

Today

- Brief midterm “discussion”
- Euler’s method

I thought the midterm
was...

- (A) ...easier than I expected.
- (B) ...pretty much what I expected.
- (C) ...harder than I expected.

Midterm 2...

Midterm 2...

- On the graphing question, did you

Midterm 2...

- On the graphing question, did you
- calculate the zeros, crit pts, potential IPS? (A) T, (B) F

Midterm 2...

- On the graphing question, did you
- calculate the zeros, crit pts, potential IPS? (A) T, (B) F
- figure out which crit pts were min/max? (A) T, (B) F

Midterm 2...

- On the graphing question, did you
- calculate the zeros, crit pts, potential IPS? (A) T, (B) F
- figure out which crit pts were min/max? (A) T, (B) F
- check that the sign of f'' changed sign at each pIP?
(A) T, (B) F

Midterm 2...

- On the graphing question, did you
- calculate the zeros, crit pts, potential IPS? (A) T, (B) F
- figure out which crit pts were min/max? (A) T, (B) F
- check that the sign of f'' changed sign at each pIP?
(A) T, (B) F
- That was all worth 11 pts. 3 pts for the actual graph.

The most useful thing I did to study was...

- (A) ...doing/reviewing WeBWork assignments.
- (B) ...doing/reviewing OSH.
- (C) ...doing practice problems from the course notes.
- (D) ...reading/annotating/taking notes on the course notes.
- (E) ...reviewing the lecture slides.

The second most useful thing I did to study was...

(A) ...doing/reviewing WeBWork assignments.

(B) ...doing/reviewing OSH.

(C) ...doing practice problems from the course notes.

(D) ...reading/annotating/taking notes on the course notes.

(E) ...reviewing the lecture slides.

The second most useful thing I did to study was...

(A) ...doing/reviewing WeBWork assignments.

(B) ...doing/reviewing OSH.

(C) ...doing practice problems from the course notes.

(D) ...reading/annotating/taking notes on the course notes.

(E) ...reviewing the lecture slides.

Other things?

Third-party prep session

(A) I attended a third-party prep session.

(B) I did not attend a third-party prep session.

Third-party prep session

- (A) I would recommend it to other students for the time spent attending it and the practice problems distributed.
- (B) I would recommend it to other students only for the time spent attending it.
- (C) I would recommend it to other students only for the practice problems distributed.
- (D) I would not recommend it.

Euler's method

Euler's method

- A "numerical" method for IVPs.

Euler's method

- A "numerical" method for IVPs.
- "Numerical" means finding a sequence of numbers that approximate $y(t)$ at specific t values.

Euler's method

- A "numerical" method for IVPs.
- "Numerical" means finding a sequence of numbers that approximate $y(t)$ at specific t values.
- e.g. Instead of the actual solution to NLC

Euler's method

- A “numerical” method for IVPs.
- “Numerical” means finding a sequence of numbers that approximate $y(t)$ at specific t values.
- e.g. Instead of the actual solution to NLC

$$T(t) = E + (T_0 - E)e^{-kt},$$

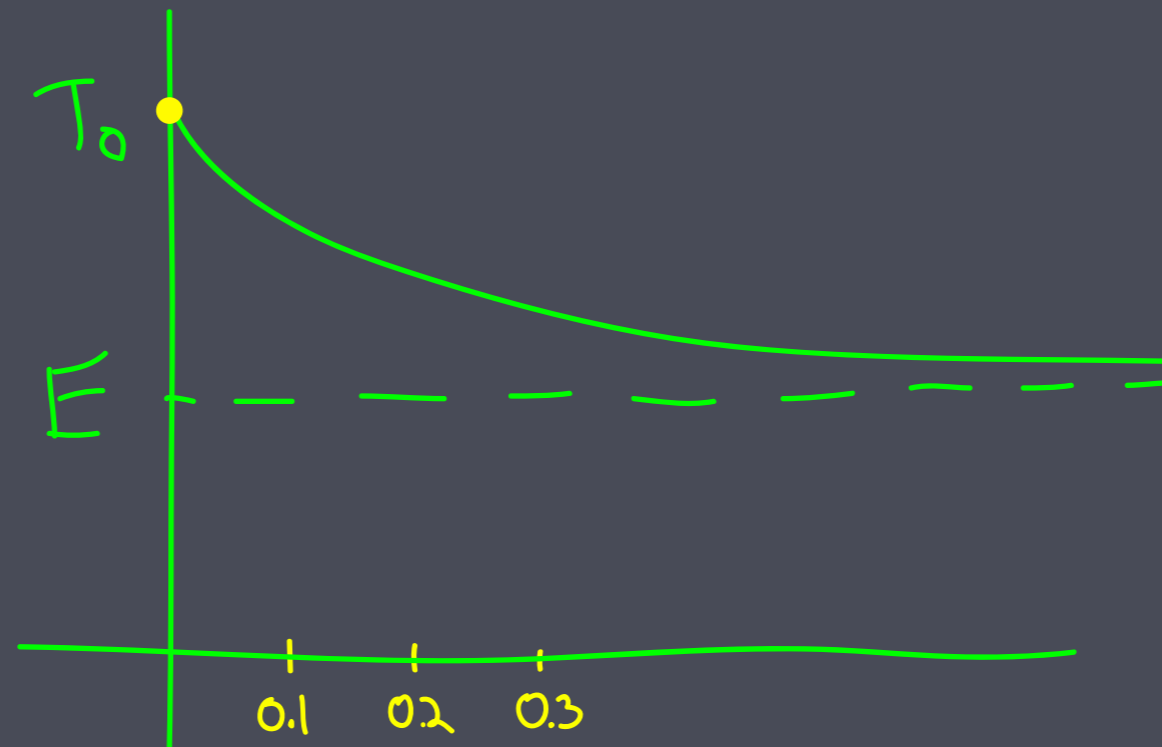
Euler's method

- A “numerical” method for IVPs.
- “Numerical” means finding a sequence of numbers that approximate $y(t)$ at specific t values.
- e.g. Instead of the actual solution to NLC

$$T(t) = E + (T_0 - E)e^{-kt},$$

we find T_1, T_2, T_3, \dots which approximate $T(0.1), T(0.2), T(0.3) \dots$

Euler's method for
 $T'(t) = 0.02(14 - T(t))$ with $T(0) = 37$.



What is the slope of the solution at $t=0$?

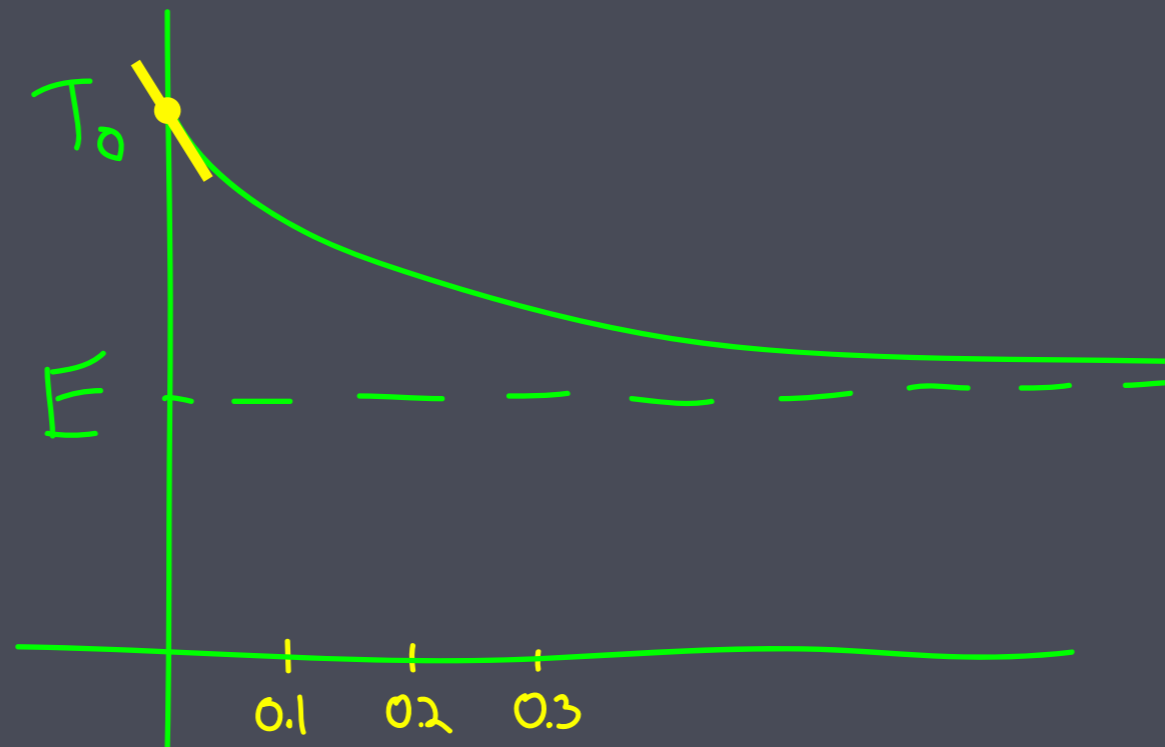
(A) -0.02

(B) -0.46

(C) 0.02

(D) -0.28

Euler's method for
 $T'(t) = 0.02(14 - T(t))$ with $T(0) = 37$.



What is the slope of the solution at $t=0$?

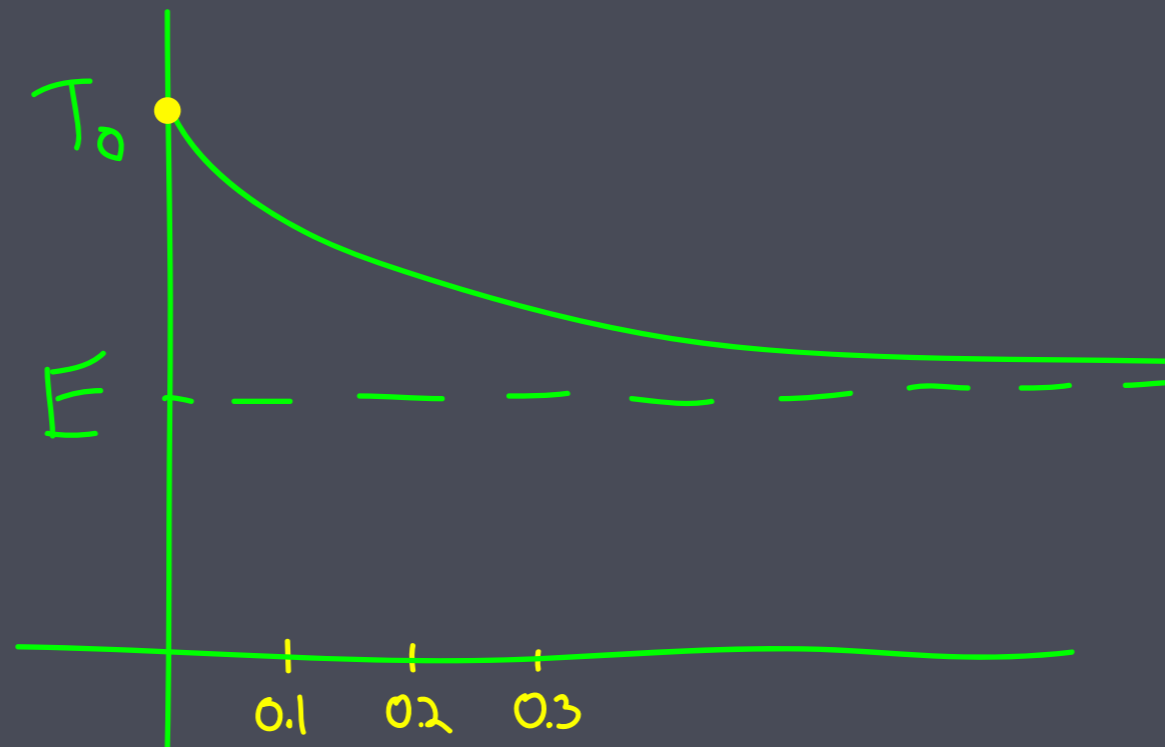
(A) -0.02

(C) 0.02

(B) -0.46

(D) -0.28

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



What is the slope of the solution at $t=0$?

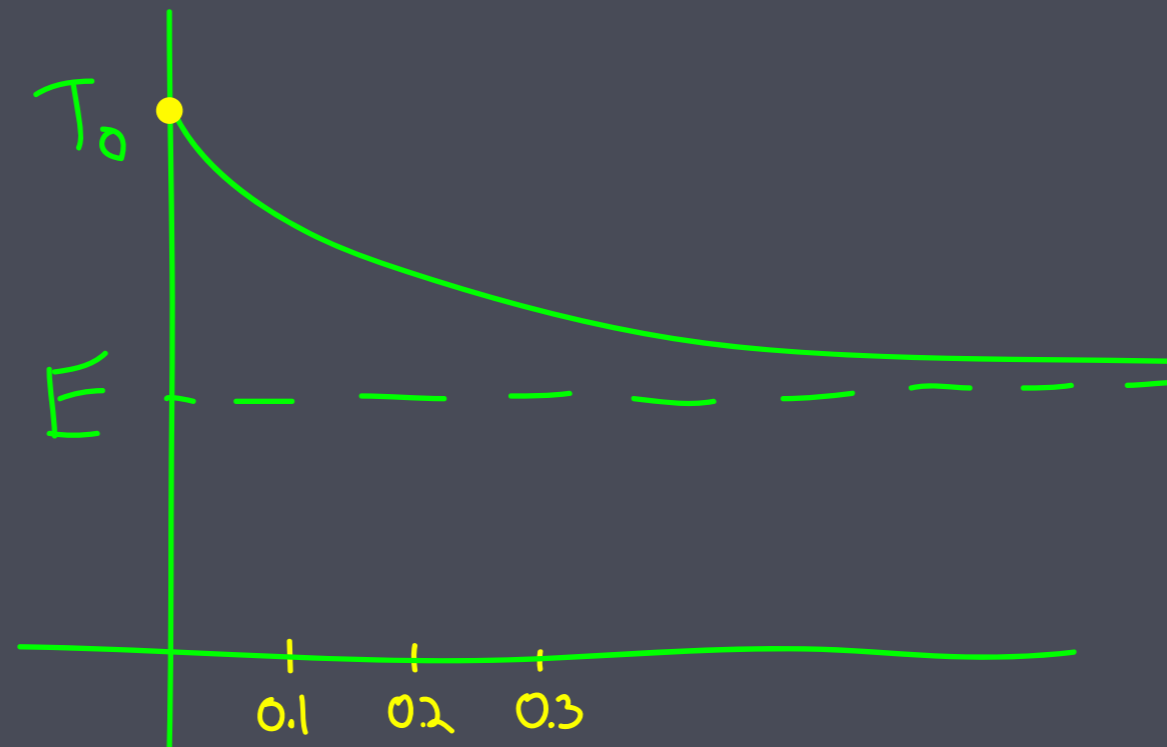
(A) $k(E - T)$

(B) $k(E - T_0)$

(C) $T'(0)$

(D) kE

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



What is the slope of the solution at $t=0$?

(A) $k(E - T)$

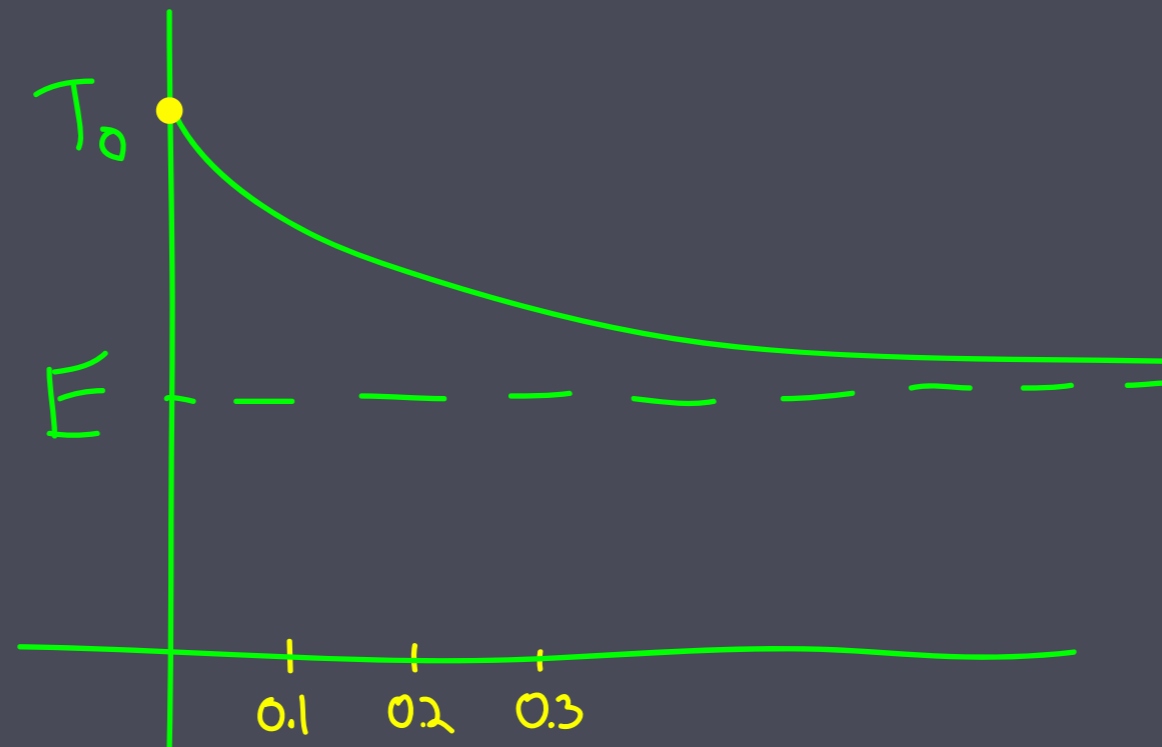
(B) $k(E - T_0)$

(C) $T'(0)$

(D) kE

← Yes, but can do better

Euler's method for
 $T'(t) = 0.02(14 - T(t))$ with $T(0) = 37$.



What is the equation of the tangent line to $T(t)$ at $t=0$?

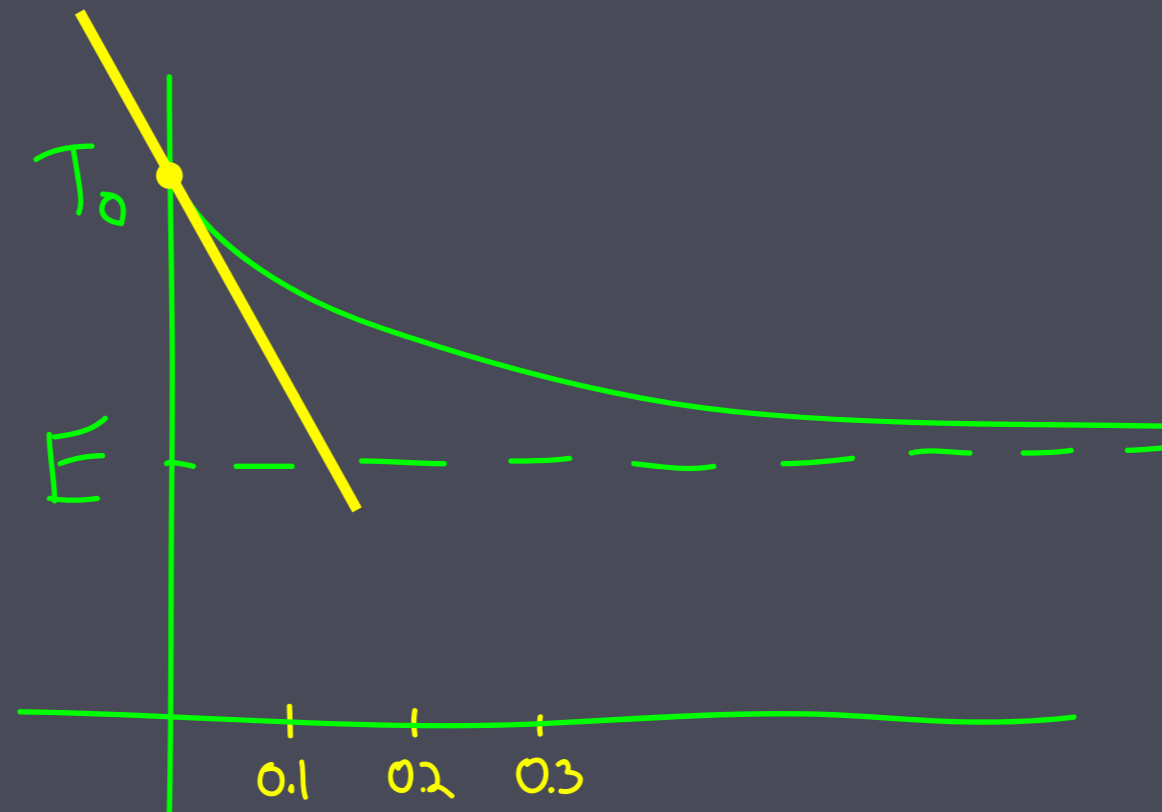
(A) $y = 37 - 0.46t$

(C) $y = 37 - 0.28t$

(B) $y = 14 + 23e^{-0.02t}$

(D) $y = 37 + 0.46t$

Euler's method for
 $T'(t) = 0.02(14 - T(t))$ with $T(0) = 37$.



What is the equation of the tangent line to $T(t)$ at $t=0$?

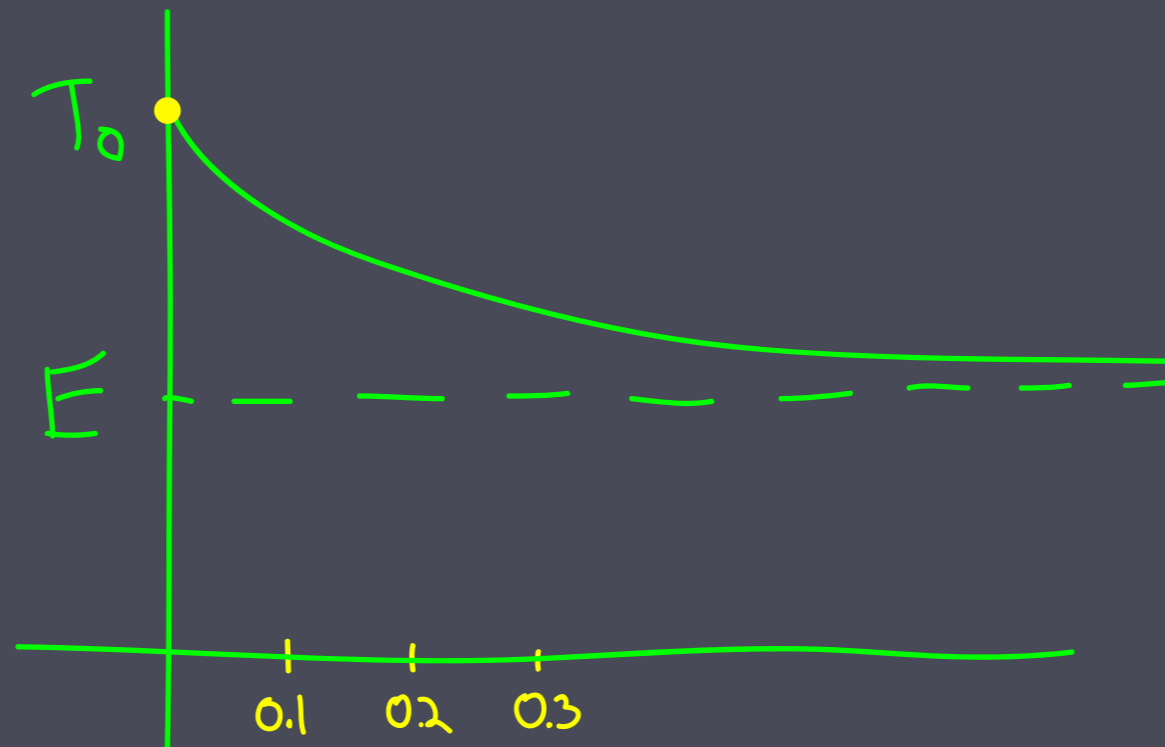
(A) $y = 37 - 0.46t$

(C) $y = 37 - 0.28t$

(B) $y = 14 + 23e^{-0.02t}$

(D) $y = 37 + 0.46t$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



What is the equation of the tangent line to $T(t)$ at $t=0$?

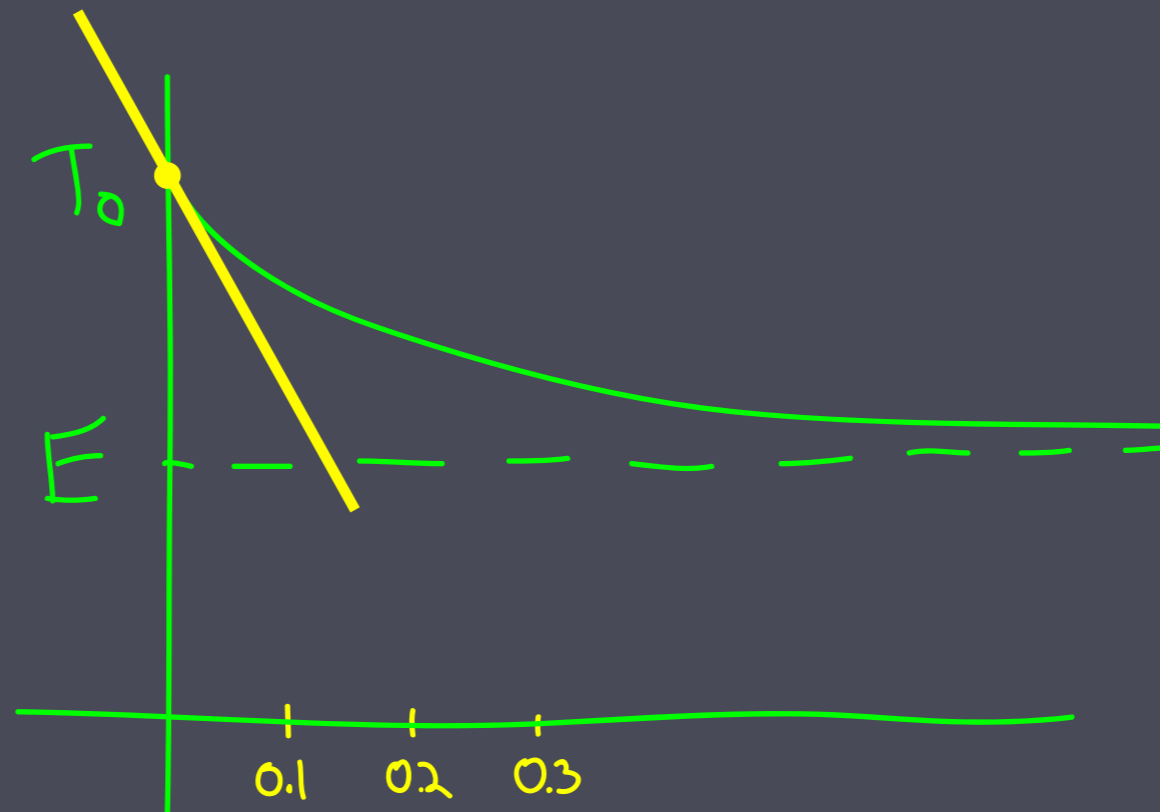
(A) $y = T(0) + T'(0)(t-0)$

(C) $y = T_0 + T_0' t$

(B) $y = E + (T_0 - E)e^{-kt}$

(D) $y = T_0 + k(E - T_0)t$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



What is the equation of the tangent line to $T(t)$ at $t=0$?

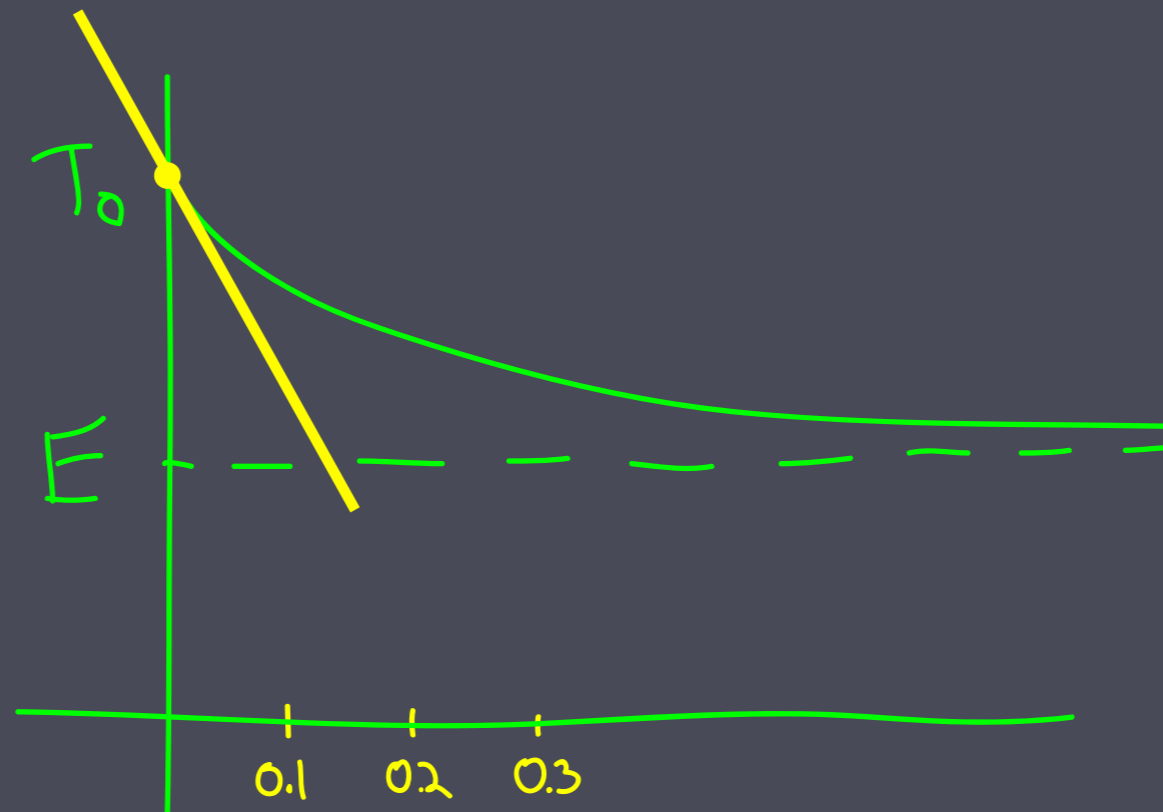
(A) $y = T(0) + T'(0)(t-0)$

(C) $y = T_0 + T_0' t$

(B) $y = E + (T_0 - E)e^{-kt}$

(D) $y = T_0 + k(E - T_0)t$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



Use the tangent line at $t=0$ to estimate $T(0.1)$:

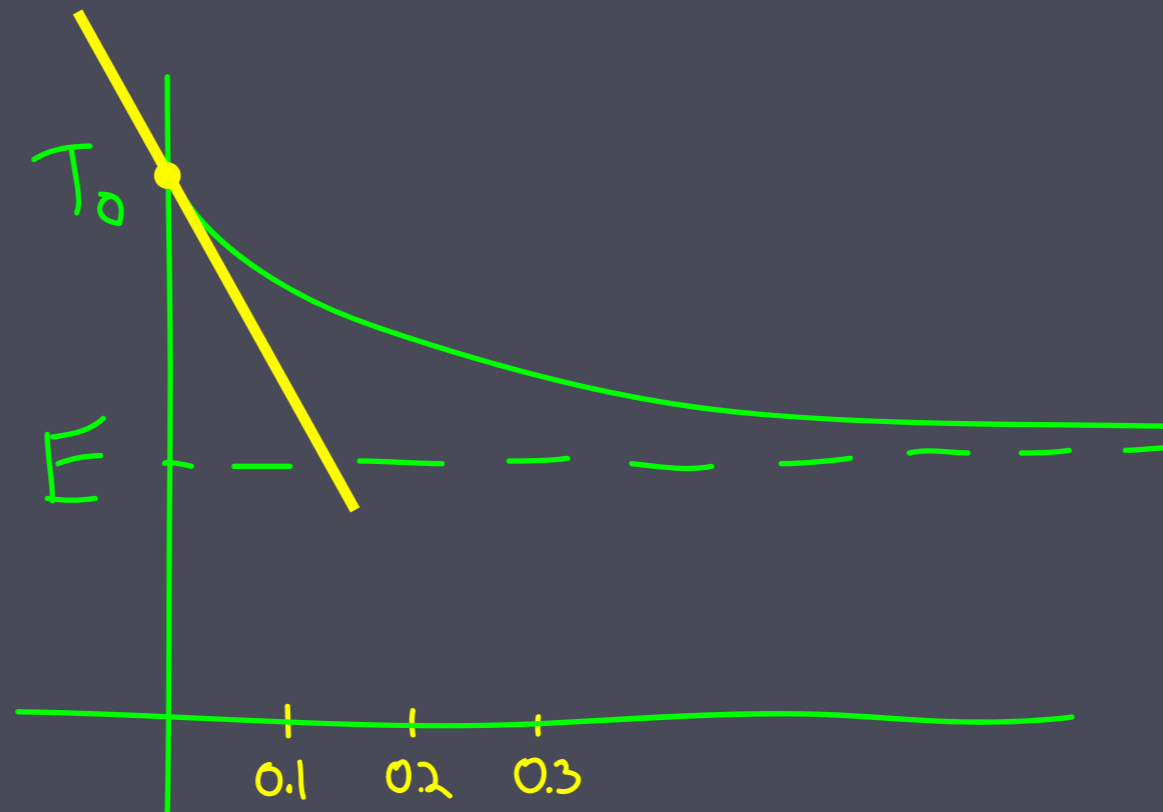
(A) $T_1 = T(0) + 0.1 T'(0)$

(B) $T_1 = T_0 + 0.1 k(E - T_0)$

(C) $T_1 = E + (T_0 - E)e^{-0.1k}$

(D) $T_1 = T_0 + 0.1 k$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



Use the tangent line at $t=0$ to estimate $T(0.1)$:

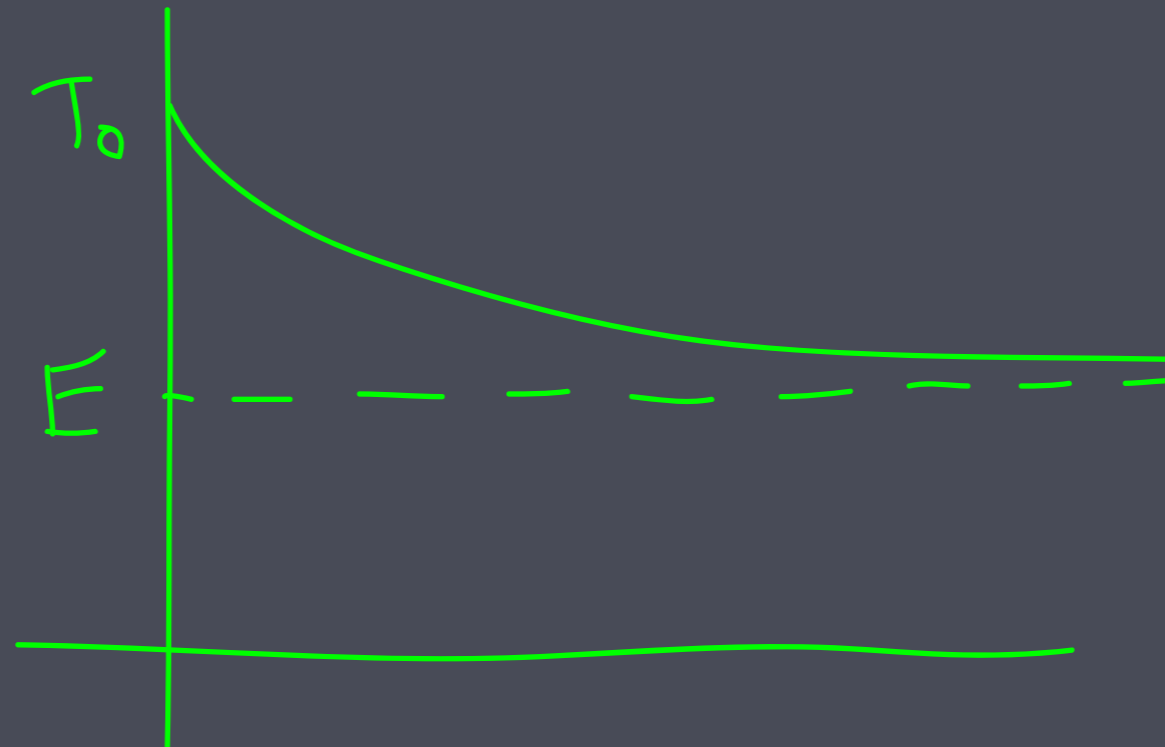
(A) $T_1 = T(0) + 0.1 T'(0)$

(C) $T_1 = E + (T_0 - E)e^{-0.1k}$

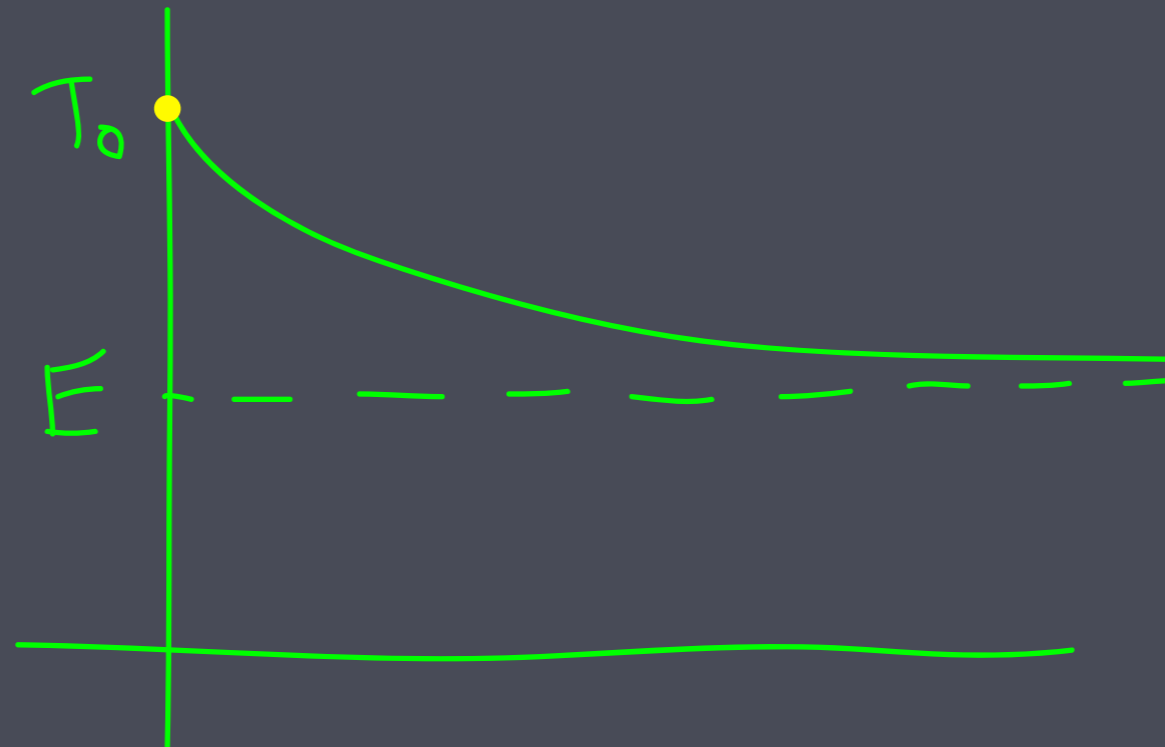
(B) $T_1 = T_0 + 0.1 k(E - T_0)$

(D) $T_1 = T_0 + 0.1 k$

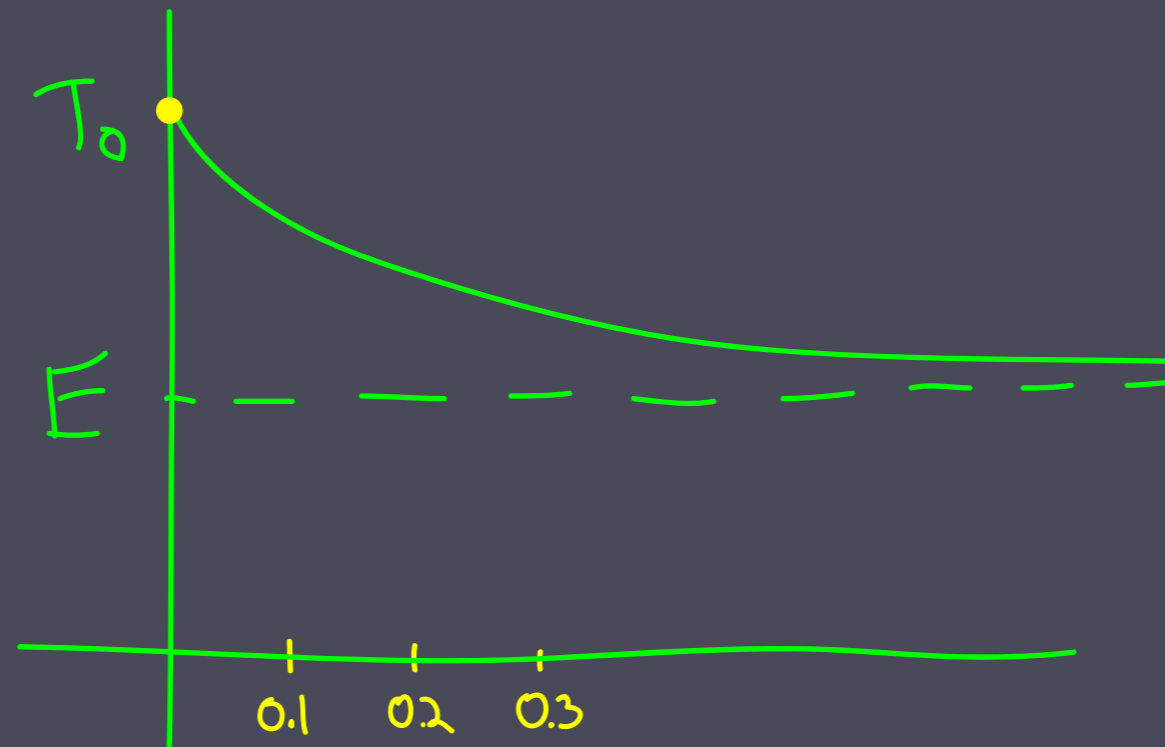
Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



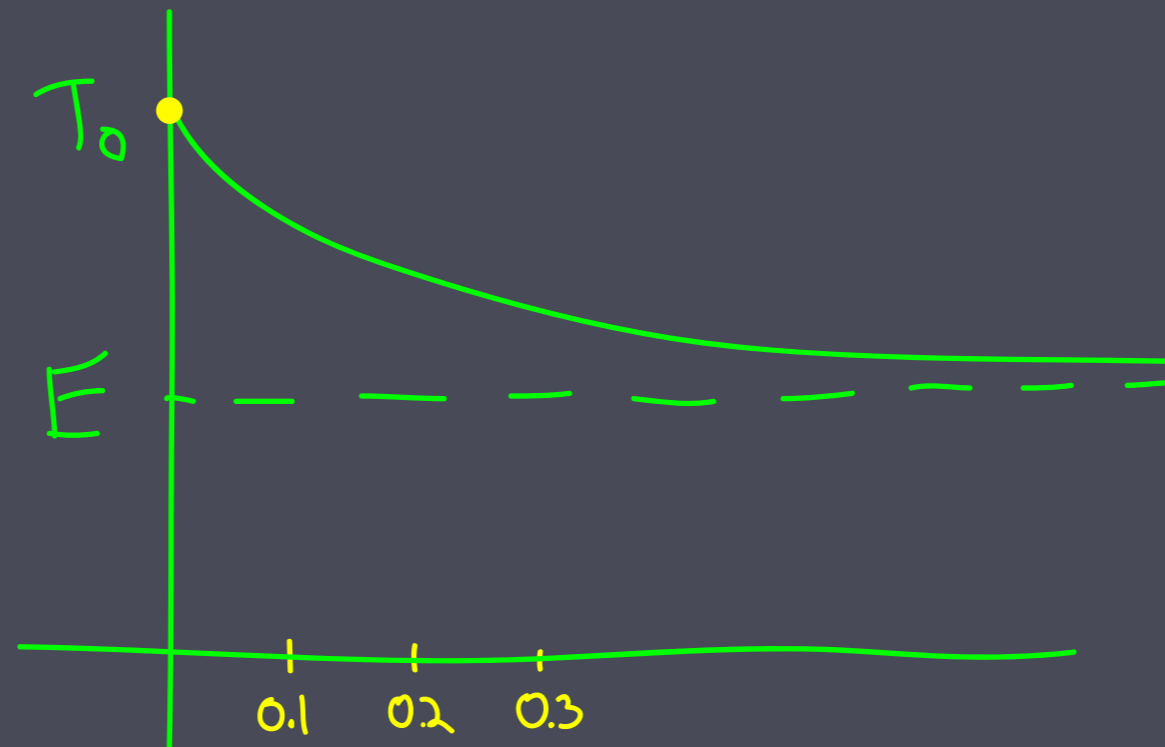
Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

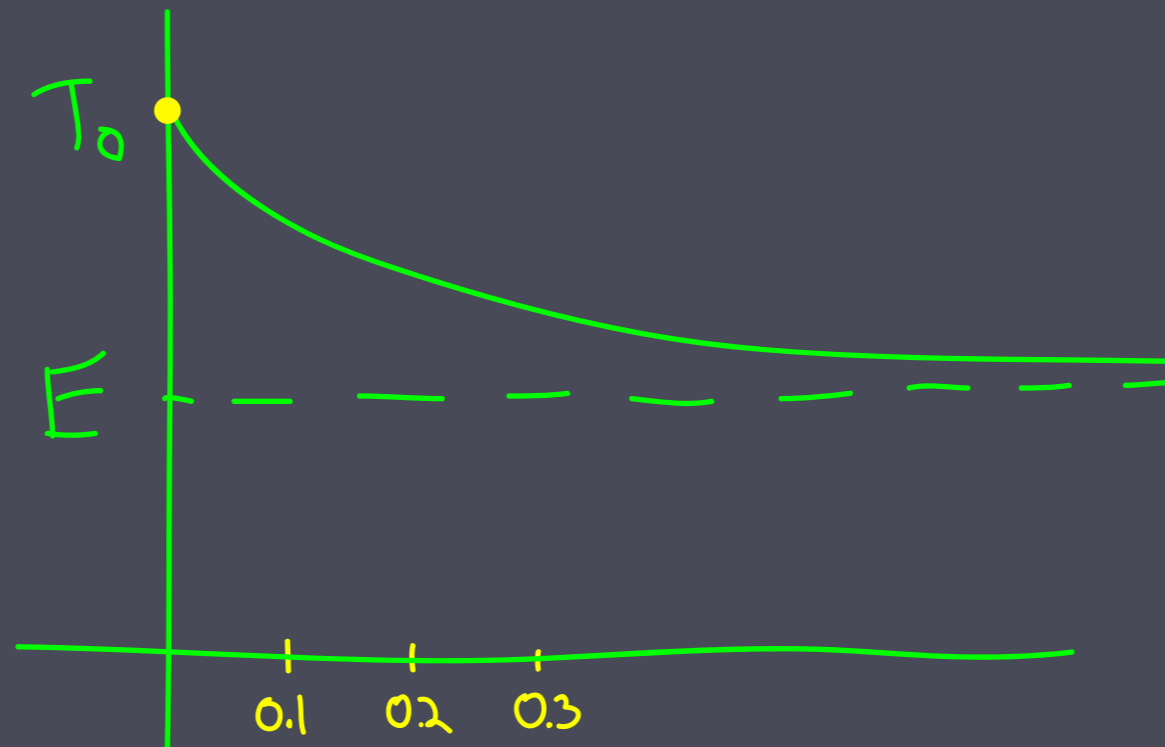


Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



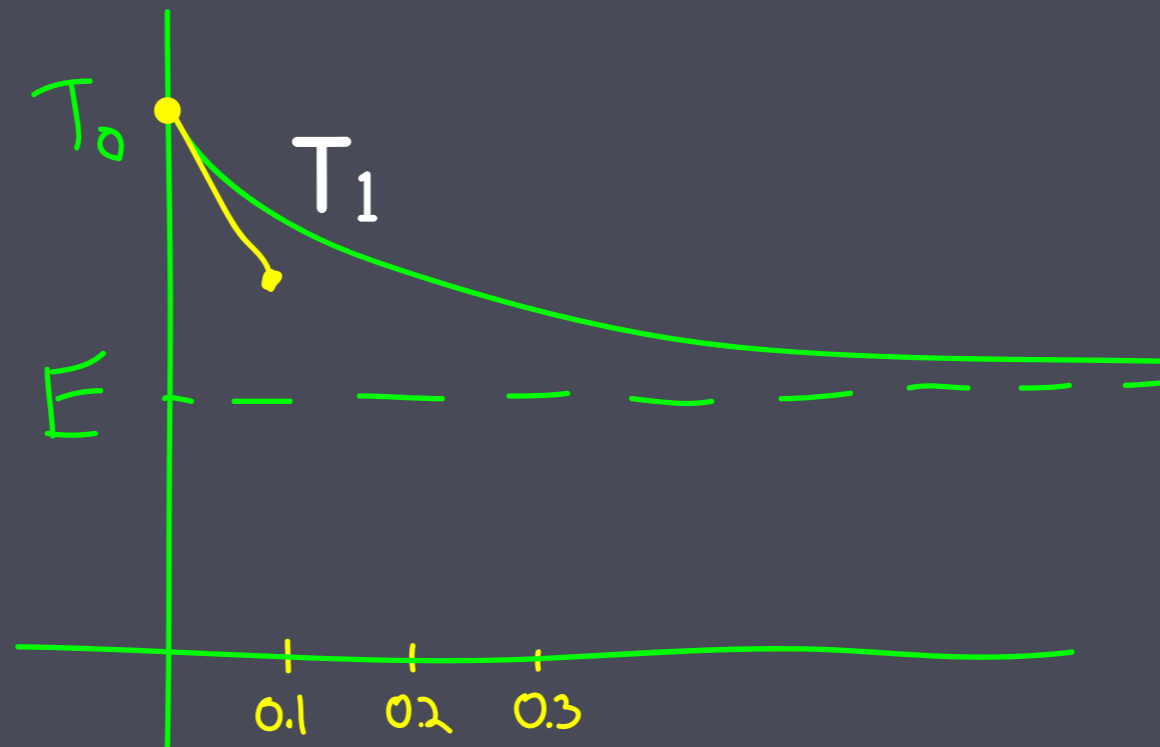
$$T(0.1) \approx T_0 + 0.1 k (E - T_0)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



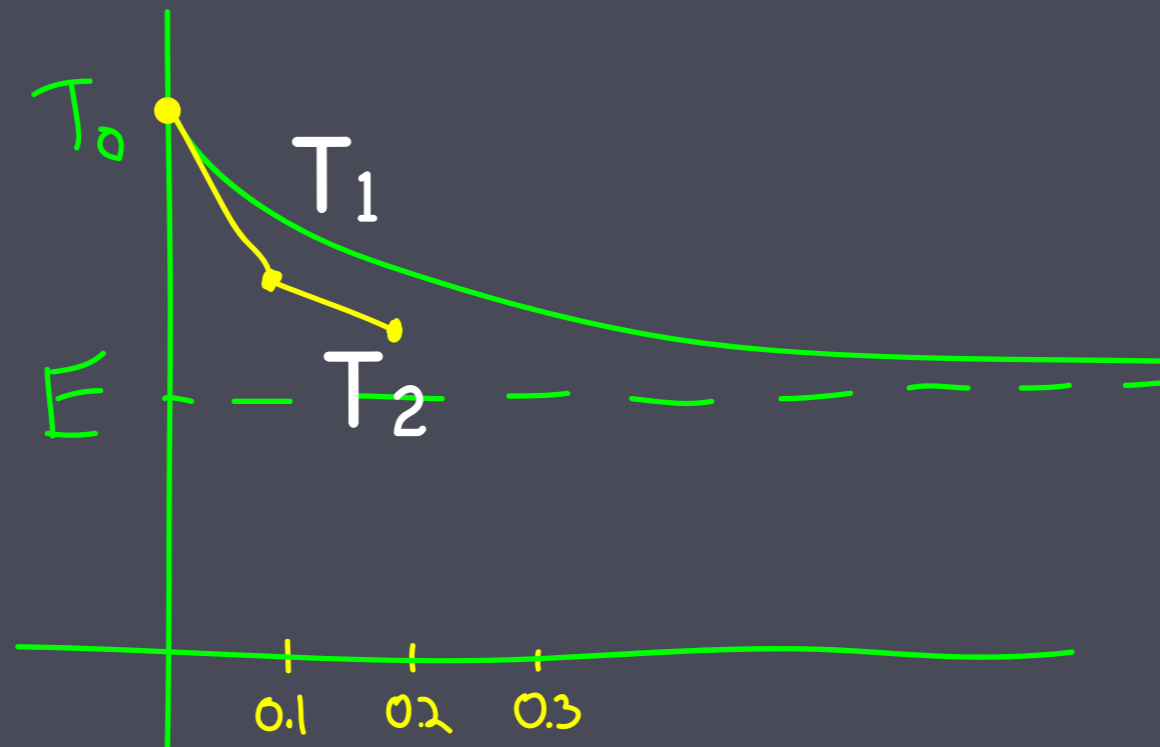
$$T(0.1) \approx T_0 + 0.1 k (E - T_0) = T_1$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



$$T(0.1) \approx T_0 + 0.1 k (E - T_0) = T_1$$

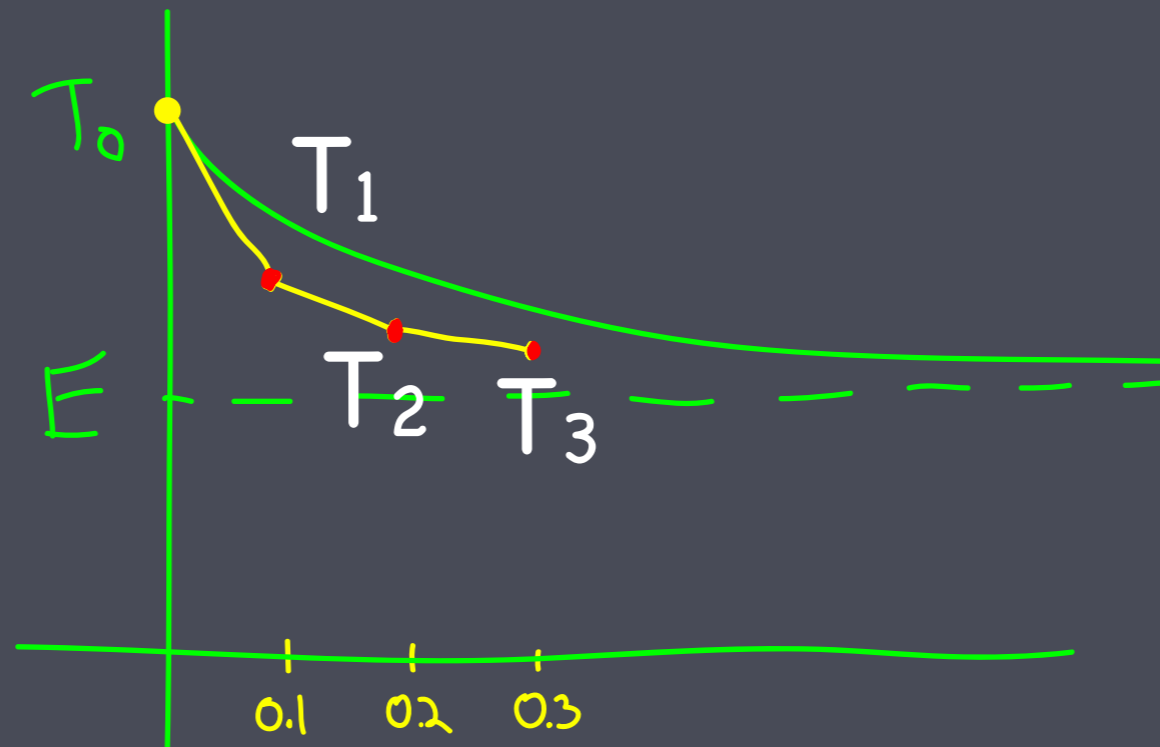
Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



$$T(0.1) \approx T_0 + 0.1 k (E - T_0) = T_1$$

$$T_2 = T_1 + 0.1 k (E - T_1)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

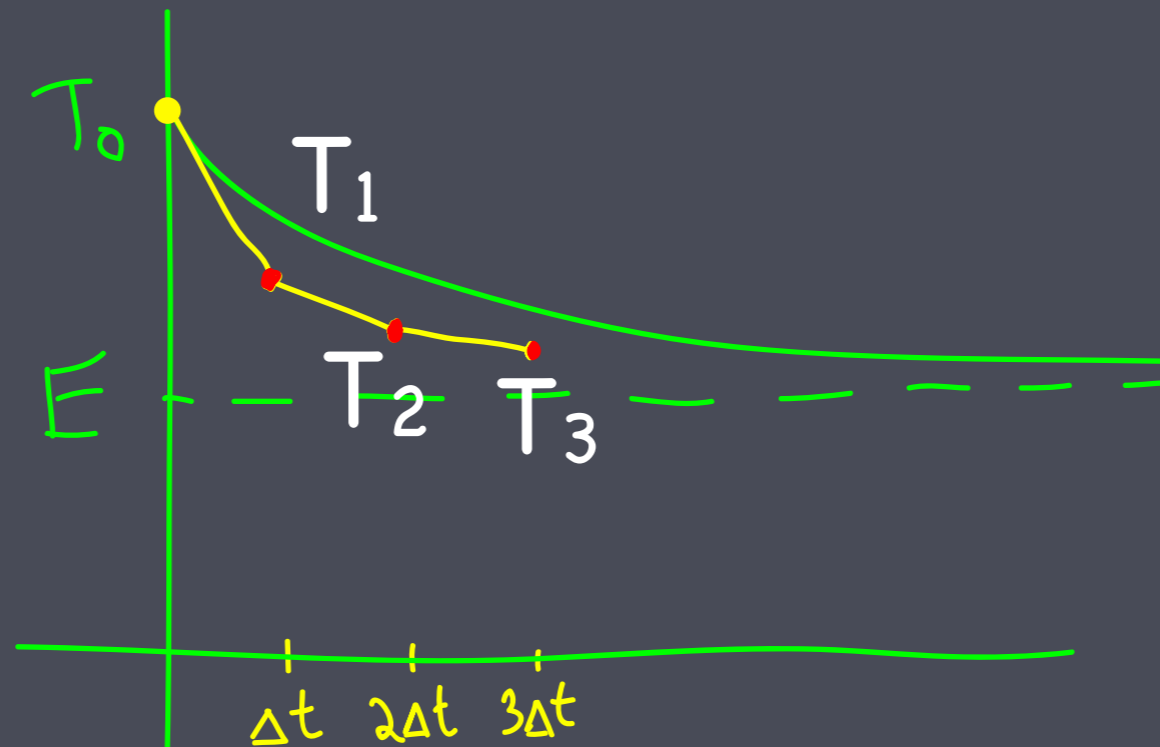


$$T(0.1) \approx T_0 + 0.1 k (E - T_0) = T_1$$

$$T_2 = T_1 + 0.1 k (E - T_1)$$

$$T_3 = T_2 + 0.1 k (E - T_2)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

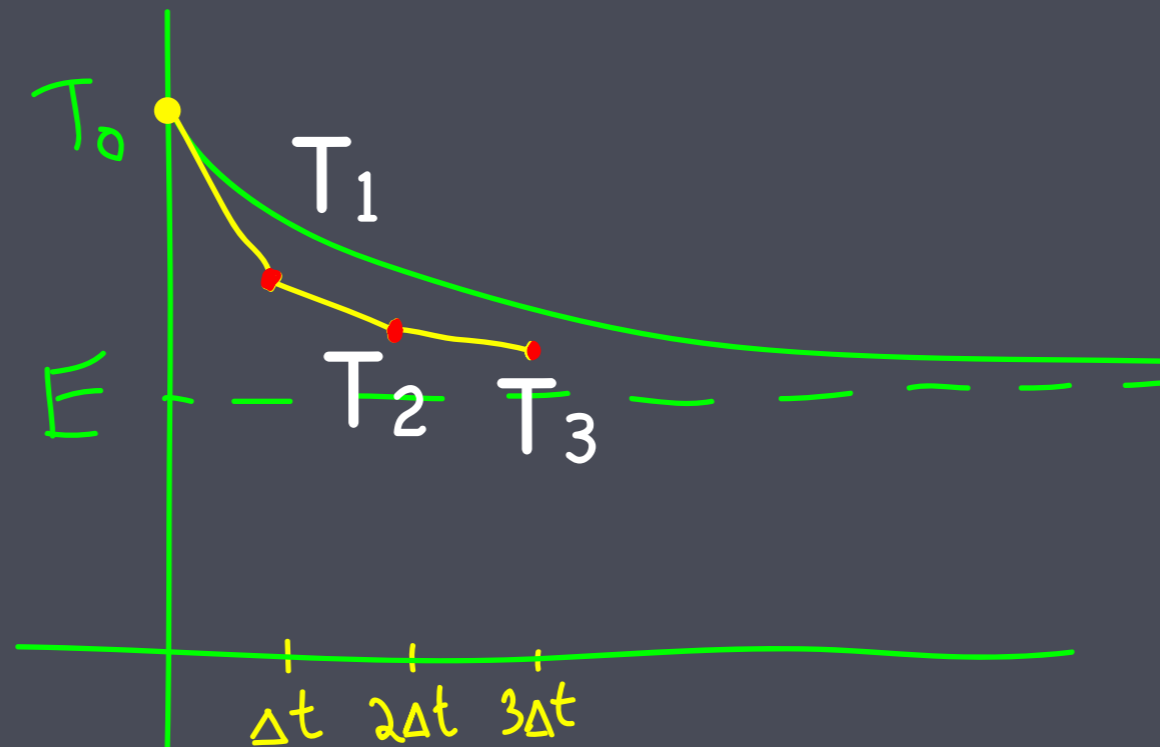


$$T(0.1) \approx T_0 + 0.1 k (E - T_0) = T_1$$

$$T_2 = T_1 + 0.1 k (E - T_1)$$

$$T_3 = T_2 + 0.1 k (E - T_2)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

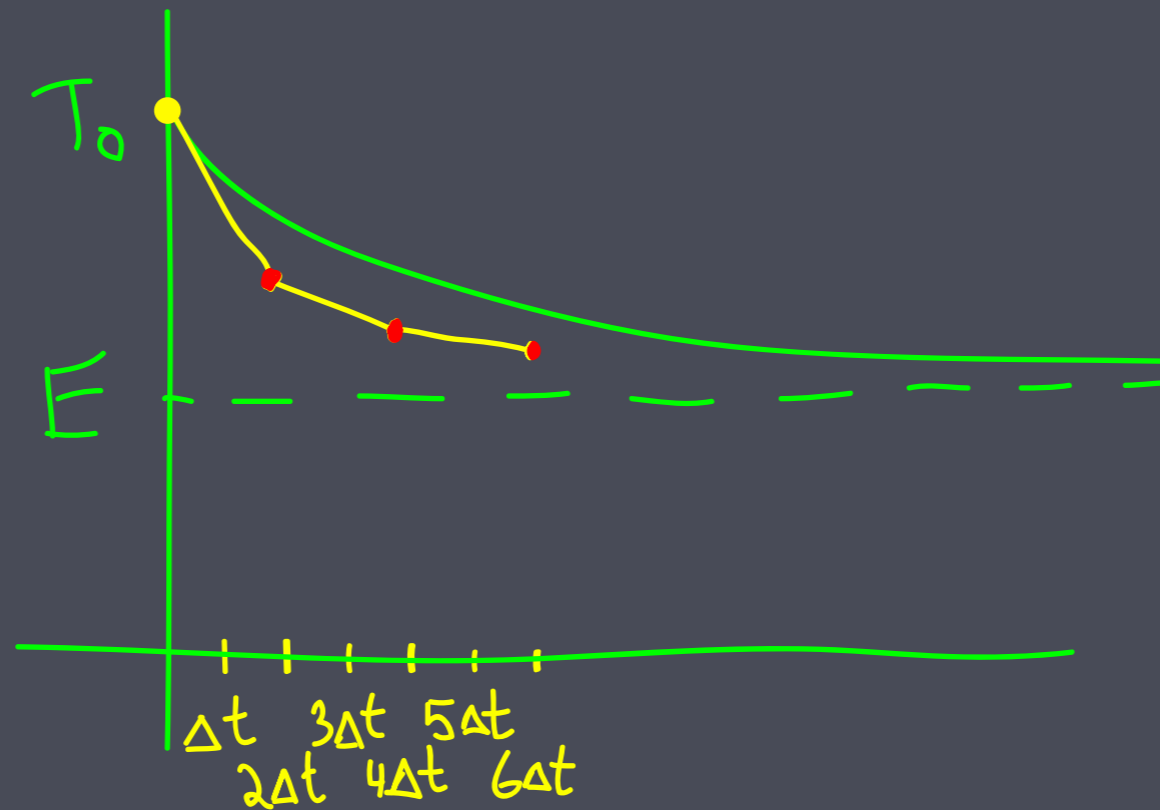


$$T(0.1) \approx T_0 + \Delta t k (E - T_0) = T_1$$

$$T_2 = T_1 + \Delta t k (E - T_1)$$

$$T_3 = T_2 + \Delta t k (E - T_2)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

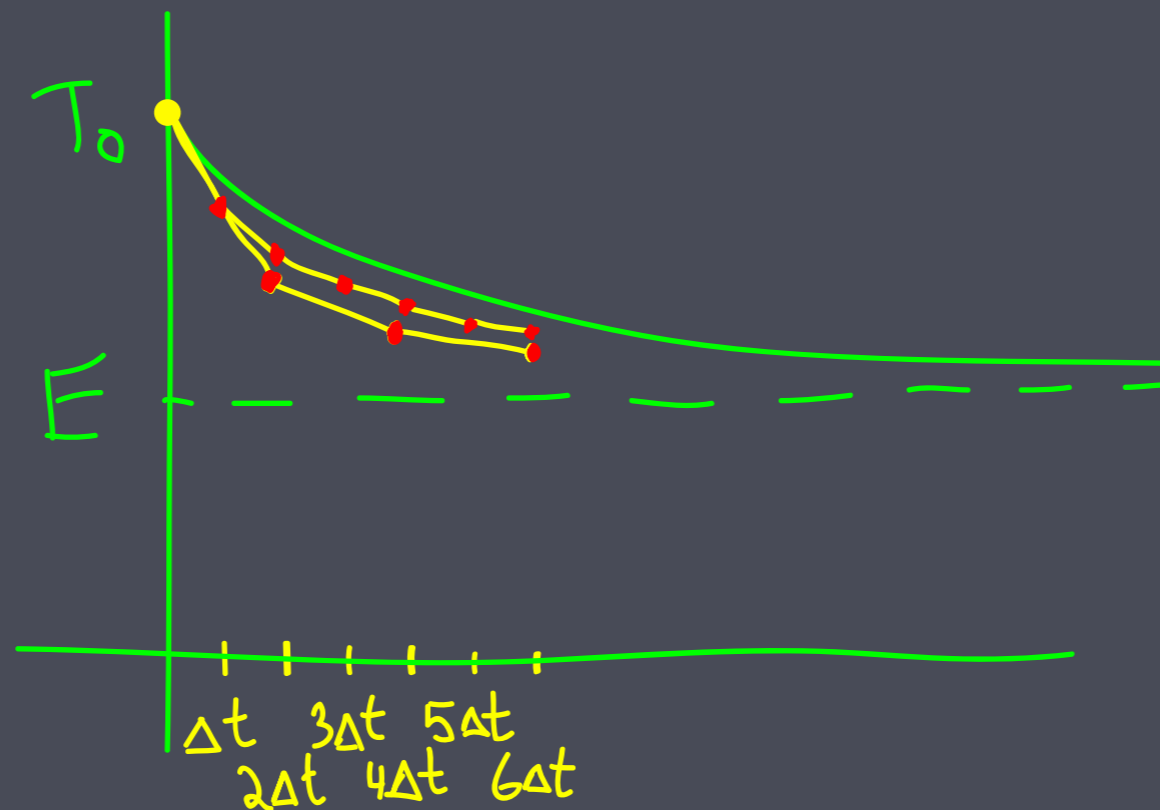


$$T(0.1) \approx T_0 + \Delta t k (E - T_0) = T_1$$

$$T_2 = T_1 + \Delta t k (E - T_1)$$

$$T_3 = T_2 + \Delta t k (E - T_2)$$

Euler's method for
 $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

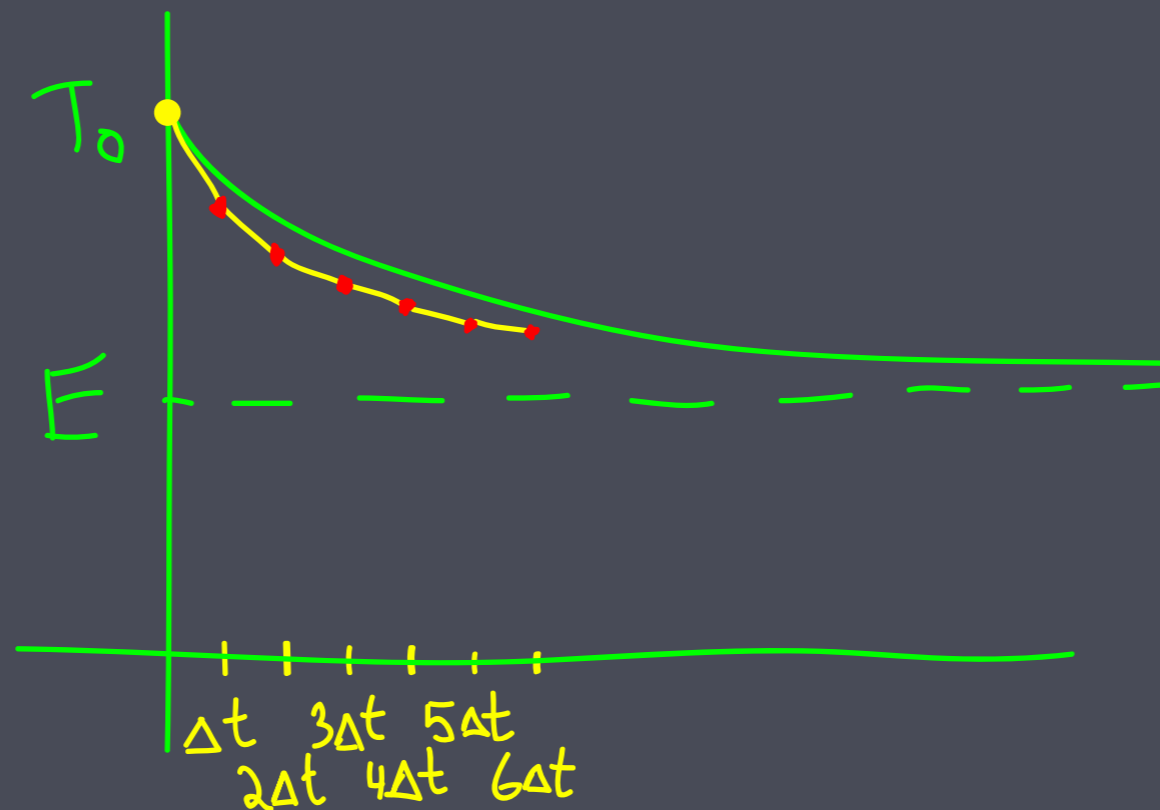


$$T(0.1) \approx T_0 + \Delta t k (E - T_0) = T_1$$

$$T_2 = T_1 + \Delta t k (E - T_1)$$

$$T_3 = T_2 + \Delta t k (E - T_2)$$

Euler's method for $T'(t) = k(E - T(t))$ with $T(0) = T_0$.

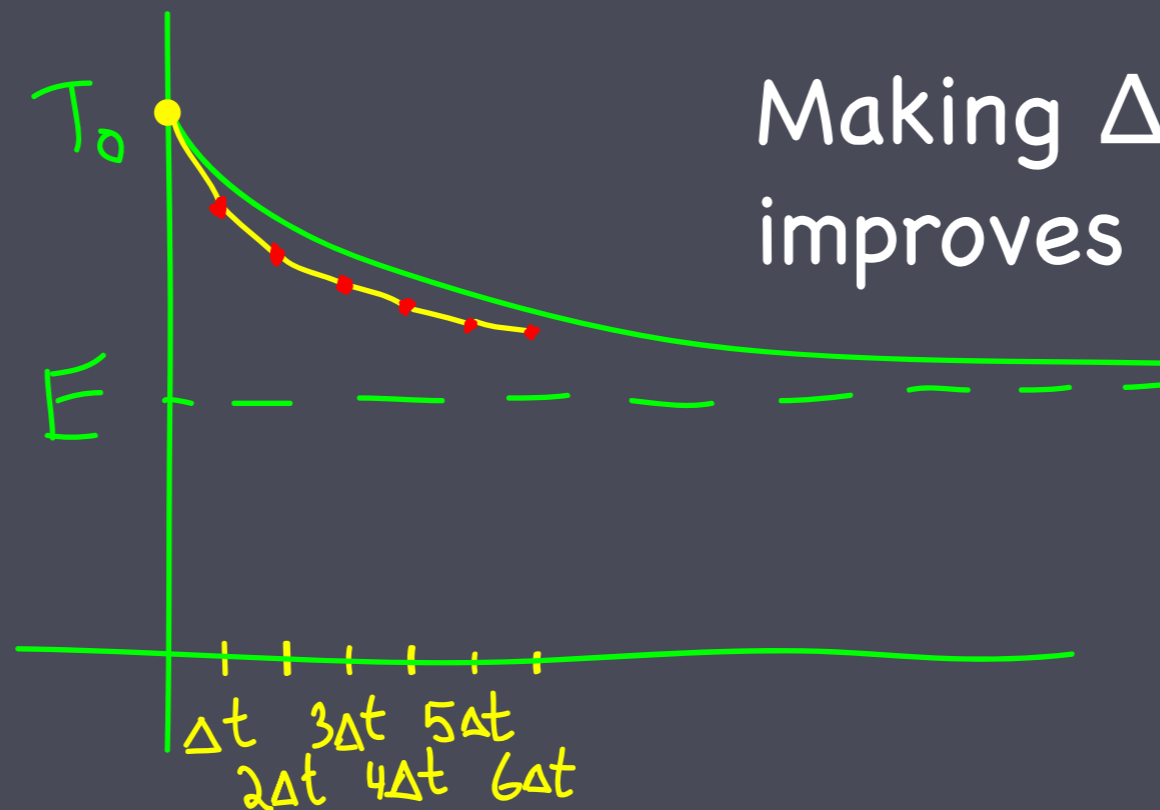


$$T(0.1) \approx T_0 + \Delta t k (E - T_0) = T_1$$

$$T_2 = T_1 + \Delta t k (E - T_1)$$

$$T_3 = T_2 + \Delta t k (E - T_2)$$

Euler's method for $T'(t) = k(E - T(t))$ with $T(0) = T_0$.



Making Δt smaller
improves the estimate.

$$T(0.1) \approx T_0 + \Delta t k (E - T_0) = T_1$$

$$T_2 = T_1 + \Delta t k (E - T_1)$$

$$T_3 = T_2 + \Delta t k (E - T_2)$$

When will Euler's method underestimate the true solution?

- (A) When the derivative of the true solution is positive.
- (B) When the derivative of the true solution is negative.
- (C) When the second derivative of the true solution is positive.
- (D) When the second derivative of the true solution is negative.