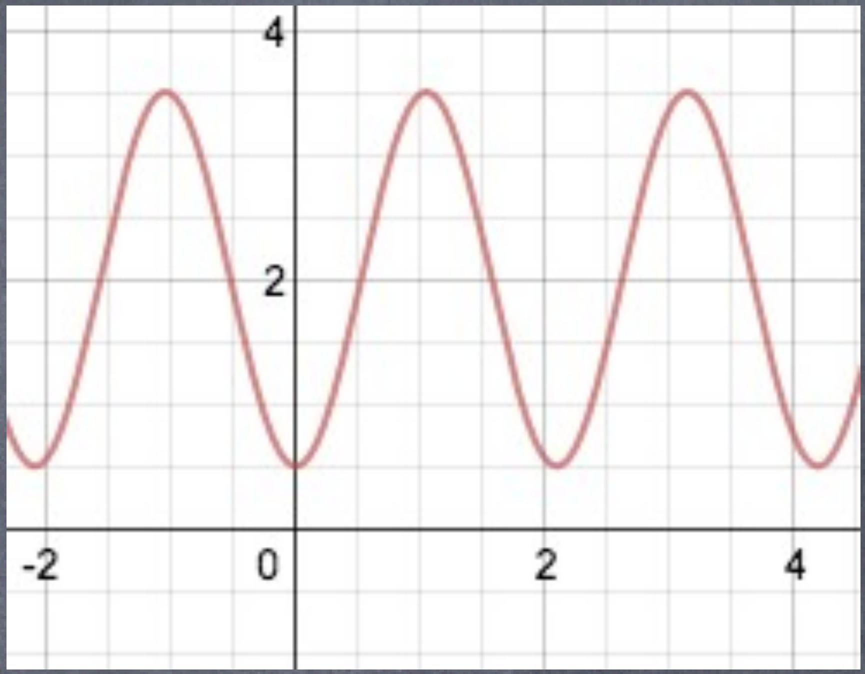


Which is the graph of $y = 2 + 1.5 \sin(3x - \pi/2)$?

(A)



(B)



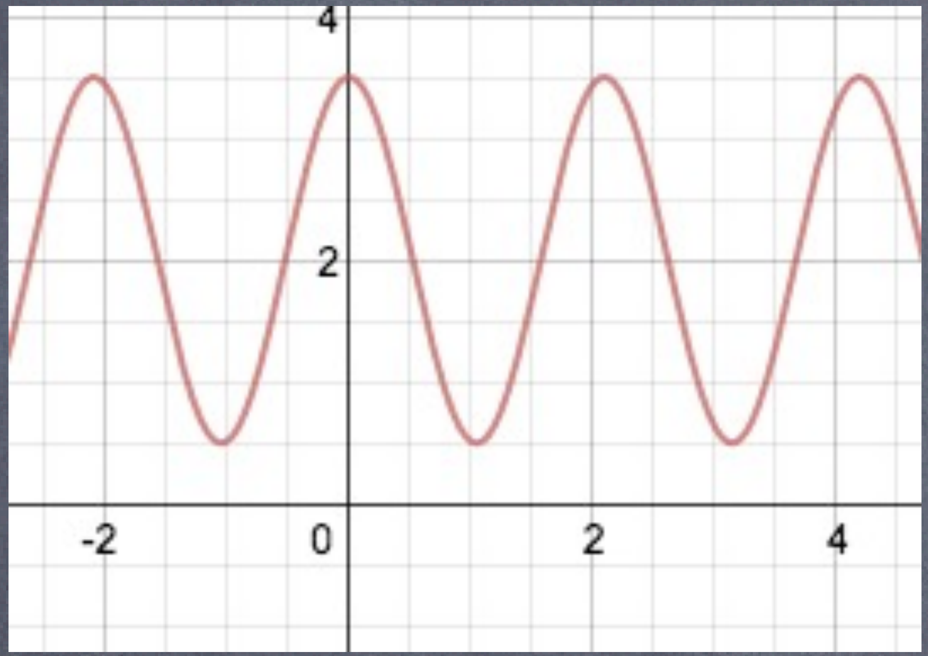
$y=2+1.5 \sin(3(x-\pi/6))$

so this is like $\sin(3x)$ shifted

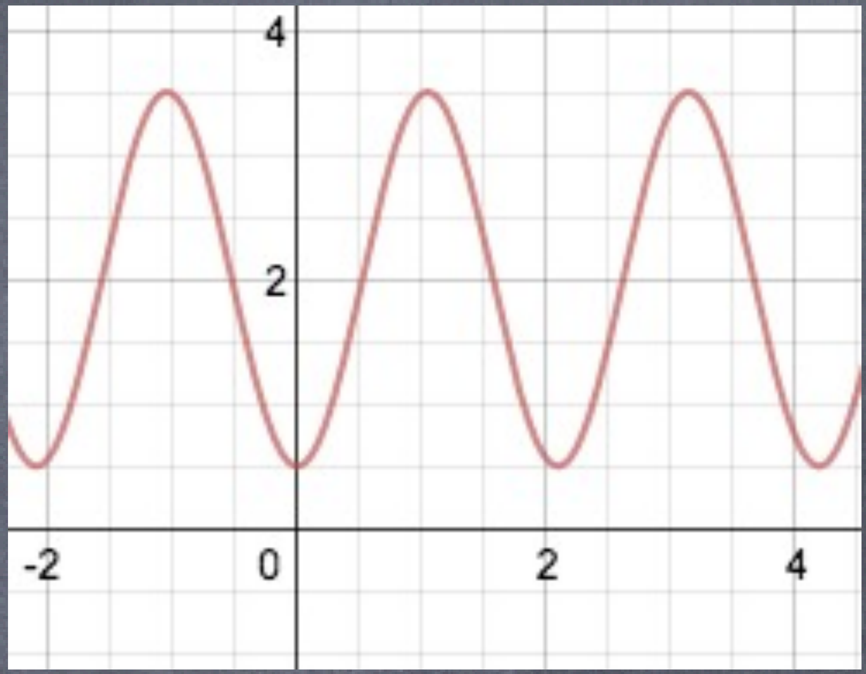
right by $\pi/6$.

Which is the graph of $y = 2 + 1.5 \sin(3x-\pi/2)$?

(A)



(B)



$y=2+1.5 \sin(3(x-\pi/6))$

so this is like $\sin(3x)$ shifted right by $\pi/6$.

Annual variation in daylight per day in Vancouver (Jan 1 --> t=0)

- (A) $L(t) = 12 + 4\cos\left(\frac{2\pi}{365}(t-172)\right)$
- (B) $L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t-172)\right)$
- (C) $L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t+80)\right)$
- (D) $L(t) = 12 - 4\sin\left(\frac{2\pi}{365}(t-80)\right)$

Annual variation in daylight per day in Vancouver (Jan 1 --> t=0)

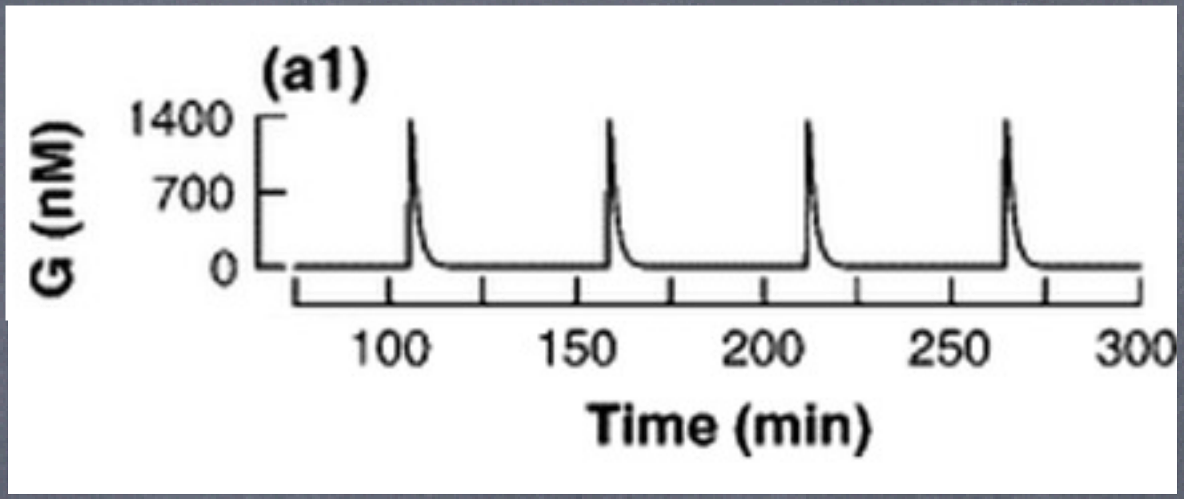
- (A) $L(t) = 12 + 4\cos\left(\frac{2\pi}{365}(t-172)\right)$
- (B) $L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t-172)\right)$
- (C) $L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t+80)\right)$
- (D) $L(t) = 12 - 4\sin\left(\frac{2\pi}{365}(t-80)\right)$

Note: t=172 is June 21; t=80 is March 21.

- The next two slides are challenging examples for you to think about...

Pulsatile release of GnRH

Which is NOT a reasonable model?



(A) $G(t) = 1400 \sin\left(\frac{2\pi}{110}(t-25)\right)^{20}$

(B) $G(t) = 1400 \sin\left(\frac{2\pi}{110}(t-25)\right)^{21}$

(C) $G(t) = 1400 \left(\frac{1}{2}\sin\left(\frac{2\pi}{55}(t+14)\right)+\frac{1}{2}\right)^{21}$

(D) $G(t) = 1400 \left(\frac{1}{2}\cos\left(\frac{2\pi}{55}t\right)+\frac{1}{2}\right)^{20}$

Length of flagpole shadow as a function of time

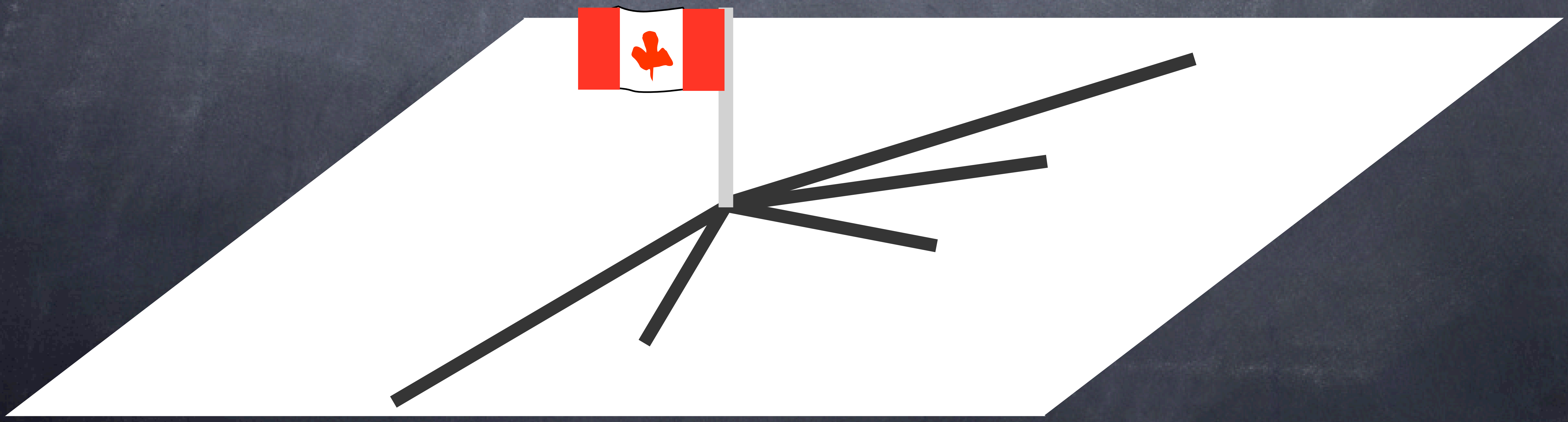
(A) sqrt(cos(2*pi/24*(t-13)))

(B) sqrt(sec(2*pi/24*(t-13)))

(C) sqrt(cos(2*pi/12*(t-13)))

(D) sqrt(sec(2*pi/12*(t-13)))

Which one is a reasonable model?



Derivative of $f(x)=\sin(x)$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h \to 0} (f(x+h) - f(x)) / h$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

$= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

  $= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

  $= \lim_{h\to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

  $= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

  $= \lim_{h\to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

  $= \lim_{h\to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

$= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h\to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h\to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

Derivative of $f(x)=\sin(x)$

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

$= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h\to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h\to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

See what $h=0.0001$ gives...

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h \to 0} (f(x+h) - f(x))/ h$

$= \lim_{h \to 0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h \to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h \to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h \to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h \to 0} \sin(h) /h$

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h \to 0} (f(x+h) - f(x))/ h$

$= \lim_{h \to 0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h \to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h \to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h \to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h \to 0} \sin(h) /h$

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

Note: this last step requires a bunch of work to show.

More details on that last step (not shown in class)

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h \to 0} (\cos(h)-1)/h$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

$= \lim_{h\to 0} (\cos^2(h)-1)/((\cos(h)+1)h)$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

$= \lim_{h\to 0} (\cos^2(h)-1)/((\cos(h)+1)h)$

$= \lim_{h\to 0} (-\sin^2(h))/((\cos(h)+1)h)$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) /h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

$= \lim_{h\to 0} (\cos^2(h)-1)/(\cos(h)+1)h)$

$= \lim_{h\to 0} (-\sin^2(h))/(\cos(h)+1)h)$

$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \sin(h)/h$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h) / h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

$= \lim_{h\to 0} (\cos^2(h)-1)/((\cos(h)+1)h)$

$= \lim_{h\to 0} (-\sin^2(h))/((\cos(h)+1)h)$

$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \sin(h)/h$

$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \lim_{h\to 0} \sin(h)/h$

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to0} \sin(h) /h$

First, we simplify $\lim_{h\to0} (\cos(h)-1)/h$

$= \lim_{h\to0} (\cos^2(h)-1)/((\cos(h)+1)h)$

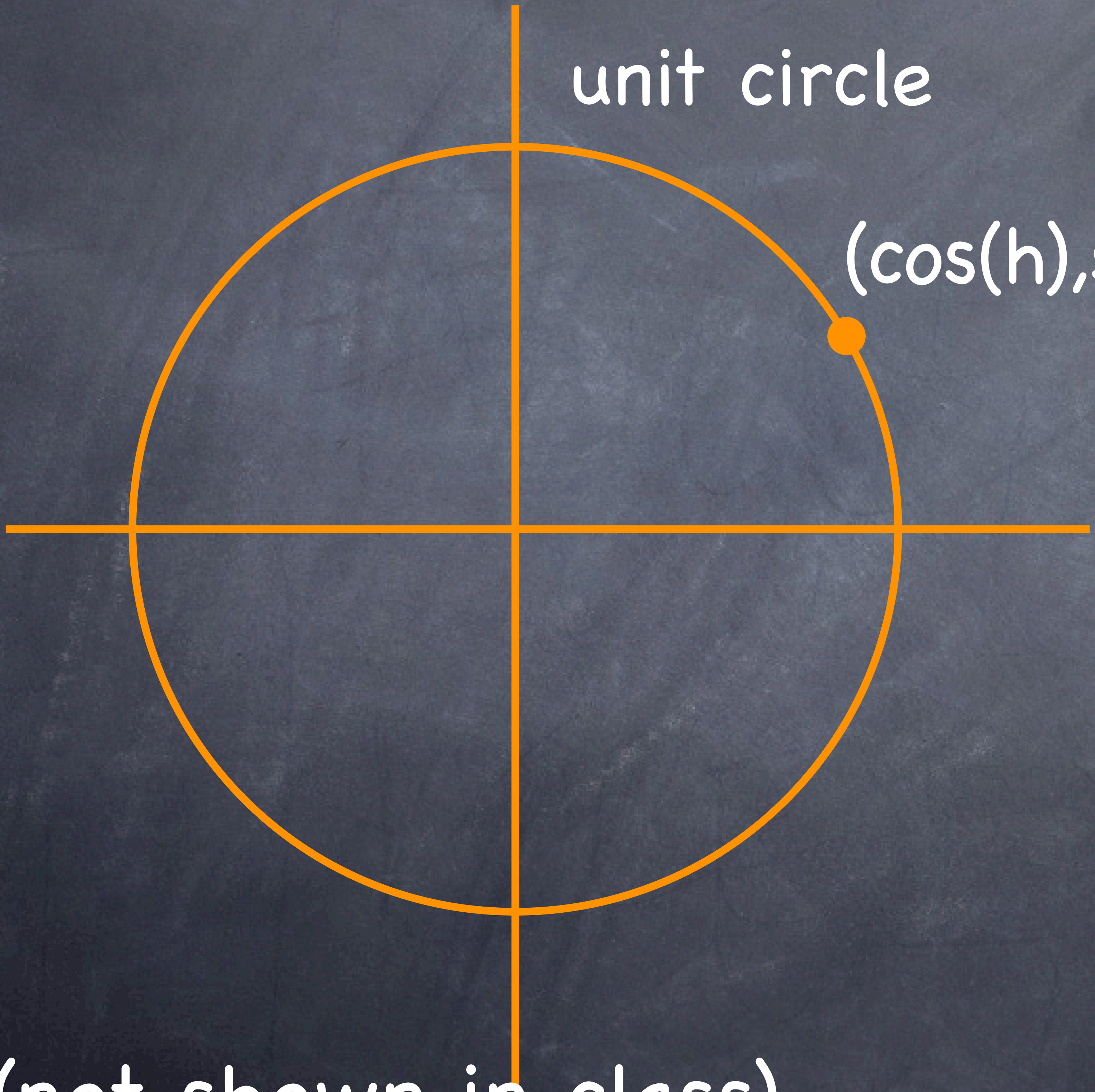
$= \lim_{h\to0} (-\sin^2(h))/((\cos(h)+1)h)$

$= \lim_{h\to0} -\sin(h)/(\cos(h)+1) \times \sin(h)/h$

$= \lim_{h\to0} -\sin(h)/(\cos(h)+1) \times \lim_{h\to0} \sin(h)/h$

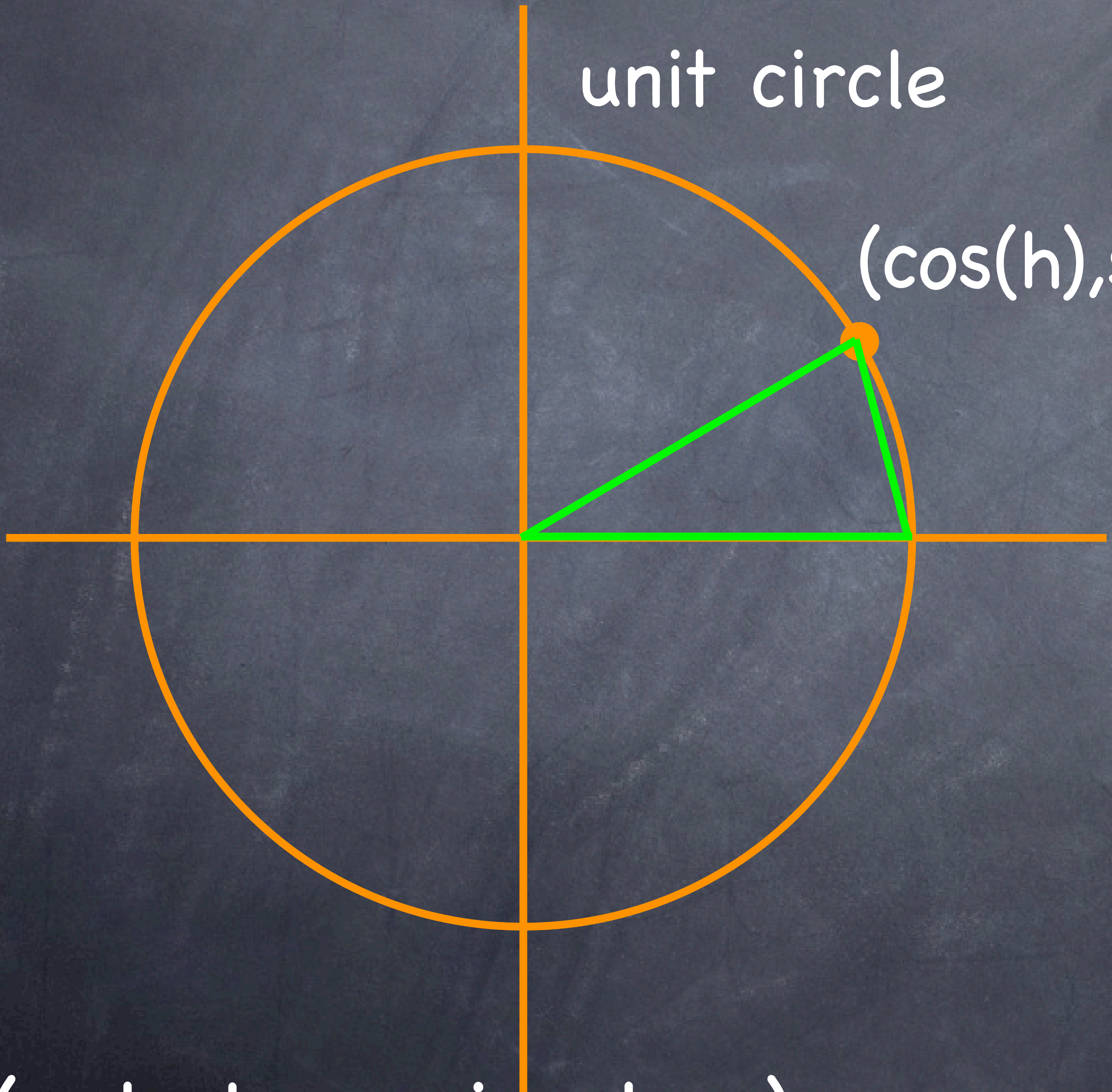
$= \quad 0 \quad \times \quad 1$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



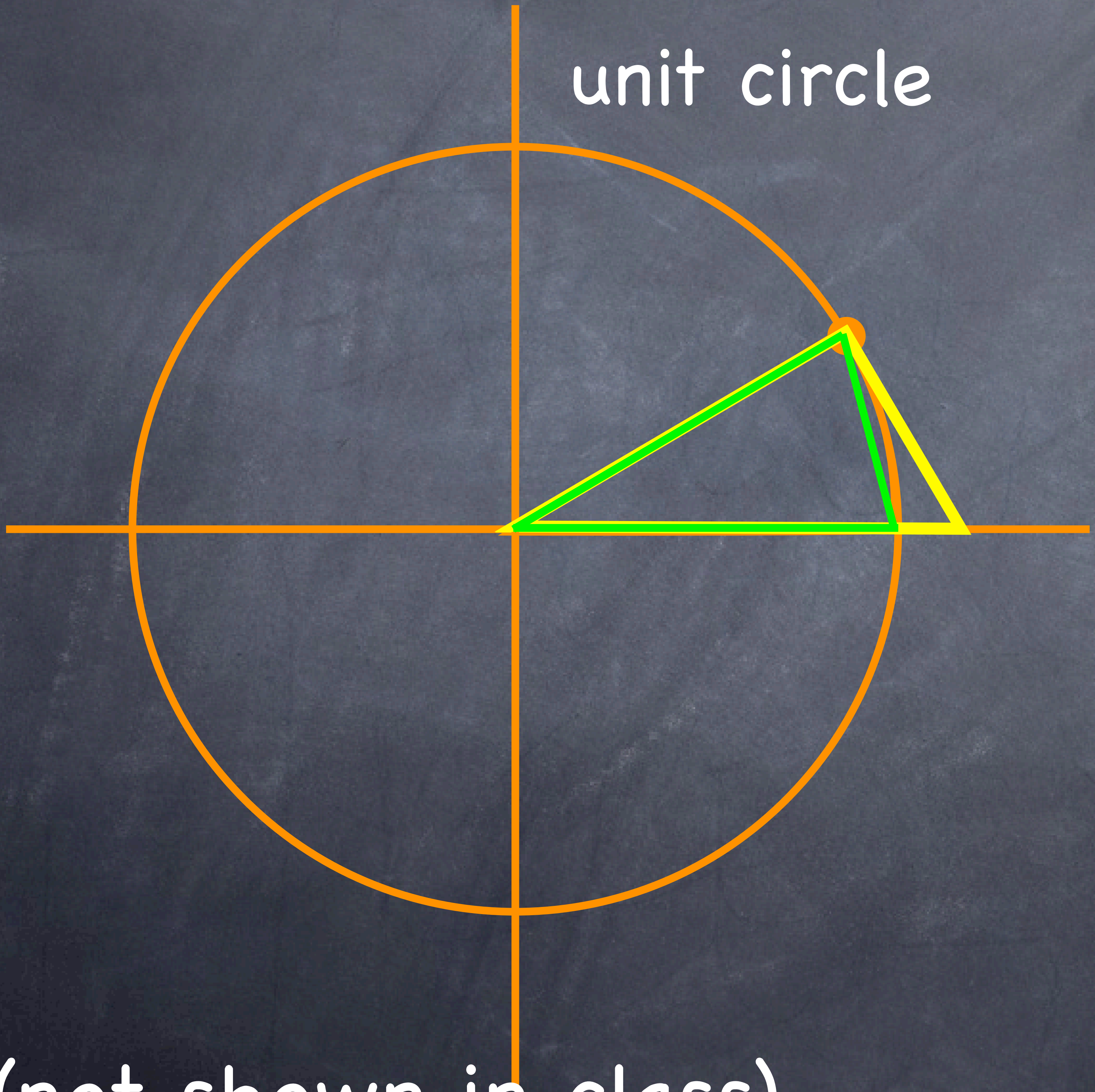
(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



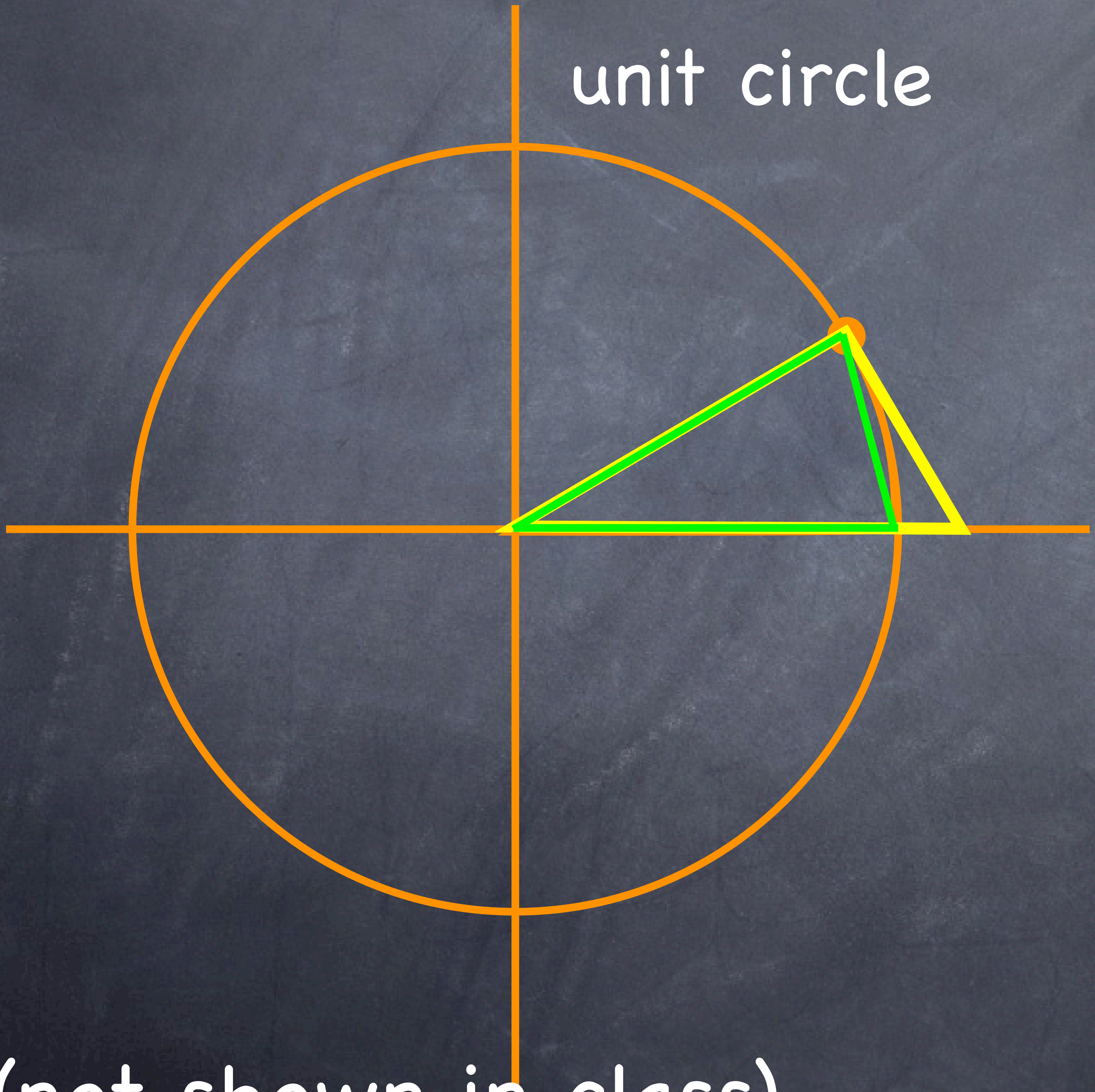
(not shown in class)

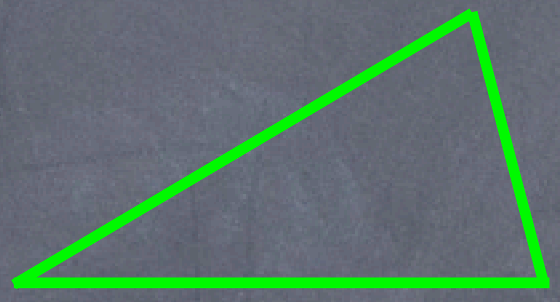
Why is $\lim_{h\to 0} \sin(h)/h = 1$?



(not shown in class)

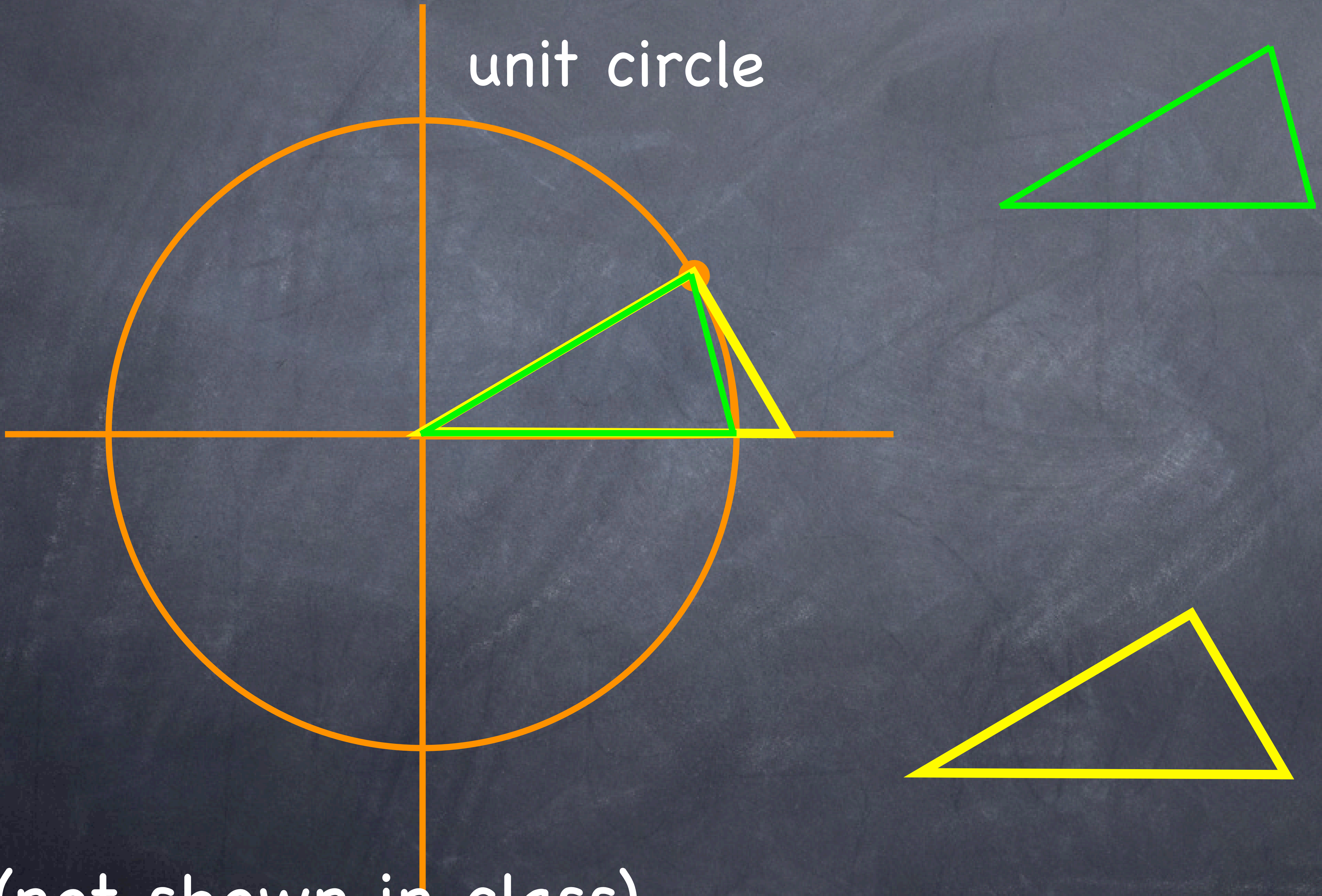
Why is $\lim_{h \to 0} \sin(h)/h = 1$?





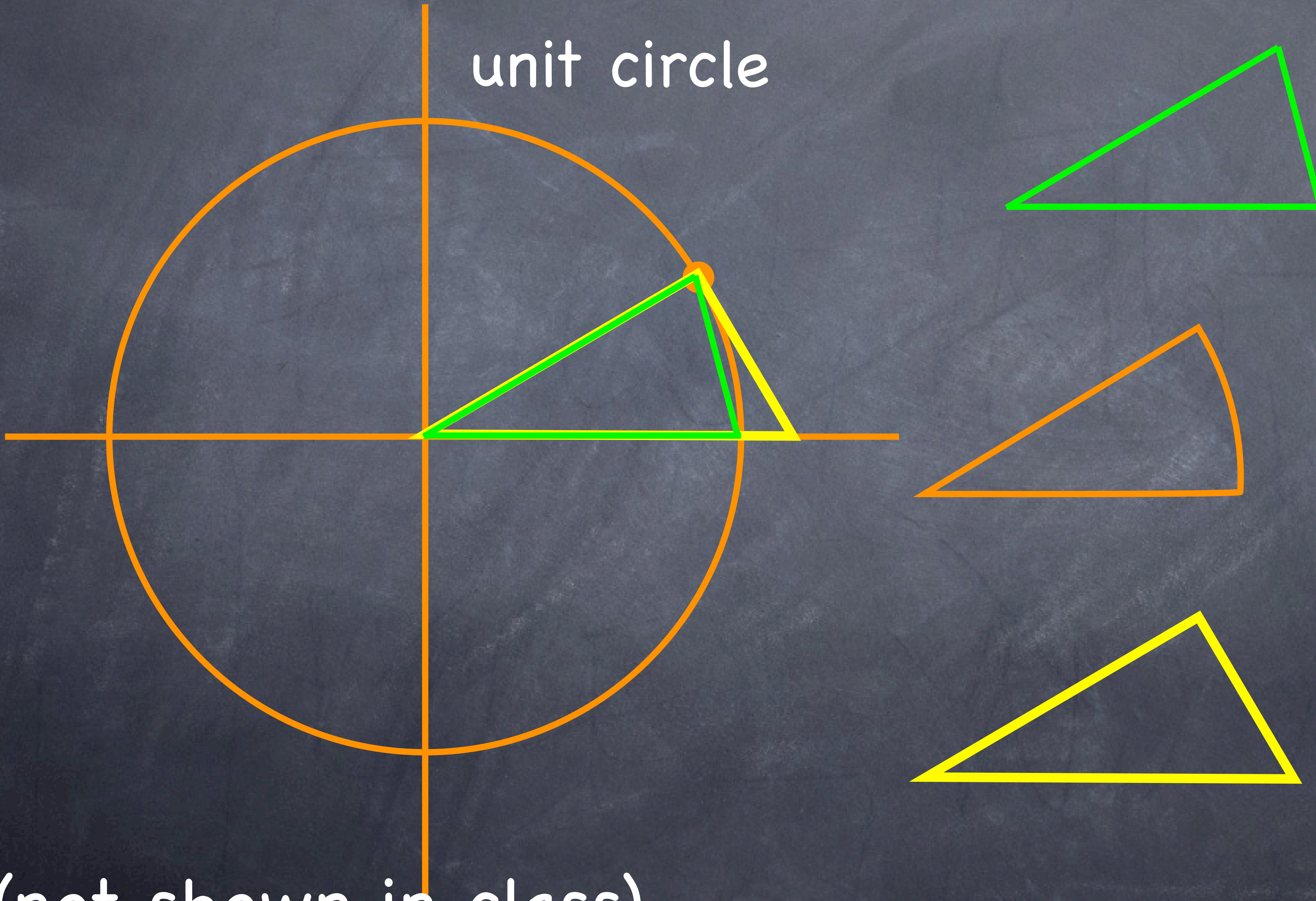
(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



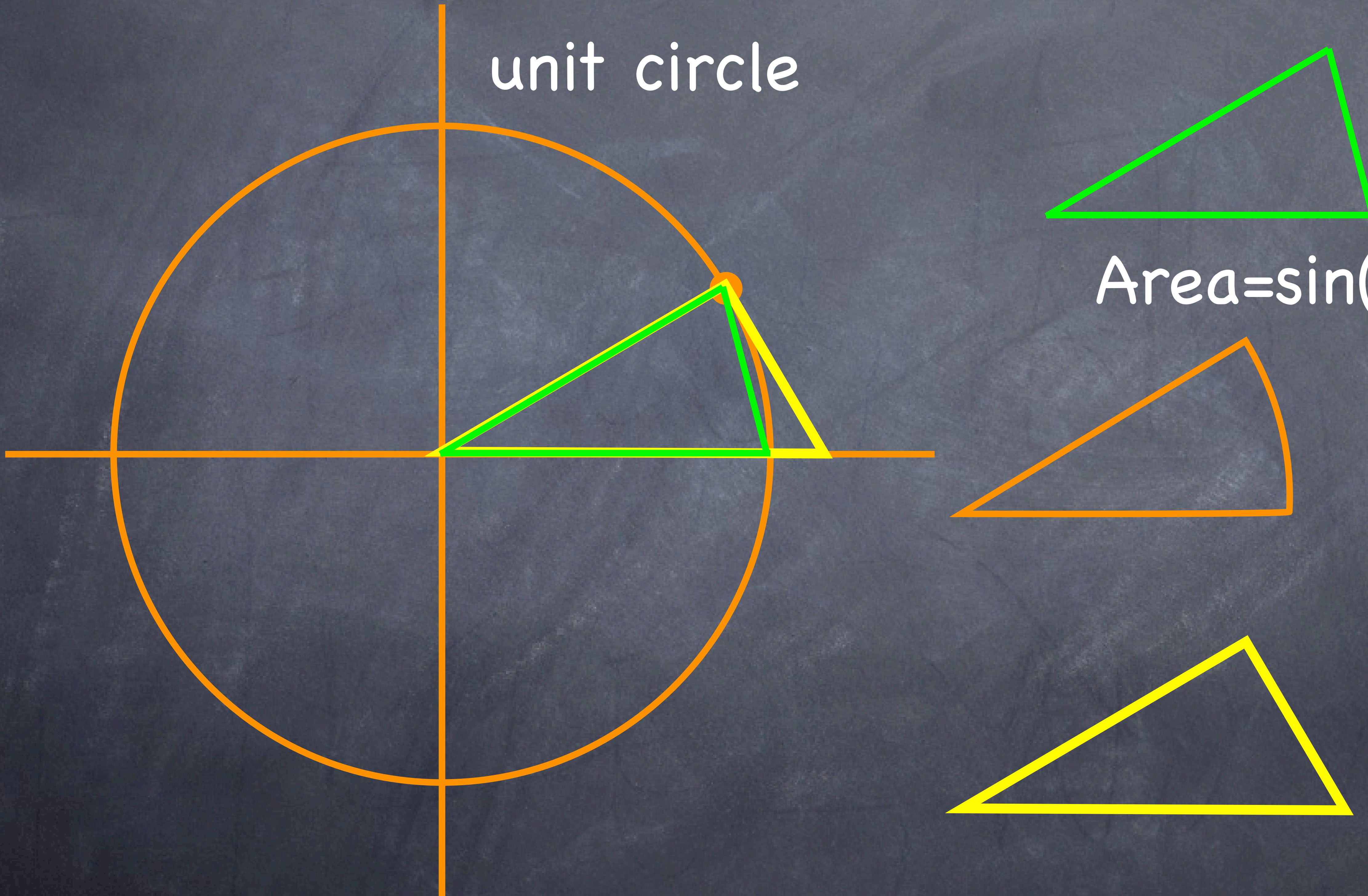
(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



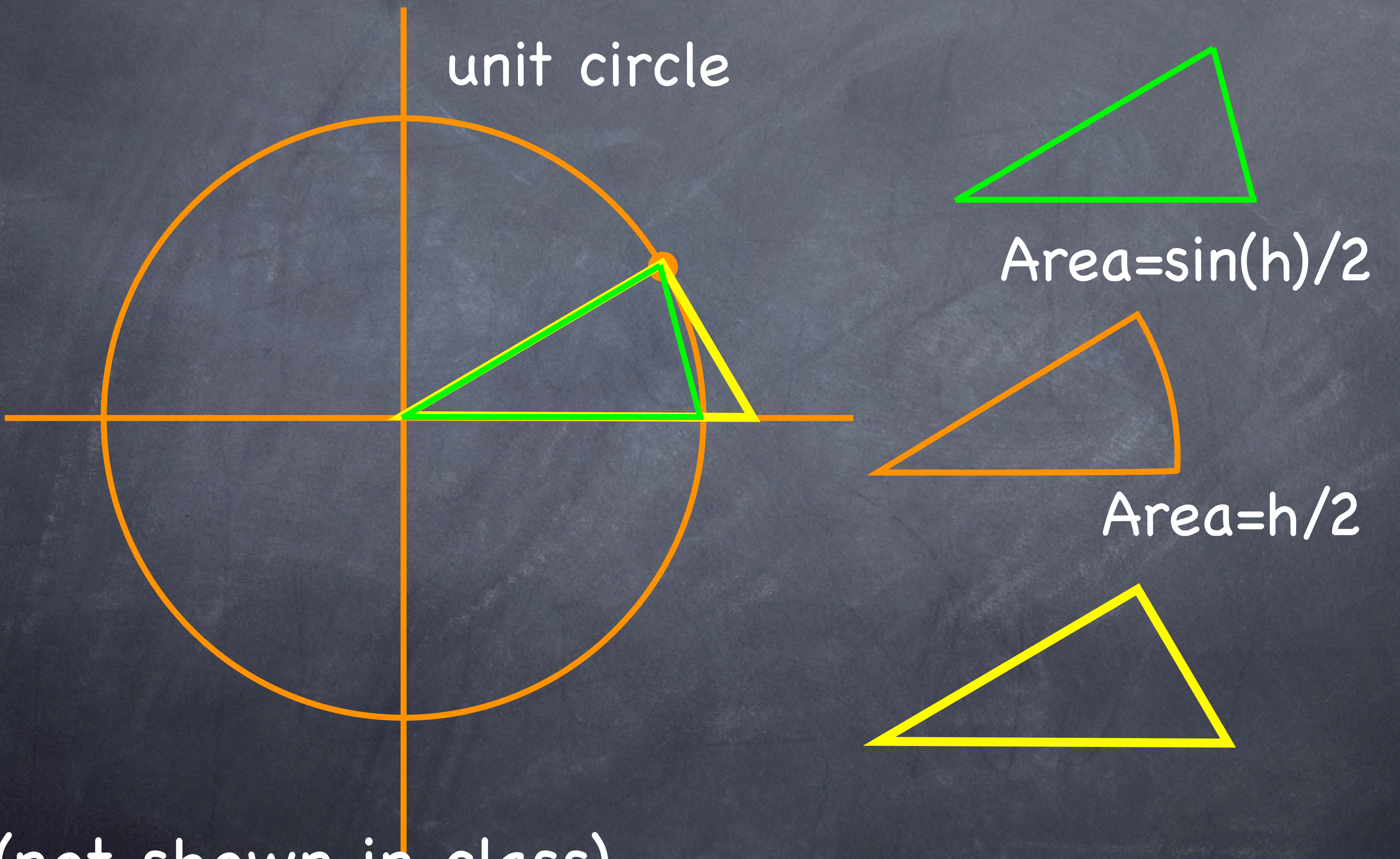
(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



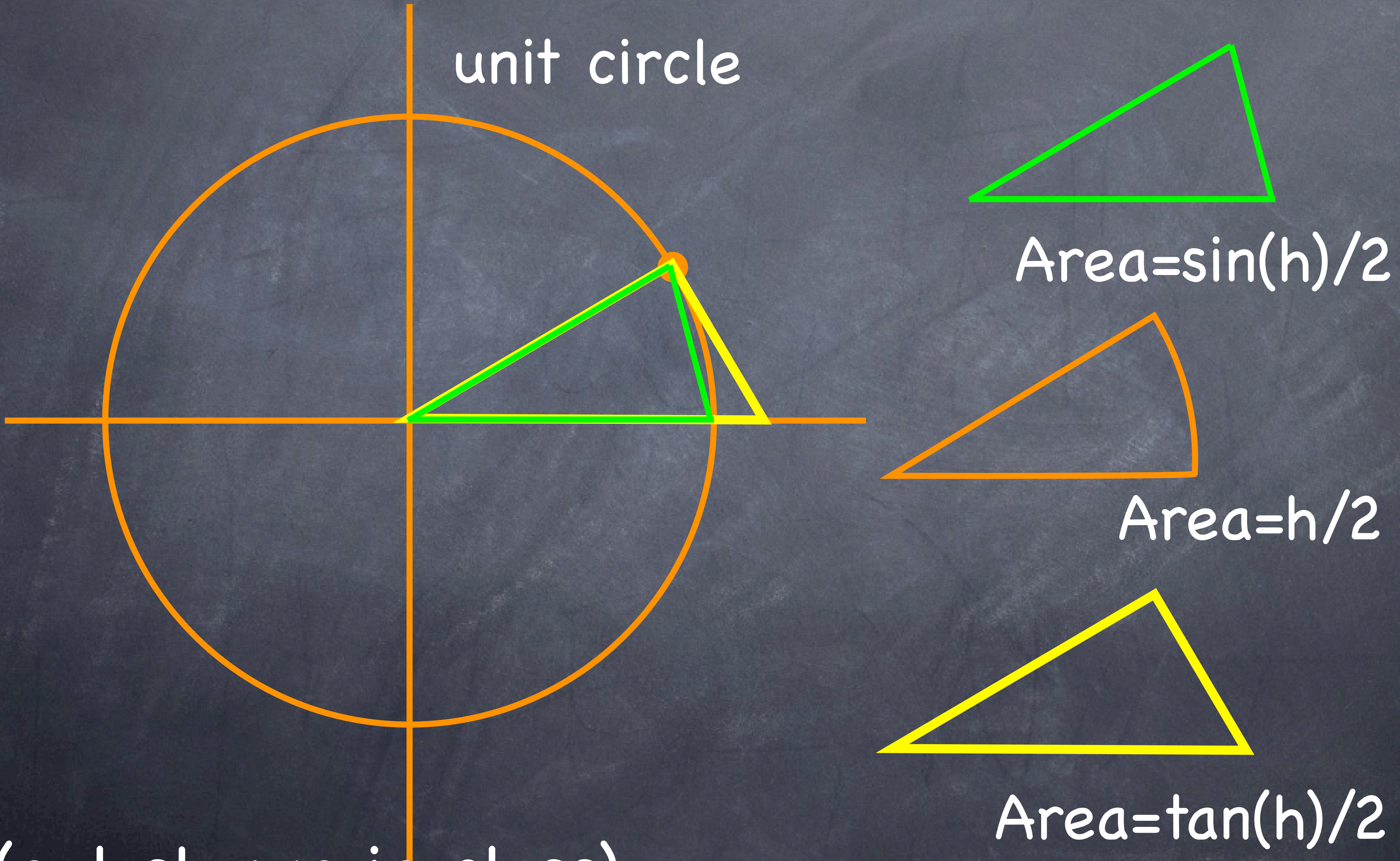
(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$:

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$: $\downarrow$

1

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$: $\cos(h) \to 1$, $1 \to 1$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$: $\cos(h) \to 1$, $1 \to 1$

sin(h)/h is stuck between!

Why is $\lim_{h\to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h\to 0}$: $\downarrow$ $\downarrow$ $\downarrow$

1 1 1

sin(h)/h is stuck between!

Derivative of $g(x)=\cos(x)$. Rewrite $\cos(x)$ as…

(A) $g(x) = \cos(x) = \sin(x-\pi/2)$

(B) $g(x) = \cos(x) = \sin(x+\pi/2)$

(C) $g(x) = \cos(x) = \sin(x+\pi)$

(D) $g(x) = \cos(x) = \sin(x-\pi)$

(E) $g(x) = \cos(x) = \sin(x+3\pi/2)$

Derivative of $g(x)=\cos(x)$. Rewrite $\cos(x)$ as...

(A) $g(x) = \cos(x) = \sin(x-\pi/2)$

(B) $g(x) = \cos(x) = \sin(x+\pi/2)$

(C) $g(x) = \cos(x) = \sin(x+\pi)$

(D) $g(x) = \cos(x) = \sin(x-\pi)$

(E) $g(x) = \cos(x) = \sin(x+3\pi/2)$

Derivative of $g(x)=\sin(x+\pi/2)$

(A) $g'(x) = \cos(x+\pi/2) = \sin(x)$

(B) $g'(x) = \cos(x+\pi/2) = -\sin(x)$

(C) $g'(x) = \cos(x+\pi/2) = \sin(x-\pi/2)$

(D) $g'(x) = \cos(x+\pi/2) = \sin(x+\pi/2)$

(E) $g'(x) = \cos(x+\pi/2) = \sin(x-3\pi/2)$

Derivative of $g(x)=\sin(x+\pi/2)$

(A) $g'(x) = \cos(x+\pi/2) = \sin(x)$

(B) $g'(x) = \cos(x+\pi/2) = -\sin(x)$

(C) $g'(x) = \cos(x+\pi/2) = \sin(x-\pi/2)$

(D) $g'(x) = \cos(x+\pi/2) = \sin(x+\pi/2)$

(E) $g'(x) = \cos(x+\pi/2) = \sin(x-3\pi/2)$

Other trig functions

The derivative of cot(x) is

(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $\csc^2(x)$

(D) $-\csc^2(x)$

(E) $\sec^2(x)$

Other trig functions

The derivative of cot(x) is

(A) csc(x)cot(x)

(B) -csc(x)cot(x)

(C) $\csc^2(x)$

(D) $-\csc^2(x)$

(E) $\sec^2(x)$

Rewrite cot(x) = cos(x)/sin(x) and use quotient rule.