Today

- Derivative of ln(x) aka log(x).
- Derivative of a^x.
- Log-log and semi-log plots.
- Exponential derivative (what is C_a?)
- Converting between a^x and e^{kx}.

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} =$

(A) 1

(B) x

(C) 1/x

(D) e

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- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

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(B) $e^y y' = 1$

(C) $e^y = x'$

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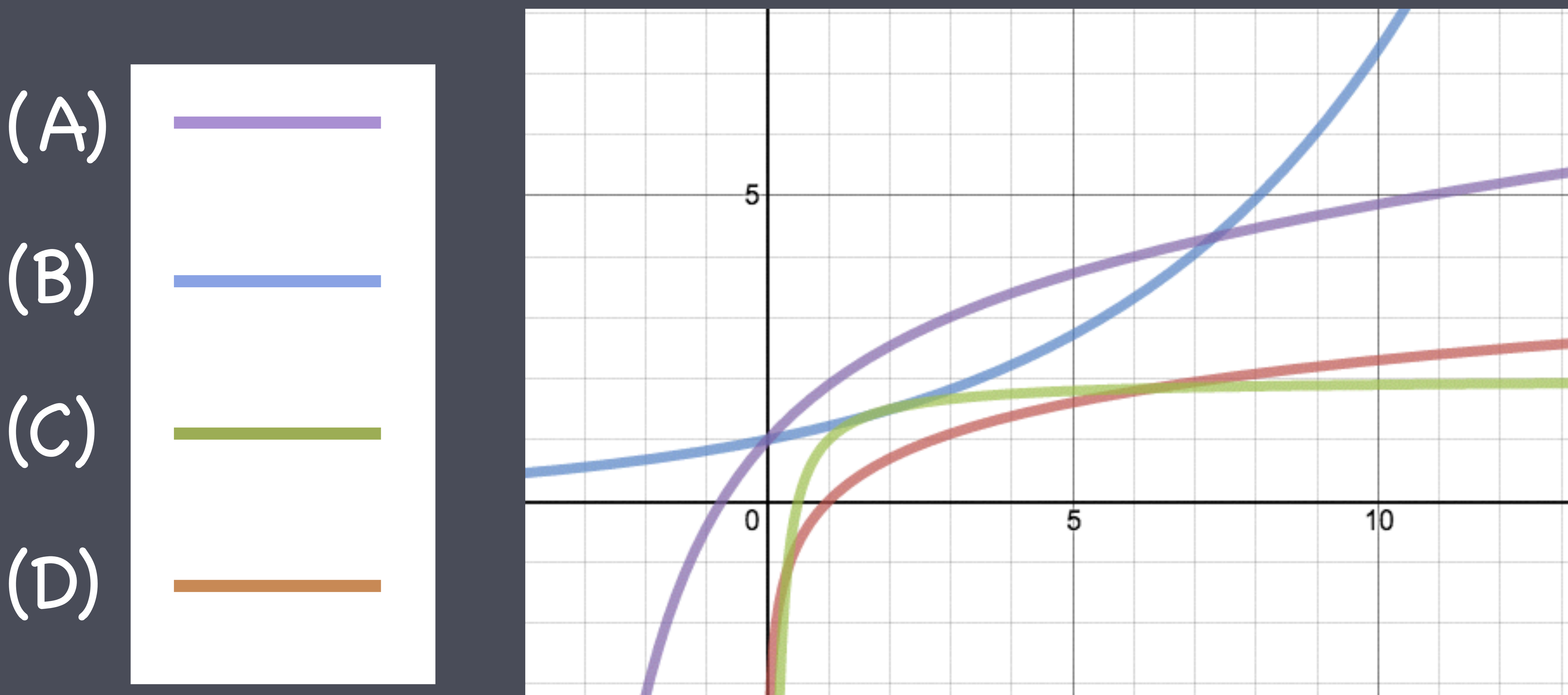
- Solve for y': $y' = e^{-y} = 1/x$

$g(x) = \ln(x)$

--> $g'(x) = 1/x$

Which of following is the graph of ln(x)?



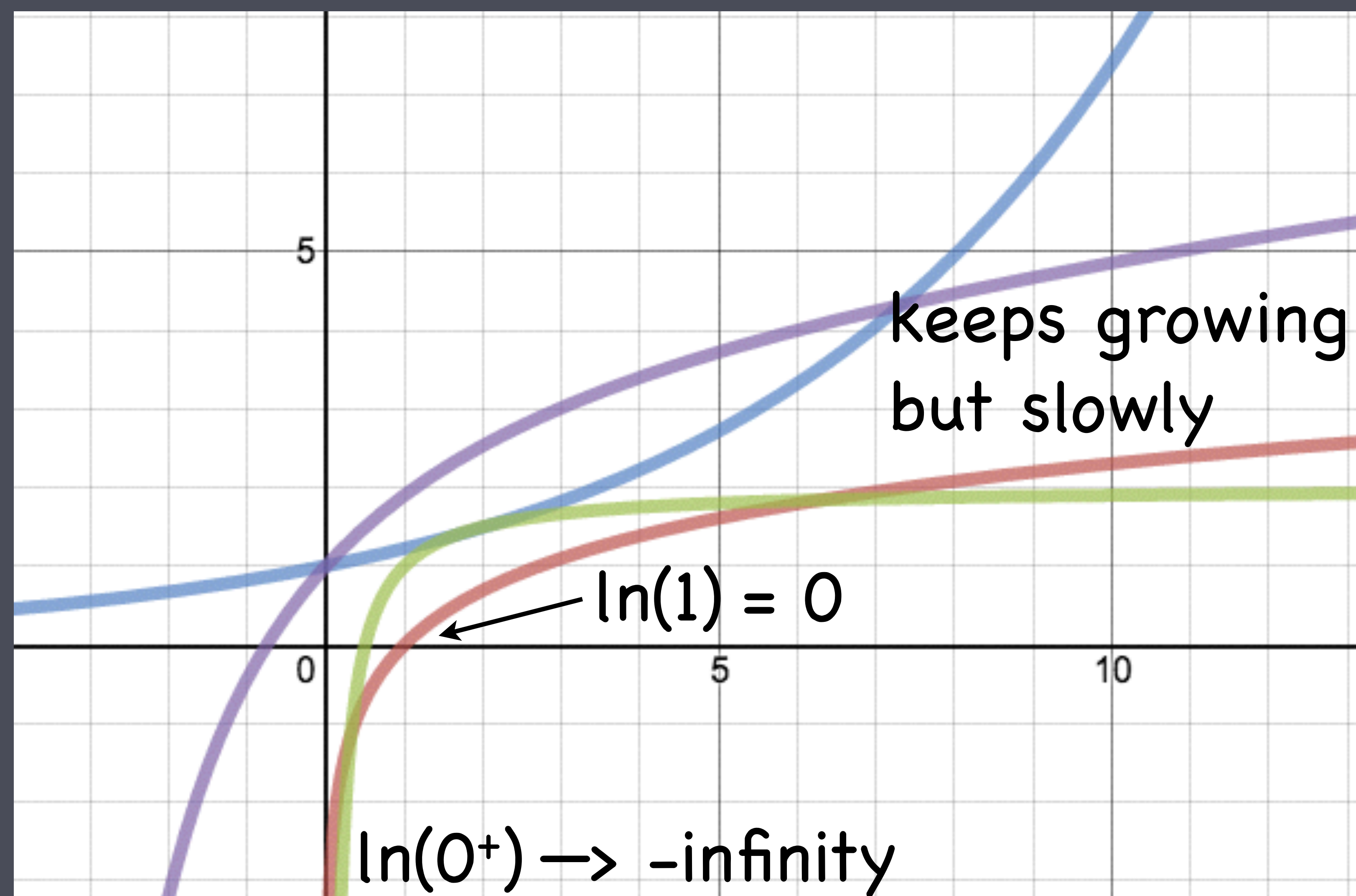
Which of following is the graph of ln(x)?

(A)

(B)

(C)

(D)



Log-log and semi-log plots

- A log-log plot is a plot on which you plot log(y) versus log(x) instead of y versus x.
- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

<---this is what OSH 5 asks you to use

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.
- $V = A + kx$ where $A=\ln(a)$.
- On a semi-log plot, $y= ae^{kx}$ looks linear.

$f(x)=a^x$. $f'(x)=C_a a^x$. $C_a=??$

- Recall that we got stuck on this derivative.
- Time to get unstuck...

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}.$

(B) $f'(x) = \ln(2)e^{\ln(2)x}.$

(C) $f'(x) = \ln(2)\cdot 1/2\cdot e^{\ln(2)x}.$

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}.$

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(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

$$f(x) = e^{\ln(2)x}.$$

(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

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(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

From the last two clicker Qs...

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.
- In general, $f(x) = a^x$ --> $f'(x) = a^x \ln(a)$.

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

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$$a^x = (e^k)^x$$
$$a = e^k$$
$$\ln(a) = \ln(e^k)$$
$$\ln(a) = k \ln(e)$$
$$\ln(a) = k$$

$f(x) = a^x = e^{\ln(a)x}$

--> $f'(x) = a^x \ln(a)$.

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

Rough estimate (no calcs, just intuition):

(A) ~1 week.

(B) ~2 weeks.

(C) ~1 month.

(D) ~1 year.

(E) ~10^4 days ≈ 27 years.

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A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $p(t) = e^{t/24}$.

(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$.

(D) $p(t) = 2^{-t/24}$.

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(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$. $= 2^t$ <-- t measured in days.

(D) $p(t) = 2^{-t/24}$.

(E) $p(t) = 2^{t/24}$. <-- t measured in hours.