Today

- Additional office hours Tuesday 1:30-3:30 pm.
- Absolute extrema.
- More on inflection points.
- Sketching using derivative information.

Absolute extrema

- A continuous function on a closed interval [a,b] takes on its highest (lowest) value either at a local maximum (minimum) or at an end point ($x=a$ or $x=b$). Call this an absolute maximum (minimum).
- When looking for absolute extrema, check critical points AND end points!

Where does $f(x)=x^3-x^2$ take on its absolute minimum on the interval [-1,2]?

(A) $x=-1$

(B) $x=0$

(C) $x=2/3$

(D) $x=2$

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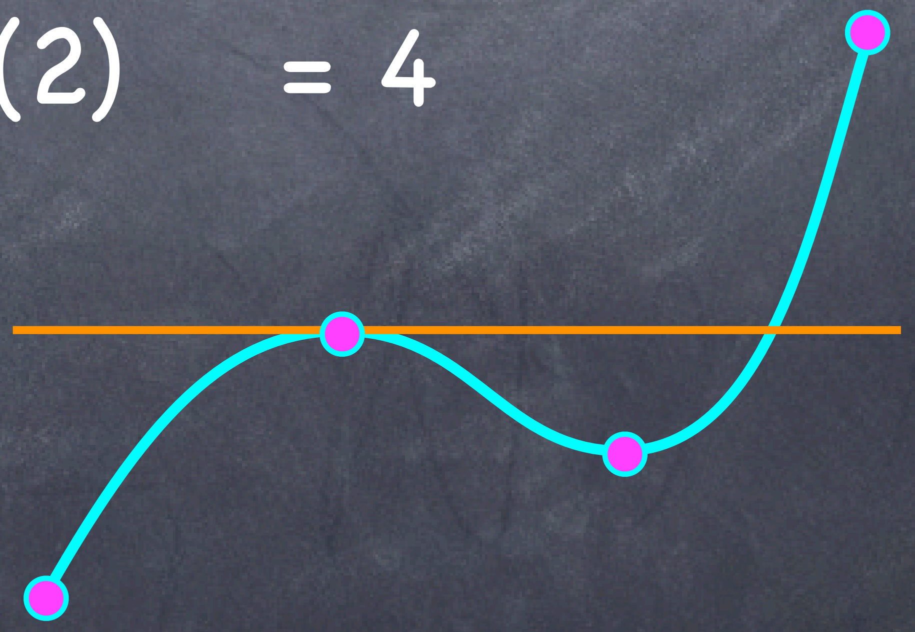
f(-1) = -2

f(0) = 0

f(2/3) = -4/27

f(2) = 4



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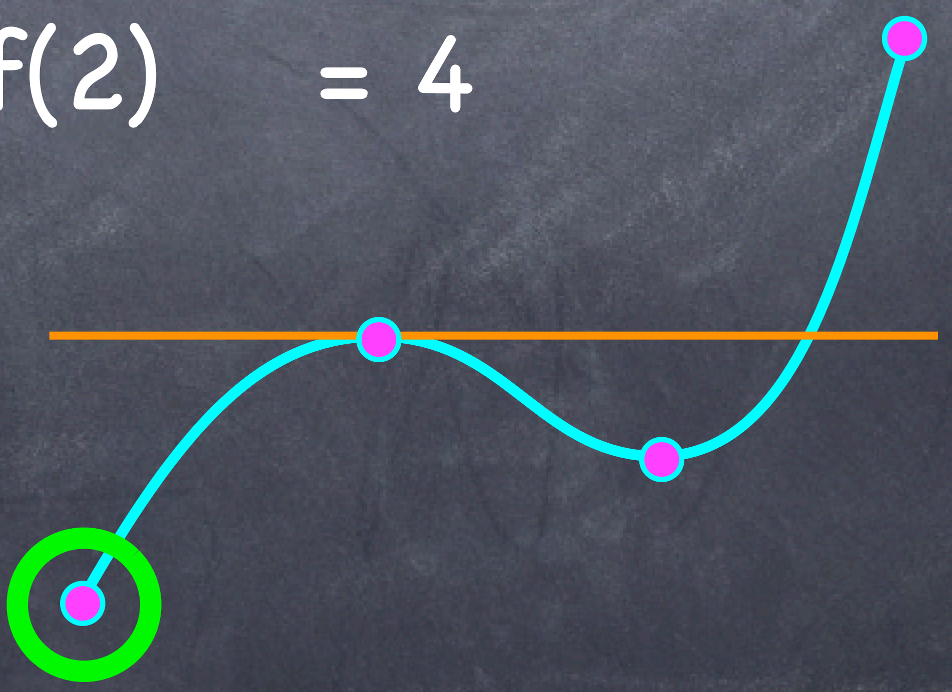

Back to $f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\,(x^3 - x^2) = 0$ --> $x=0$, $x=1$.

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- $f''(x) = 12\ (3x^2 - 2x)$.

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- $f''(0) = 0$ --> inflection point? maybe!!!

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    - --> $f'(x)$ is increasing near $x=1$.

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  - --> $f'(x)$ is increasing near $x=1$.
  - --> slope of $f(x)$ is increasing near $x=1$.

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- $f''(x) = 12 (3x^2 - 2x)$.
- $f''(0) = 0$ --> inflection point? maybe!!!
- $f''(1) = 1 > 0$
  - --> $f'(x)$ is increasing near $x=1$.
  - --> slope of $f(x)$ is increasing near $x=1$.
  - --> $f(x)$ has a minimum at $x=1$.

Is x=0 an inflection point?

(A) Yes because $f''(0)=0$.

(B) Yes because $f''(0)=0$ and $f'''(0)<0$.

(C) No because $f''(-1)=60$ and $f''(1)=12$.

(D) Yes because $f''(-1)=60$ and $f''(1/2)=-3$.

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x					
$f''(x)$					

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x	$(-\infty,0)$	0	$(0,2/3)$	$2/3$	$(2/3,\infty)$
$f''(x)$		0		0	

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x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
$f''(x)$	+	0		0	

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x	$(-\infty,0)$	0	$(0,2/3)$	$2/3$	$(2/3,\infty)$
$f''(x)$	+	0	-	0	+

Is x=0 an inflection point?

(A) Yes because f″(0)=0.

(B) Yes be ... .

DANGER - f″ might also change sign at a vertical asymptote or a point at which f′ or f″ DNE.

(C) No be ... 2.

(D) Yes because f″(-1)=60 and f″(1/2)=-3.

x	$(-\infty,0)$	0	(0,2/3)	2/3	$(2/3,\infty)$
f″(x)	+	0	-	0	+

Is x=0 an inflection point?

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$f''(x)$	+	0	-	0	+

$f'''(0)<0$

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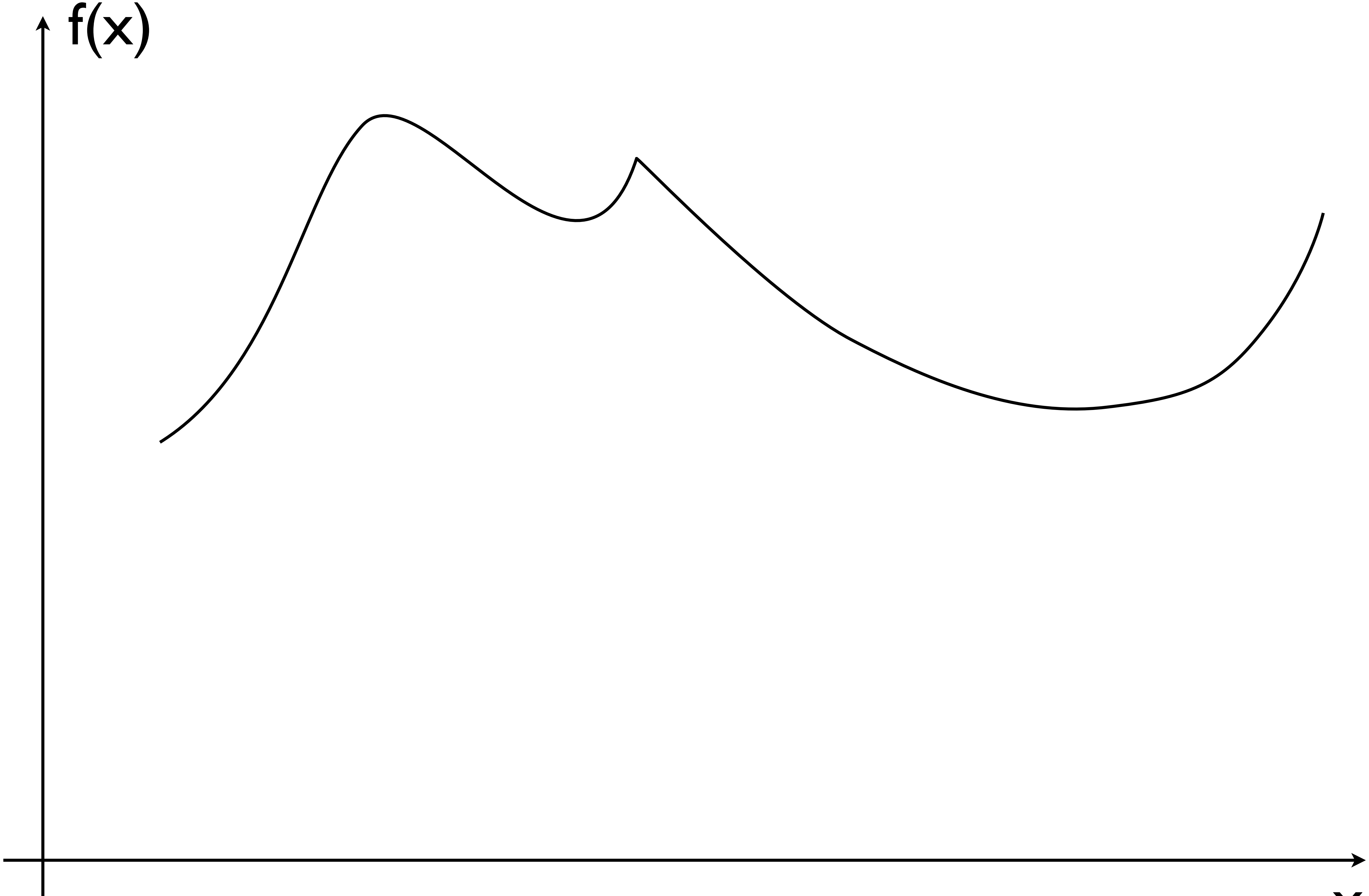
$f'''(0)<0$

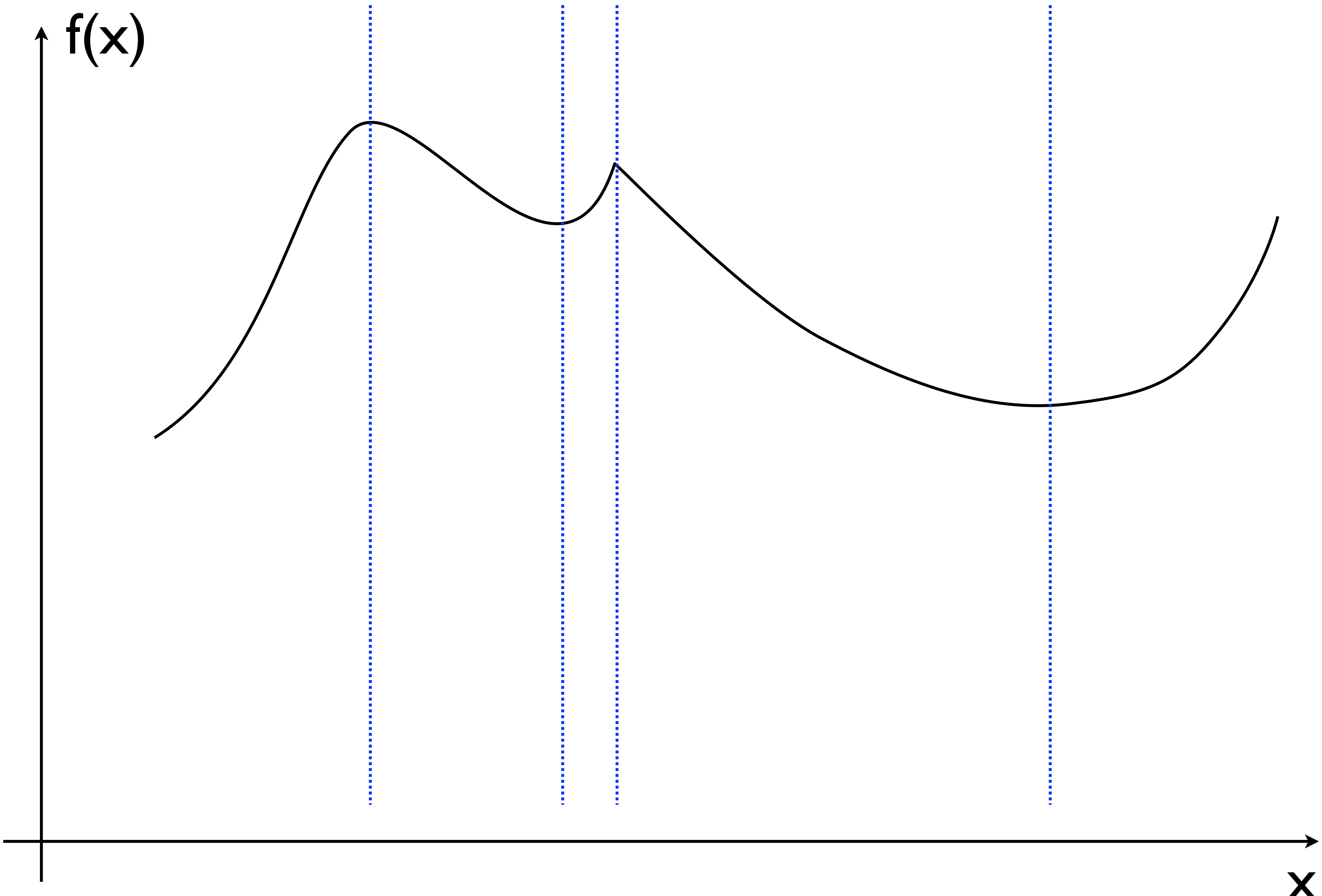
Using f, f′ and f″ to graph f

Annotating the graph of f(x) with f'(x) info



What is the function doing?

Annotating the graph of f(x) with f'(x) info



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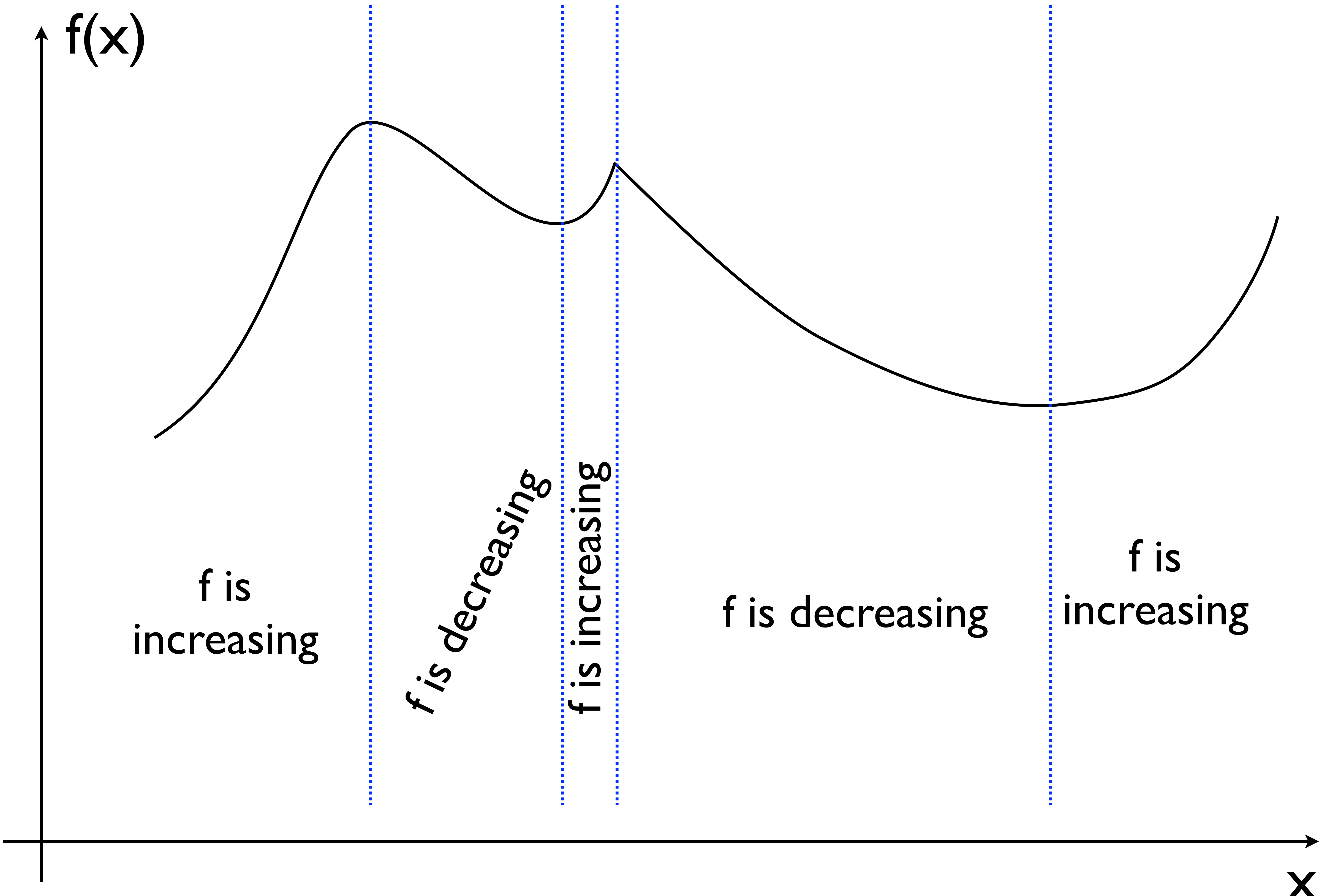

What does that mean for the first derivative?

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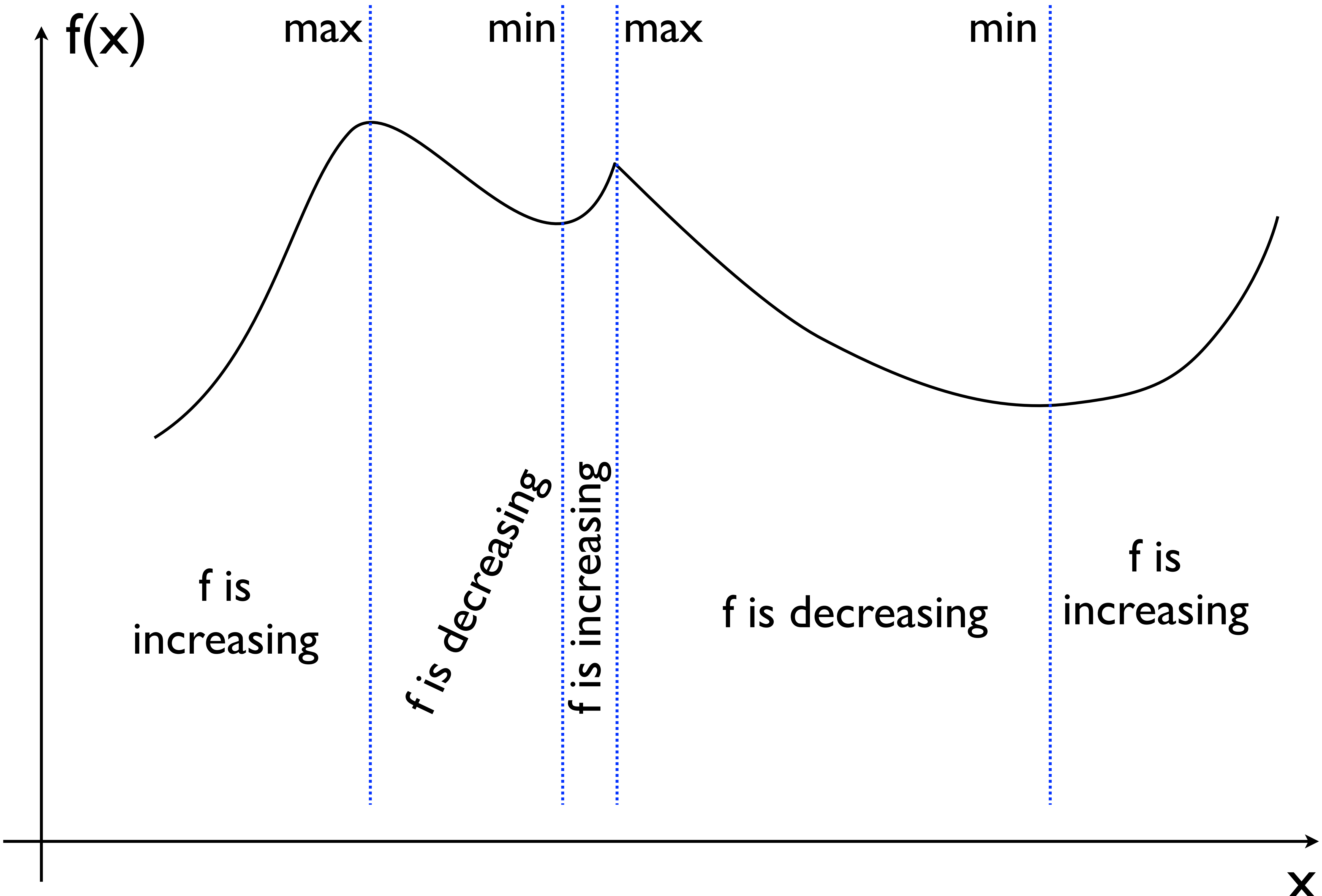

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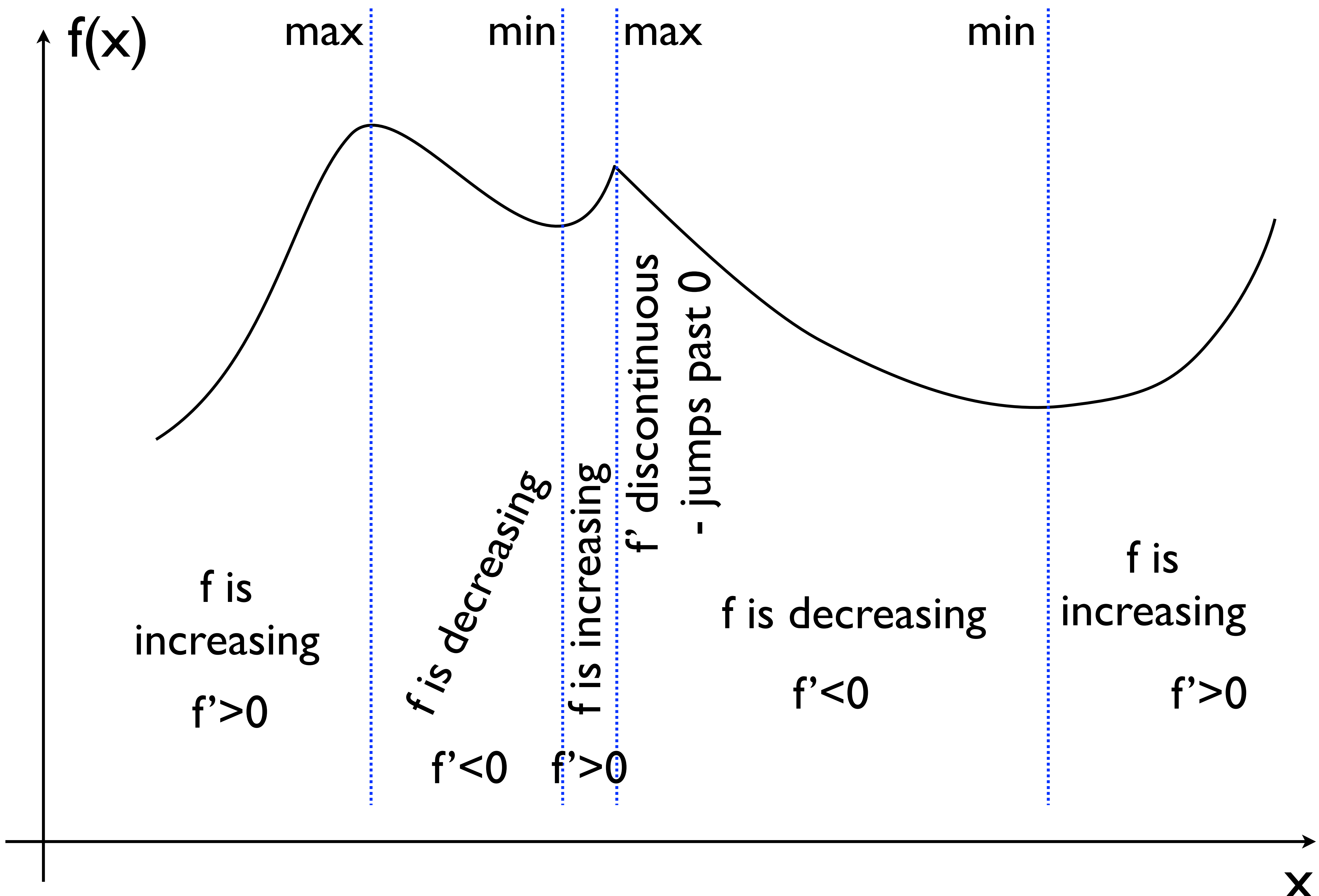

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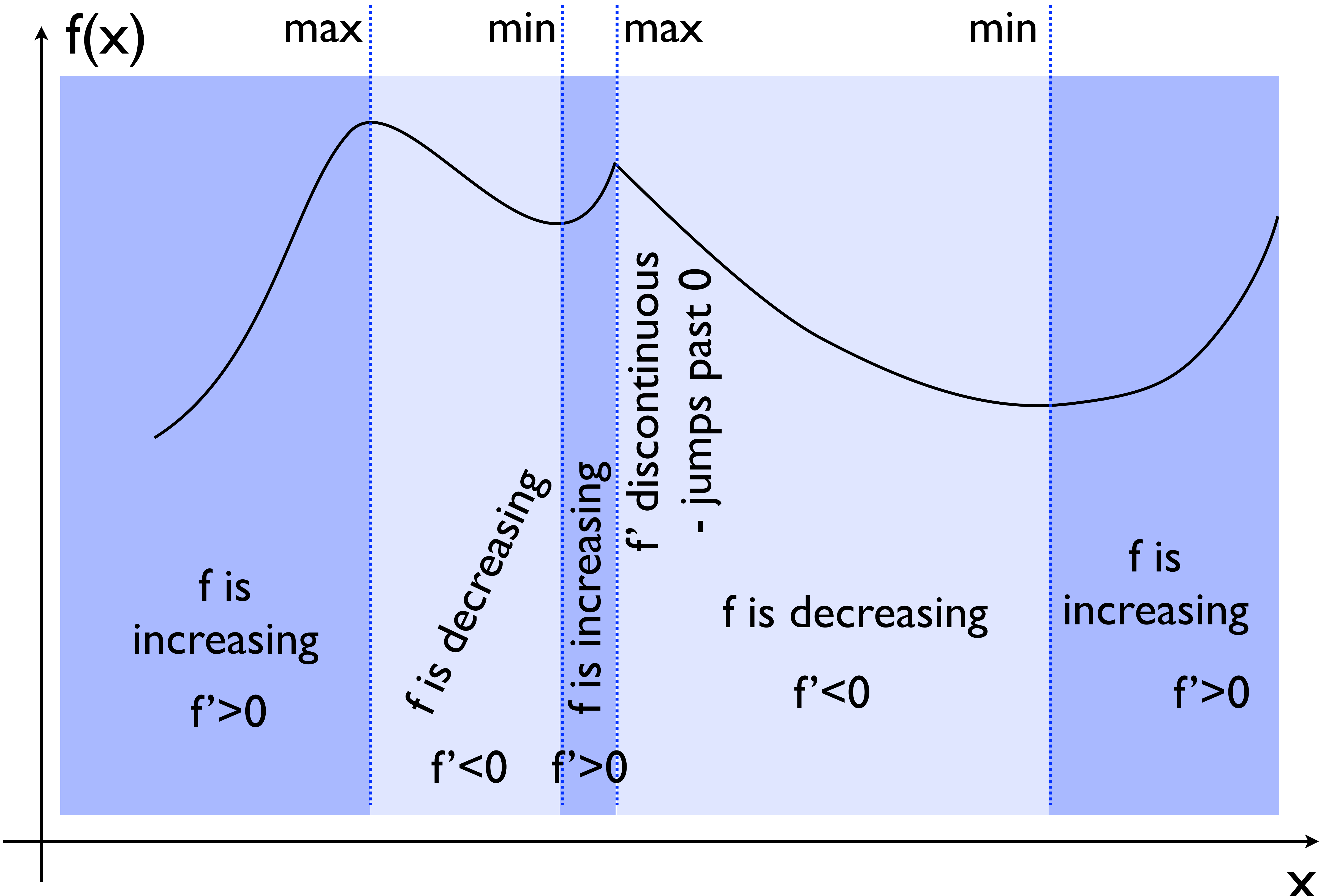

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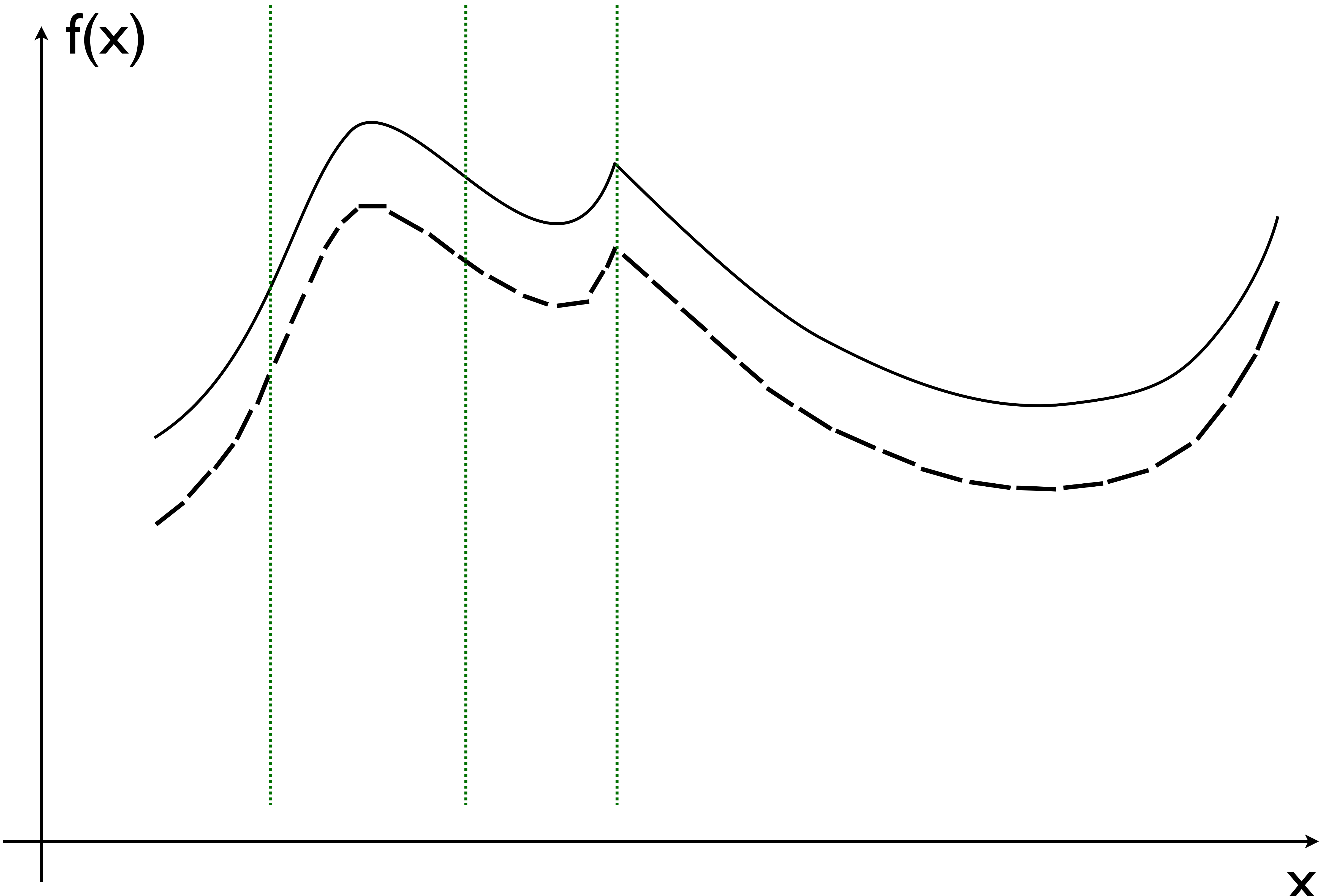

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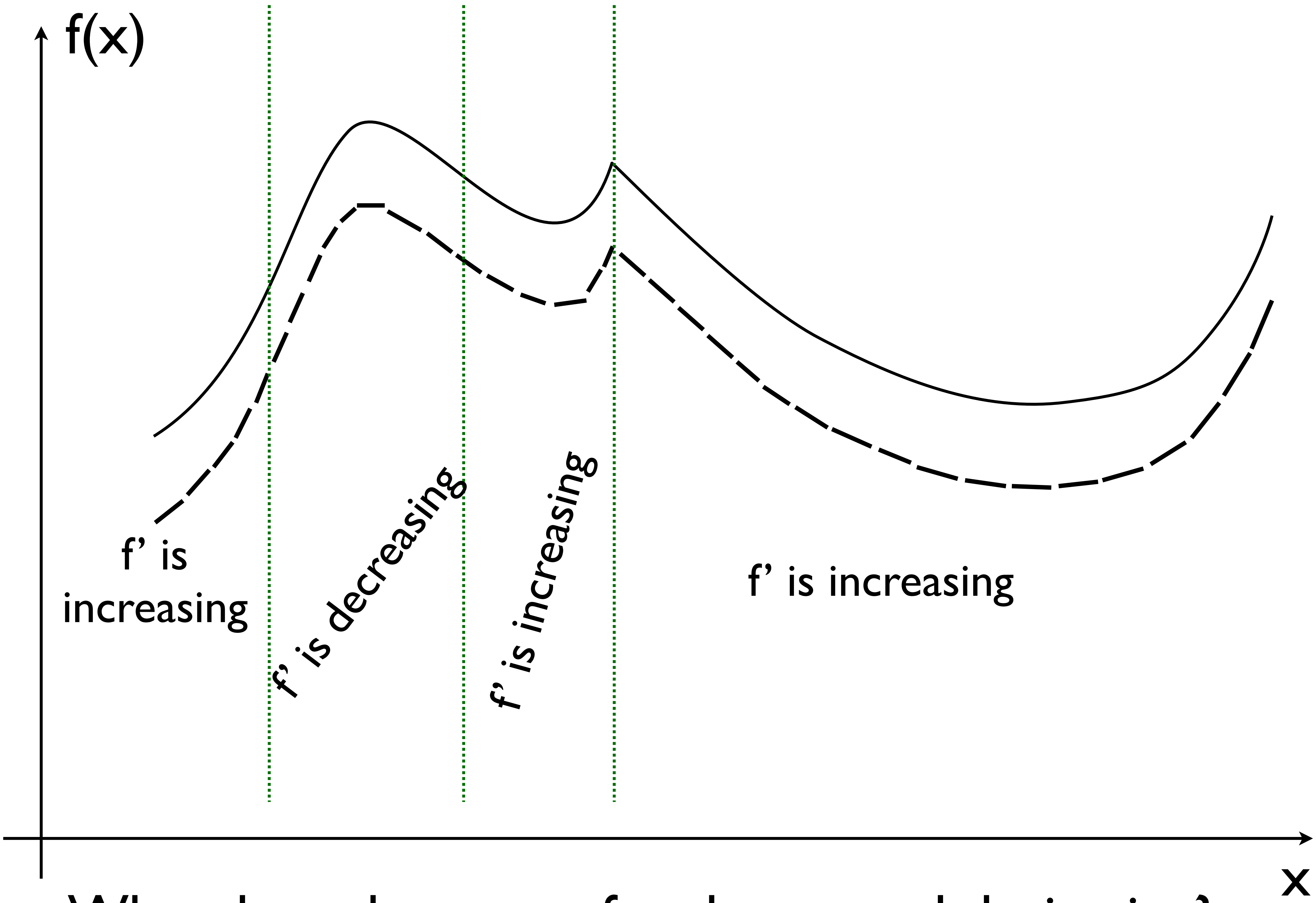

What does that mean for the second derivative?

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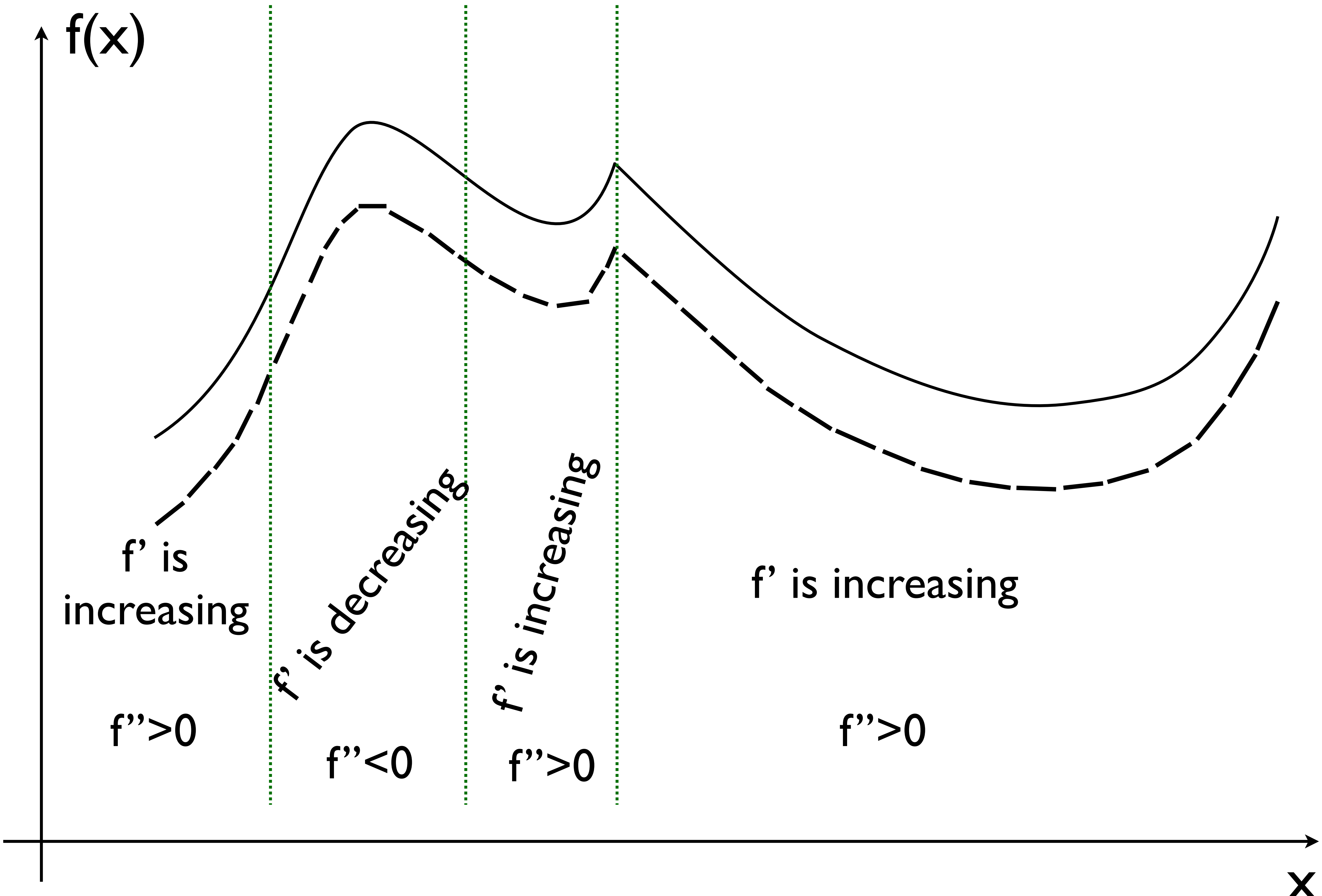

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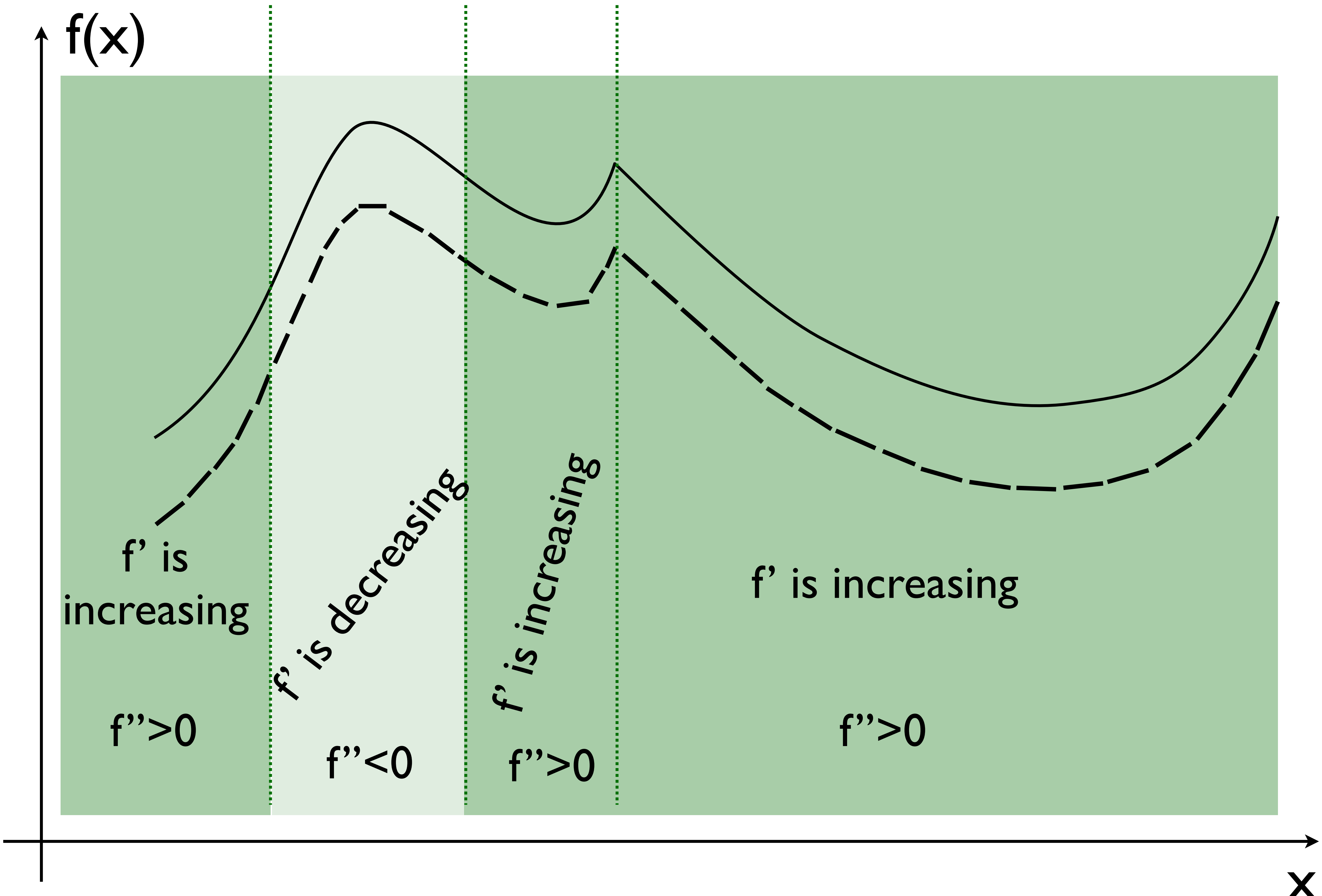

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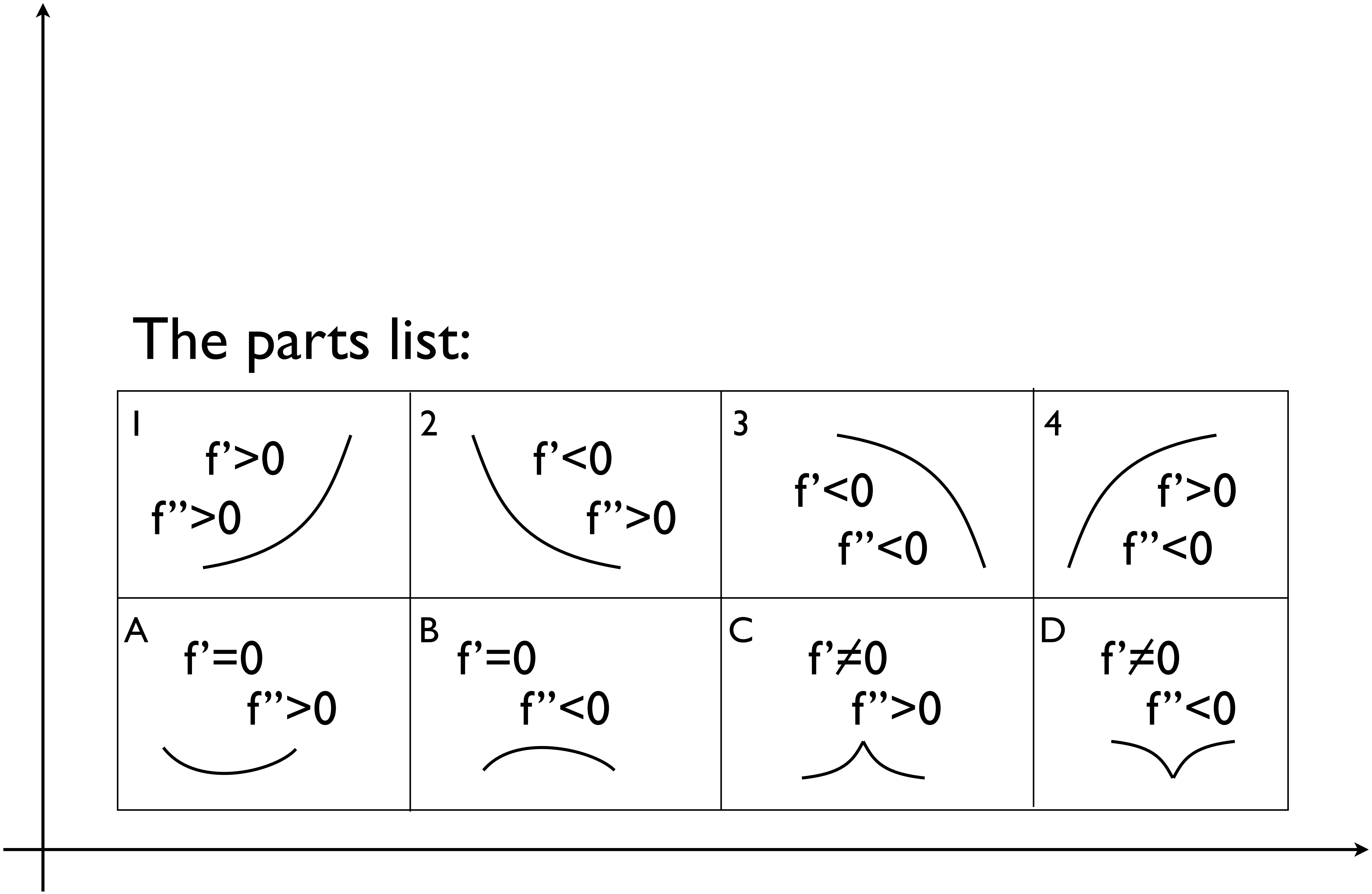
What you have to do to graph it.



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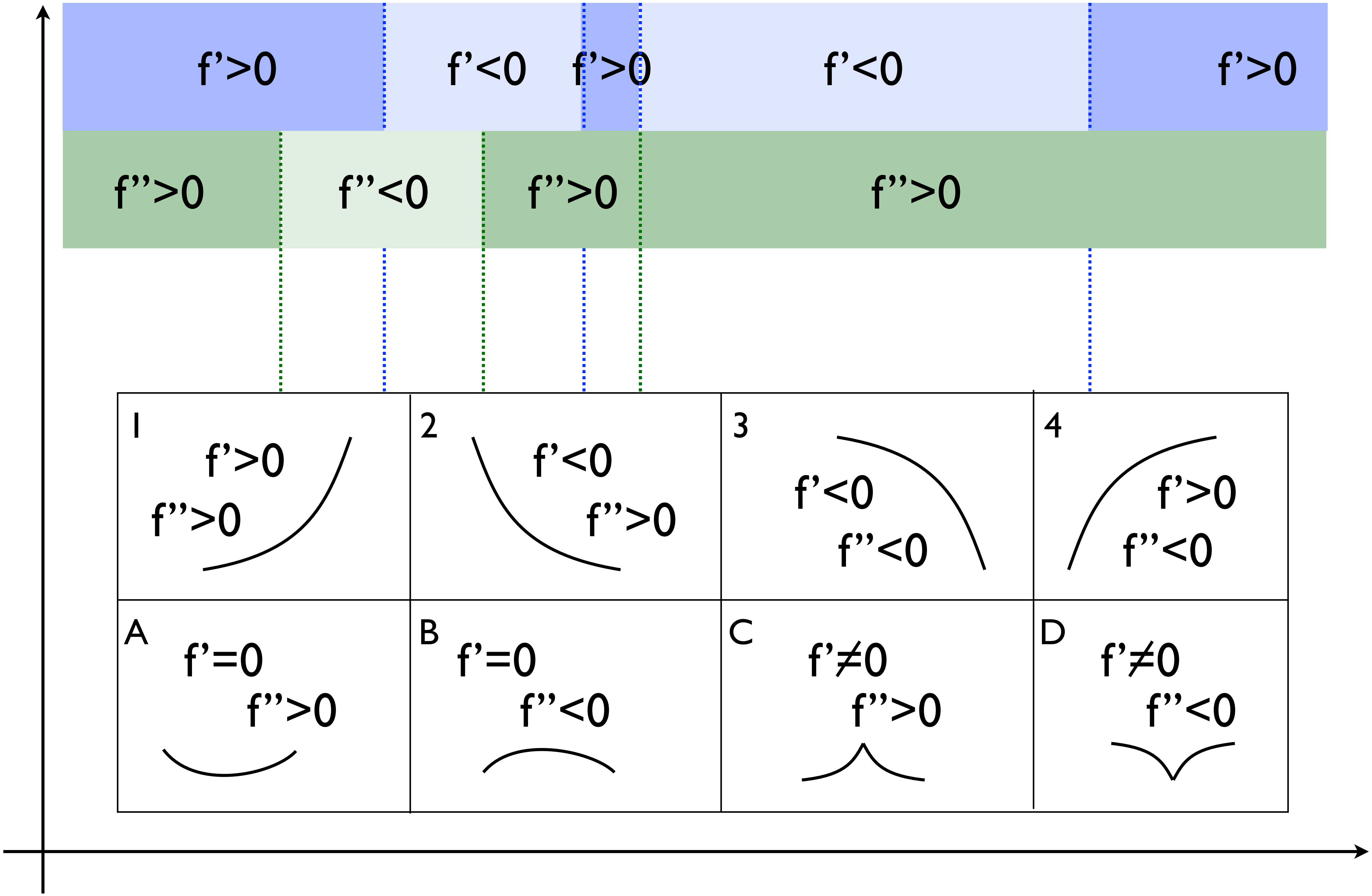

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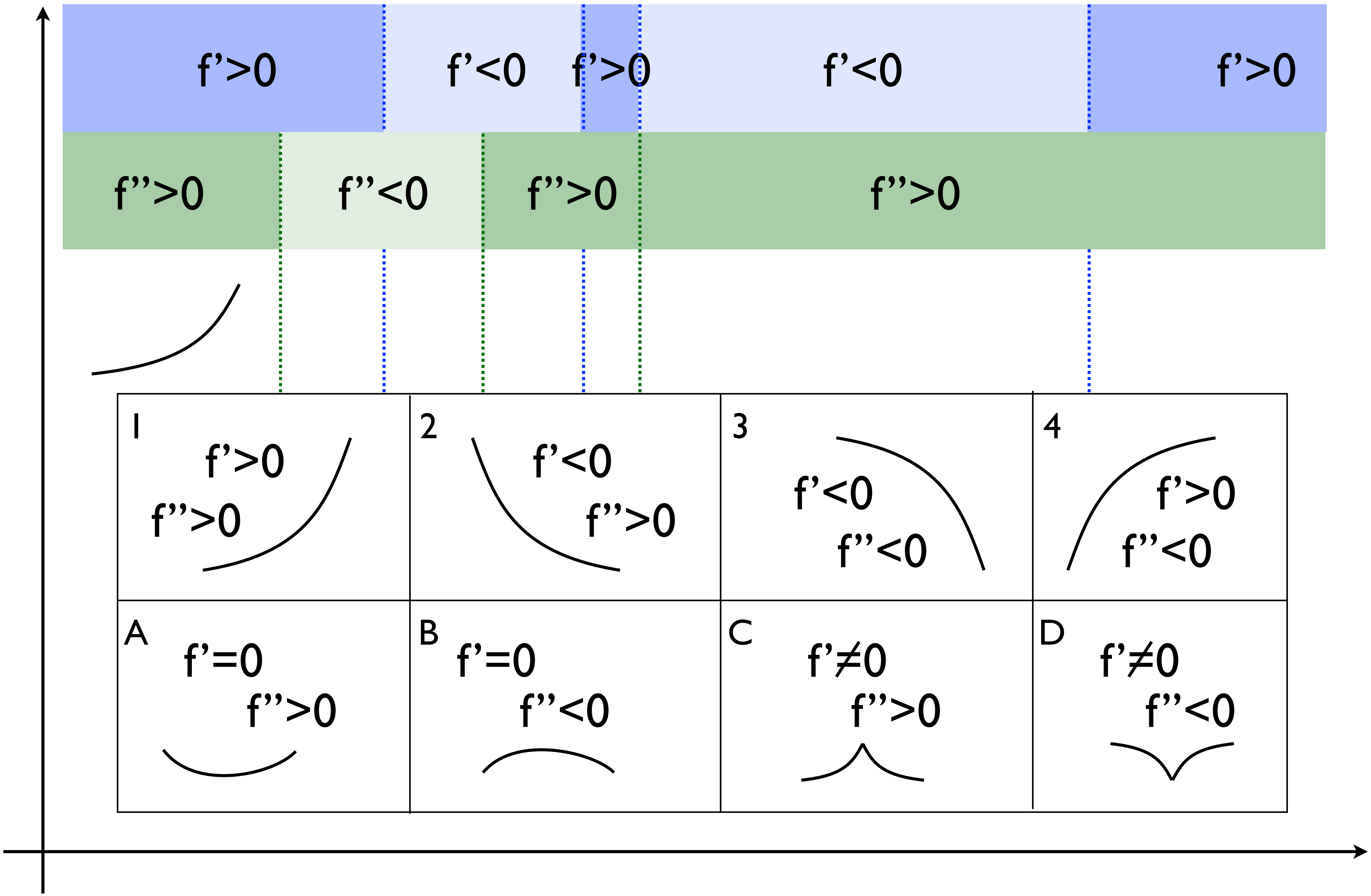

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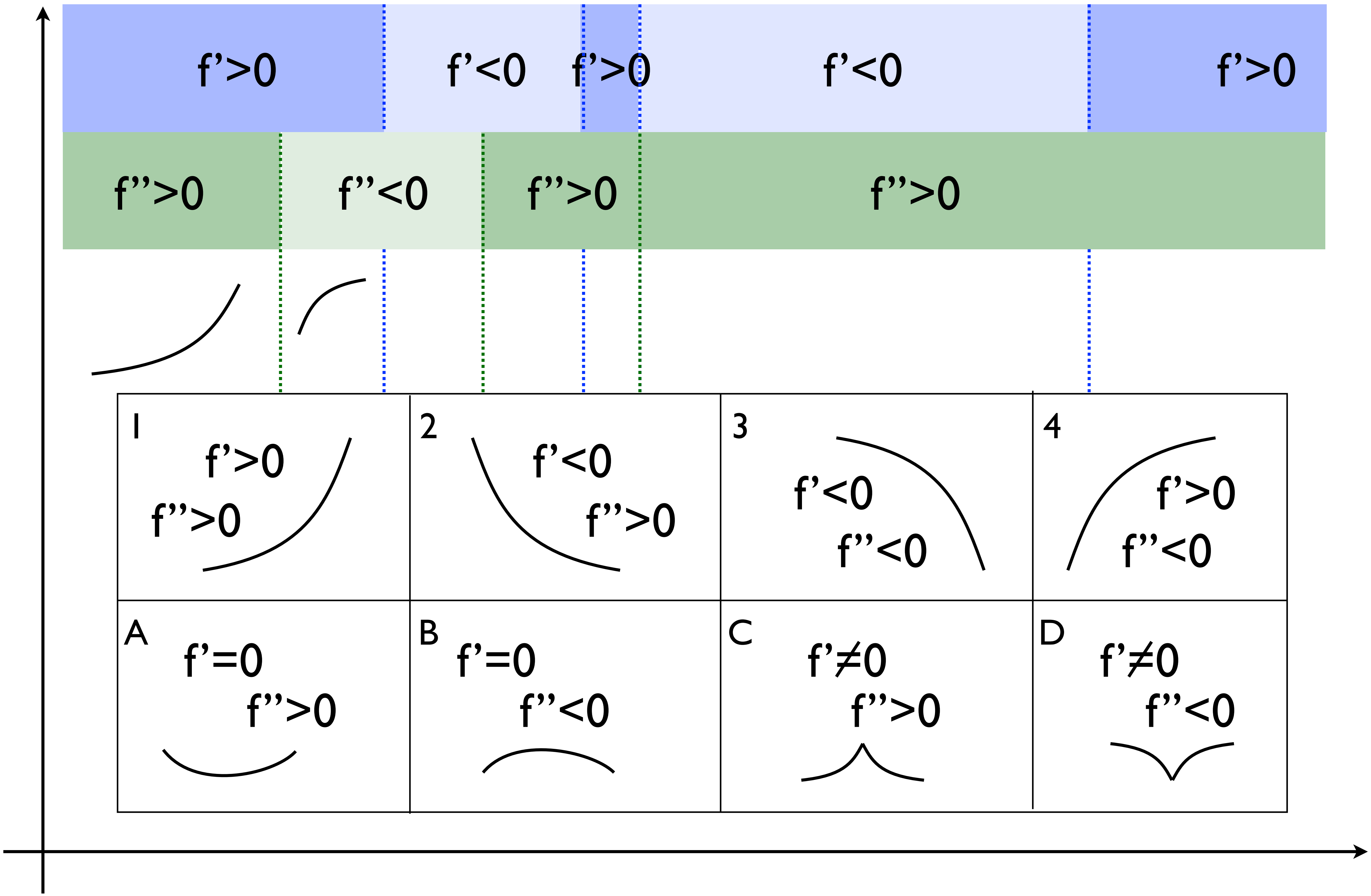

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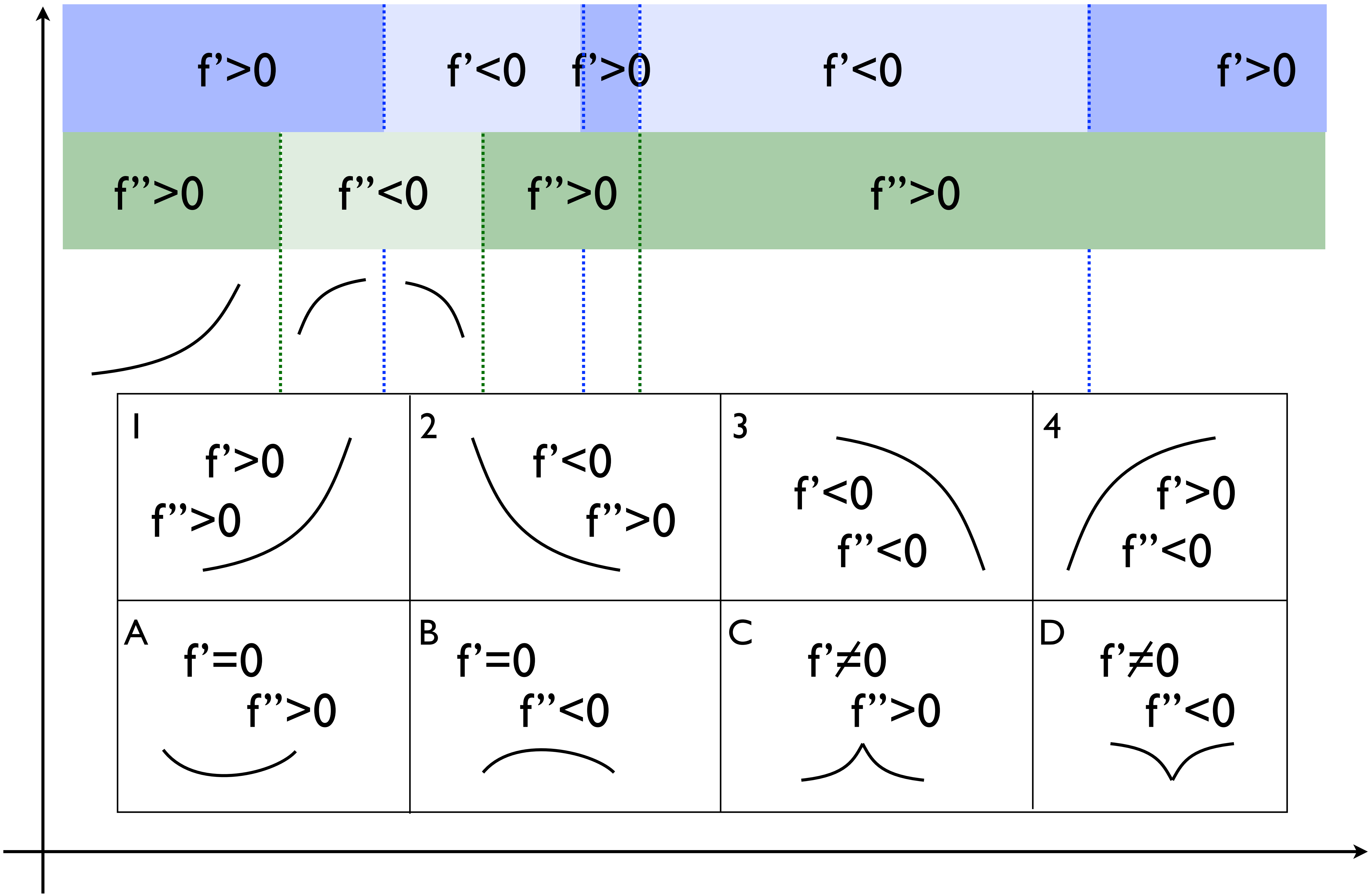

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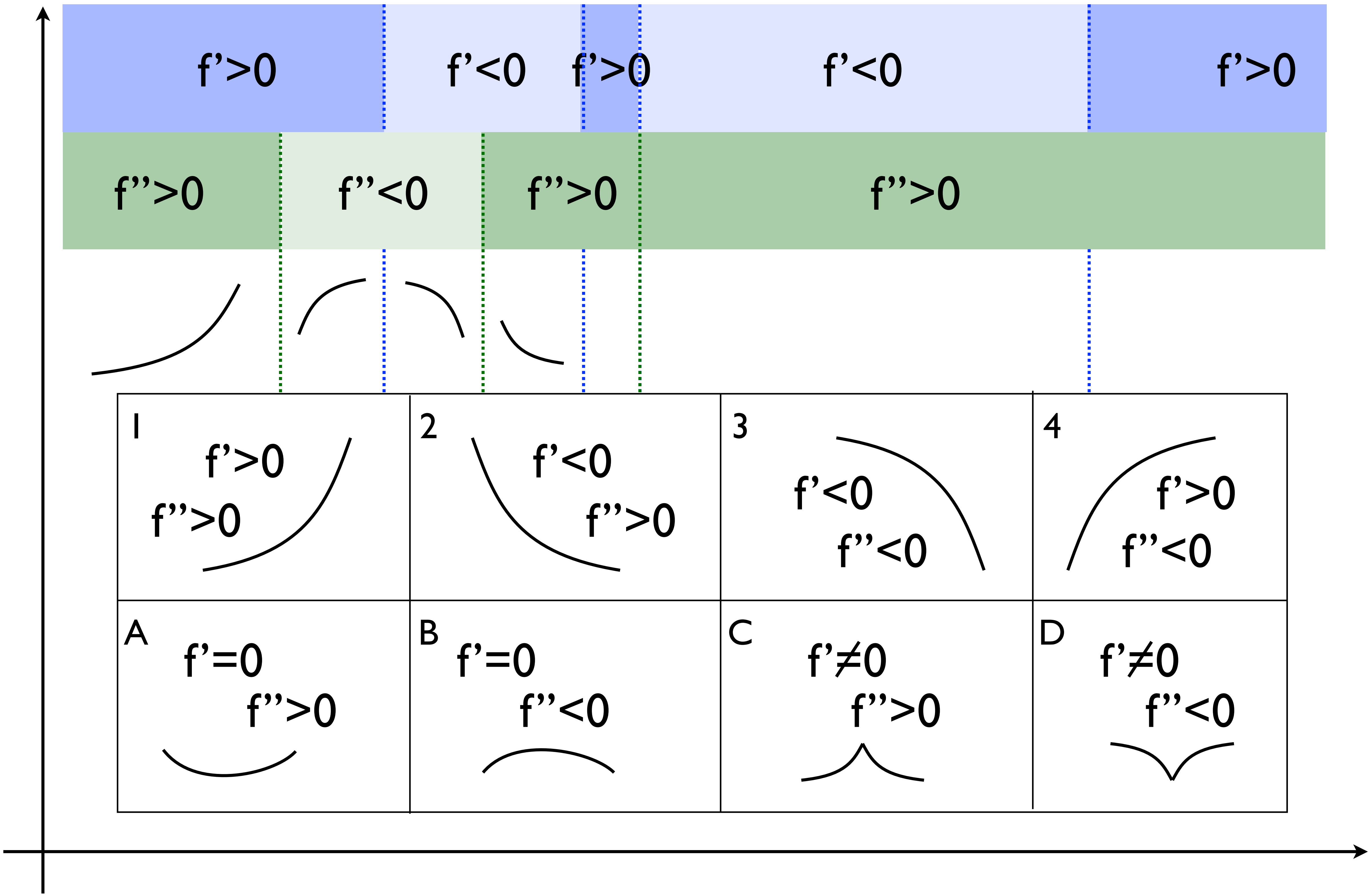

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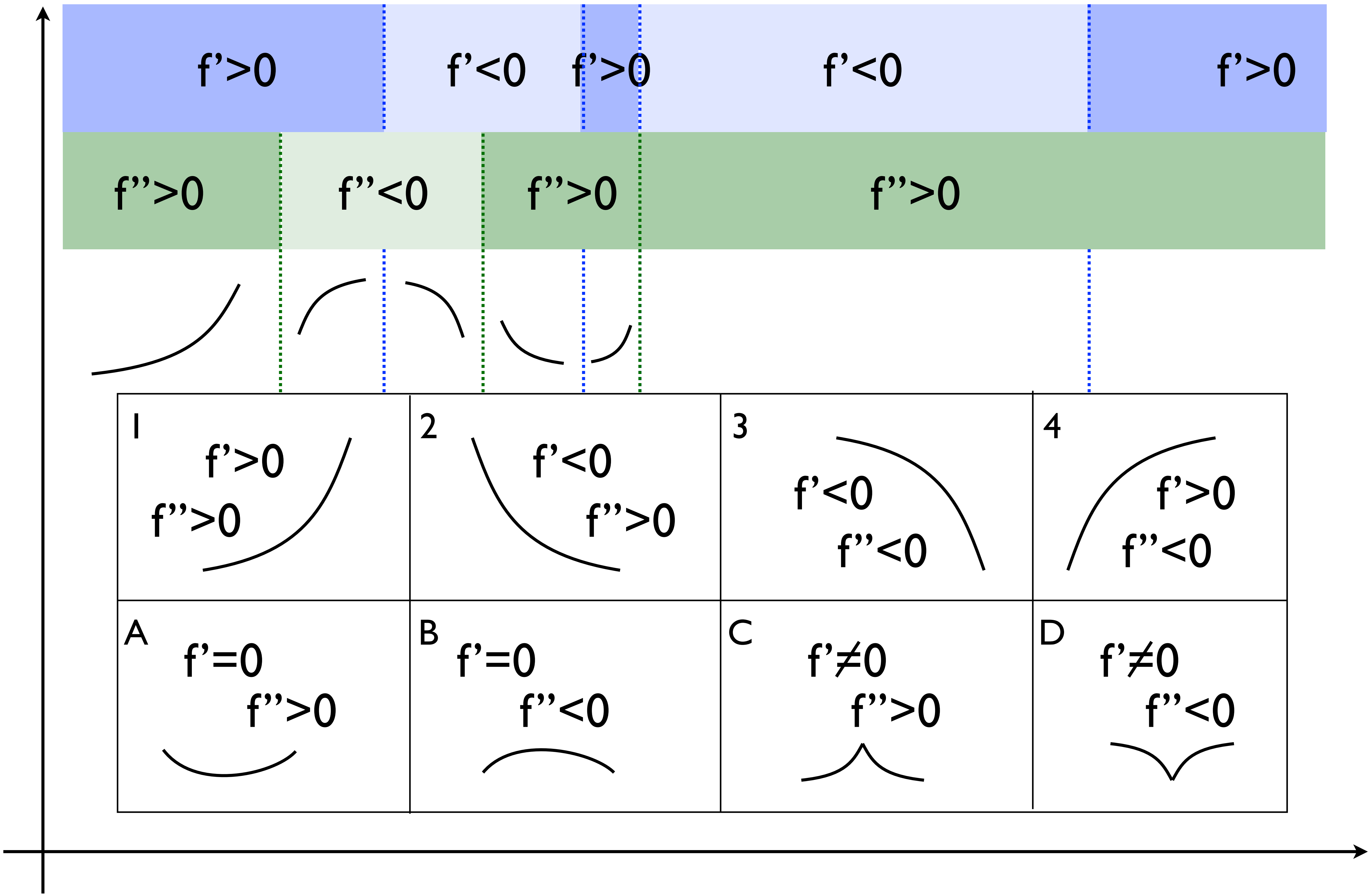

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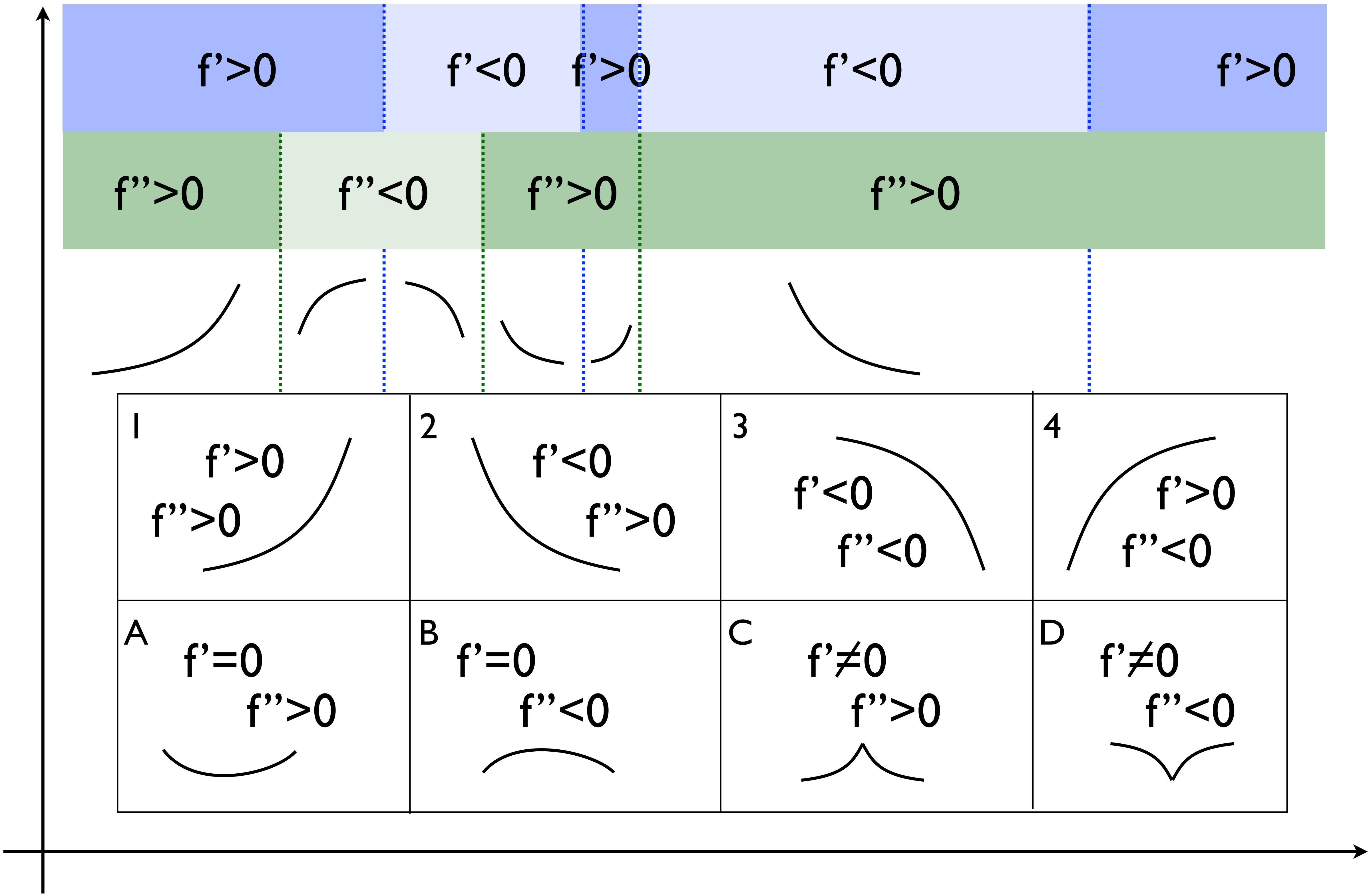

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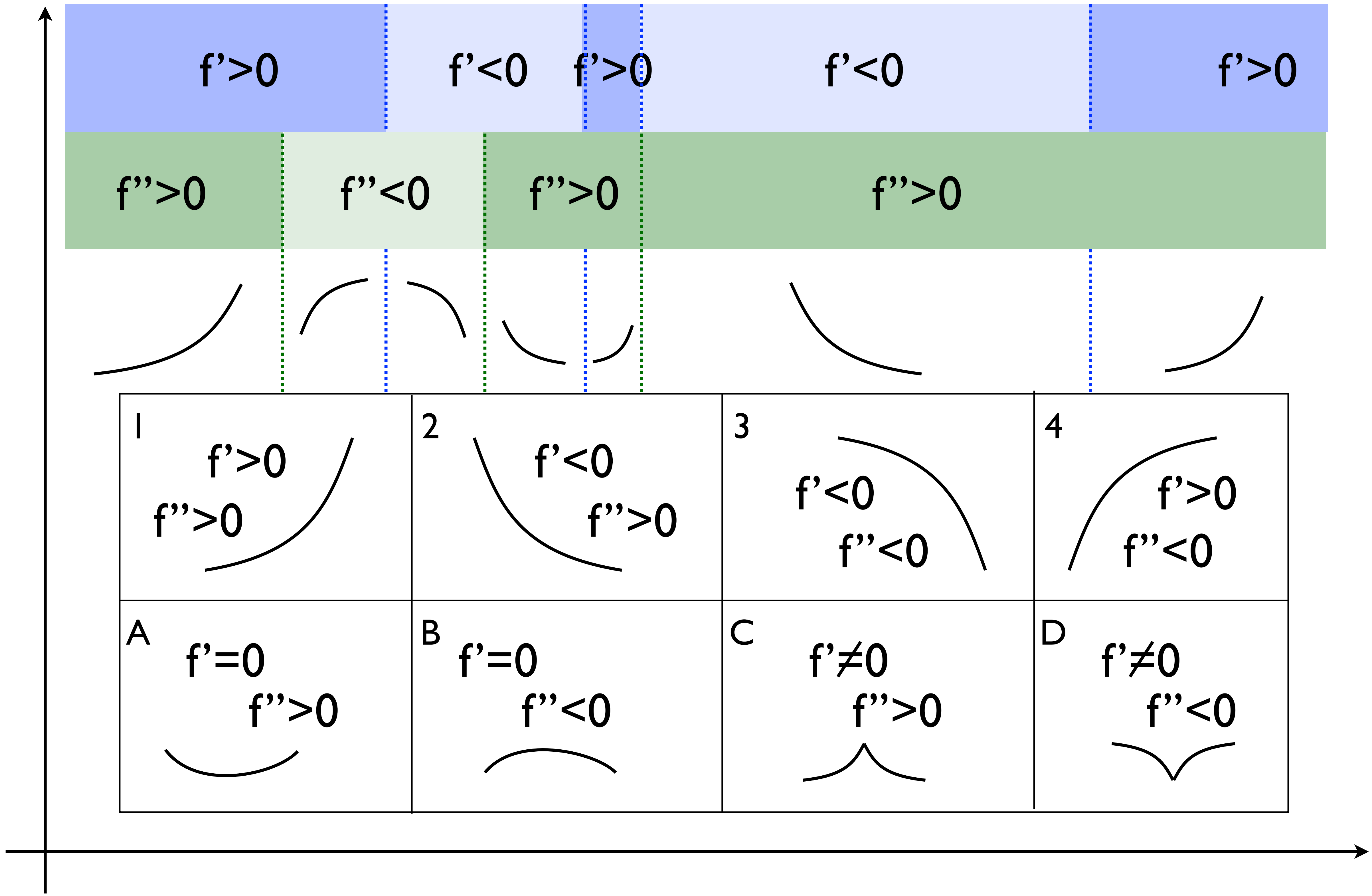

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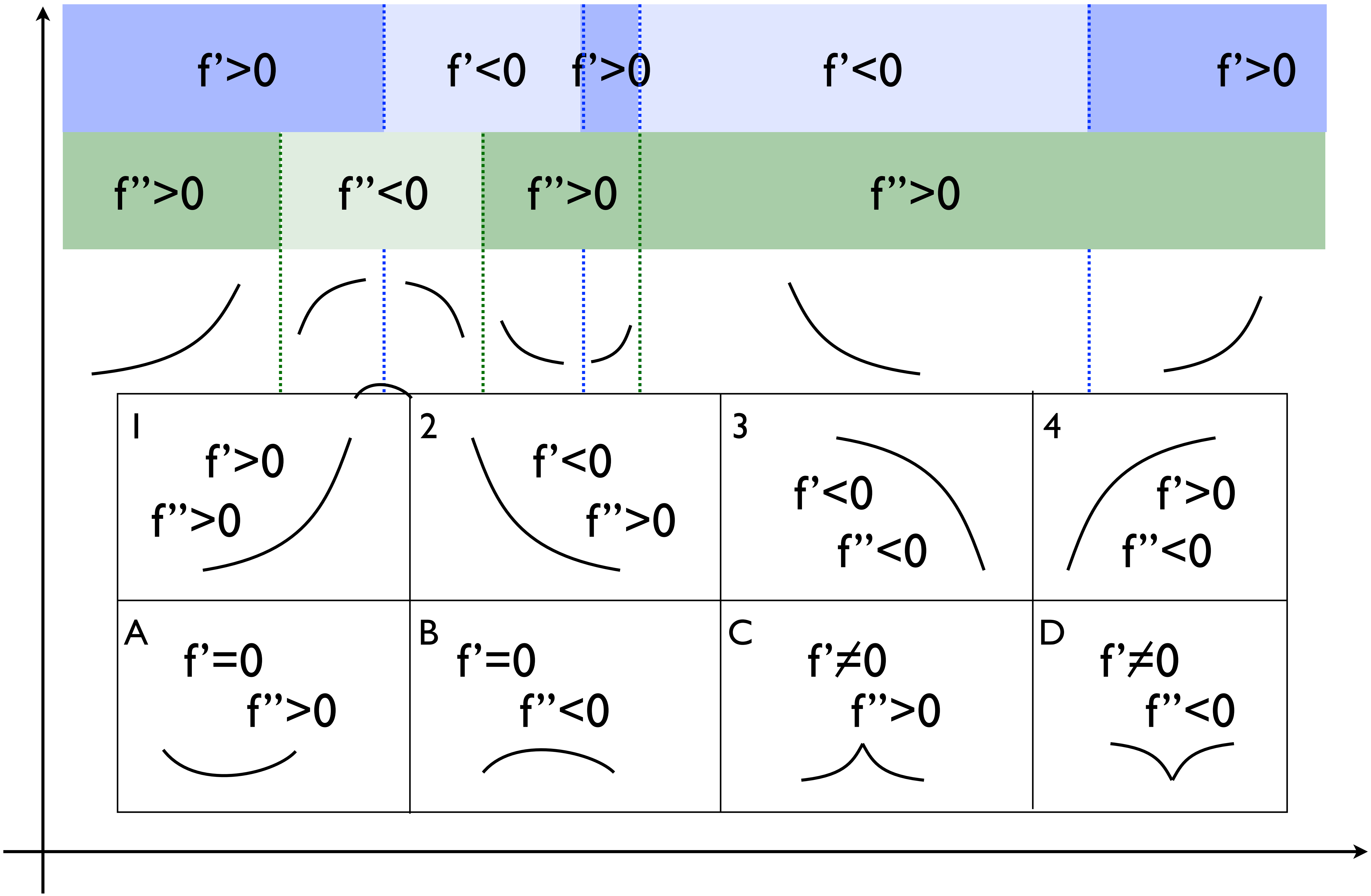

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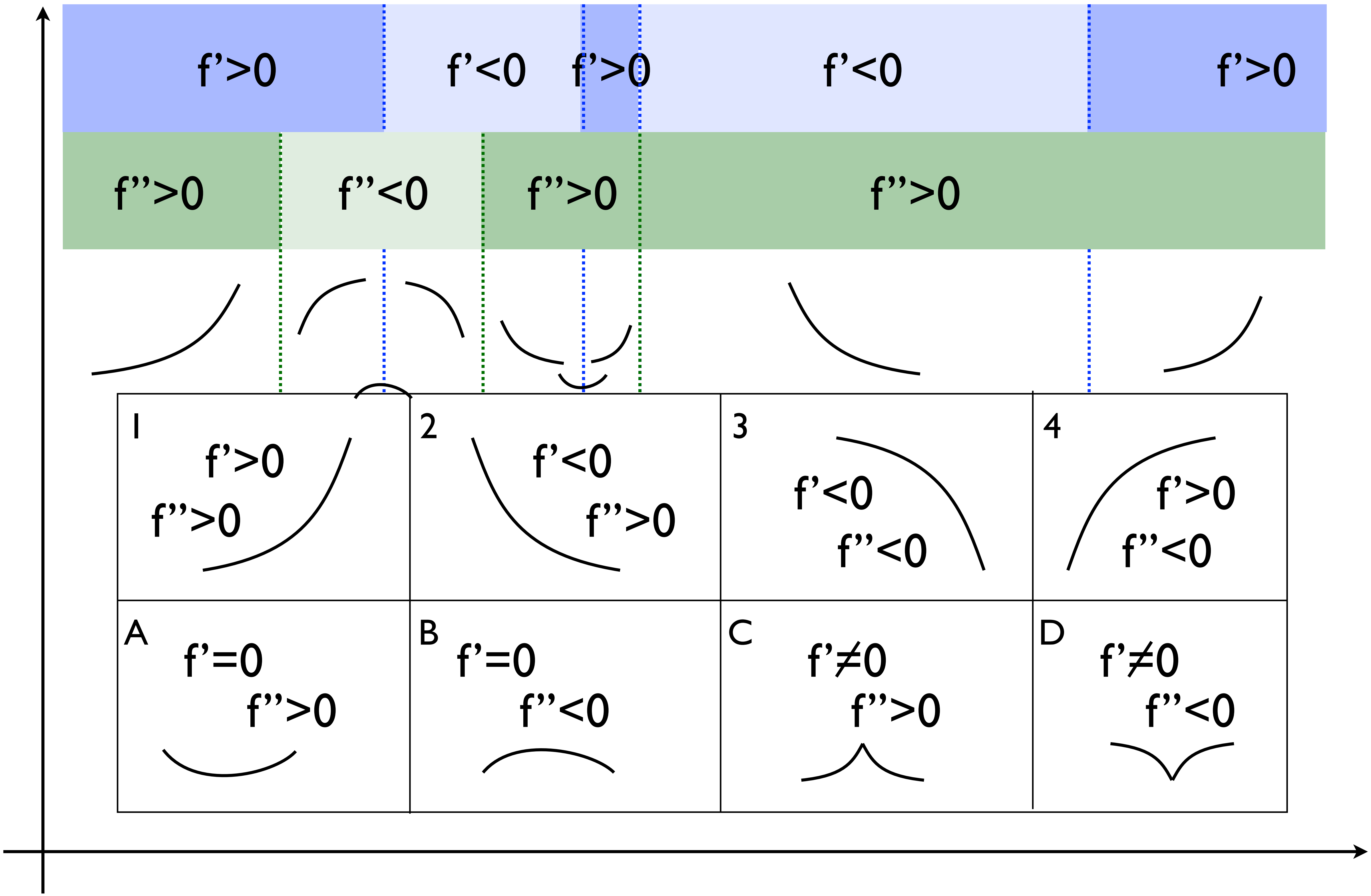

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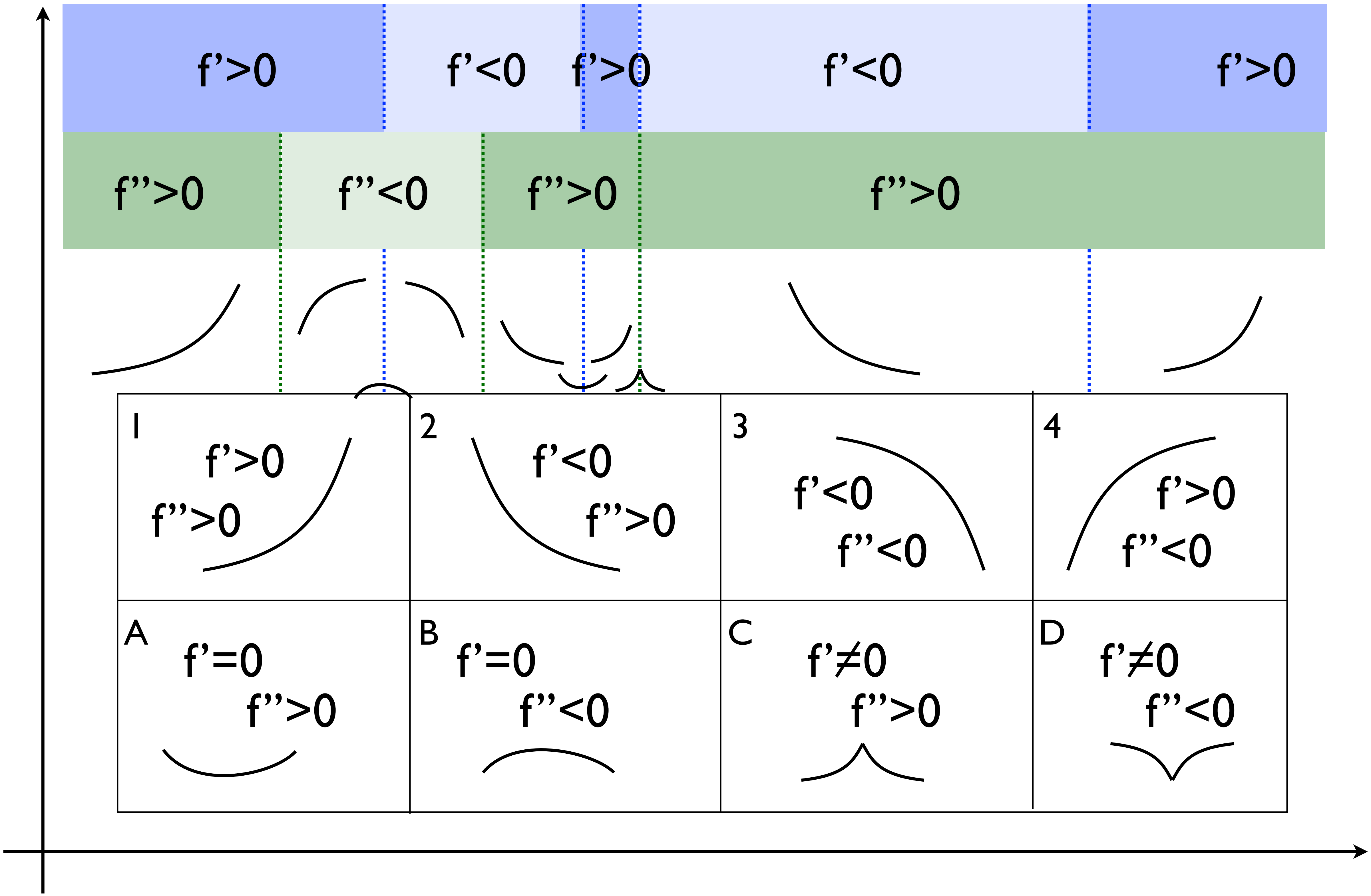

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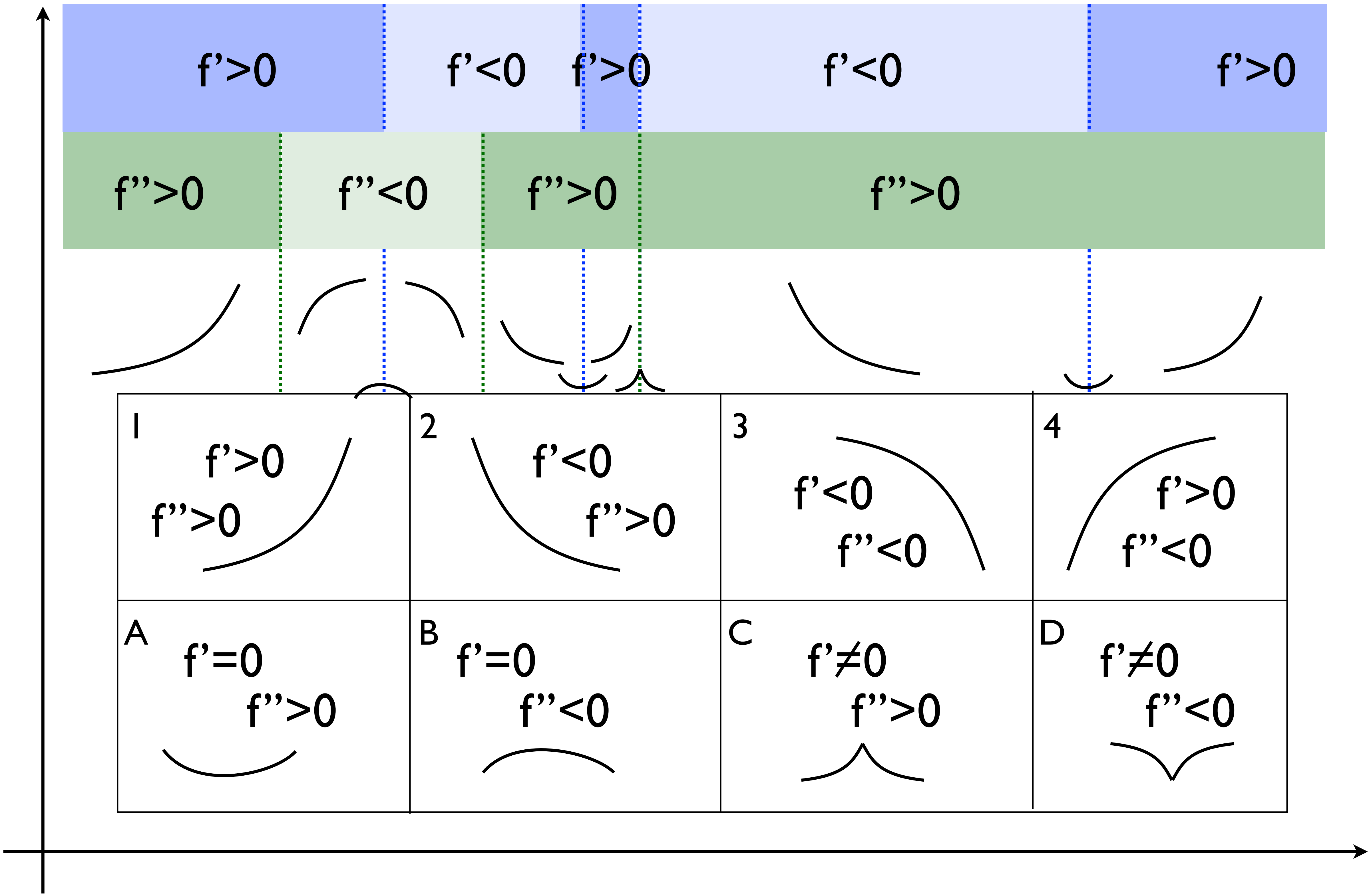

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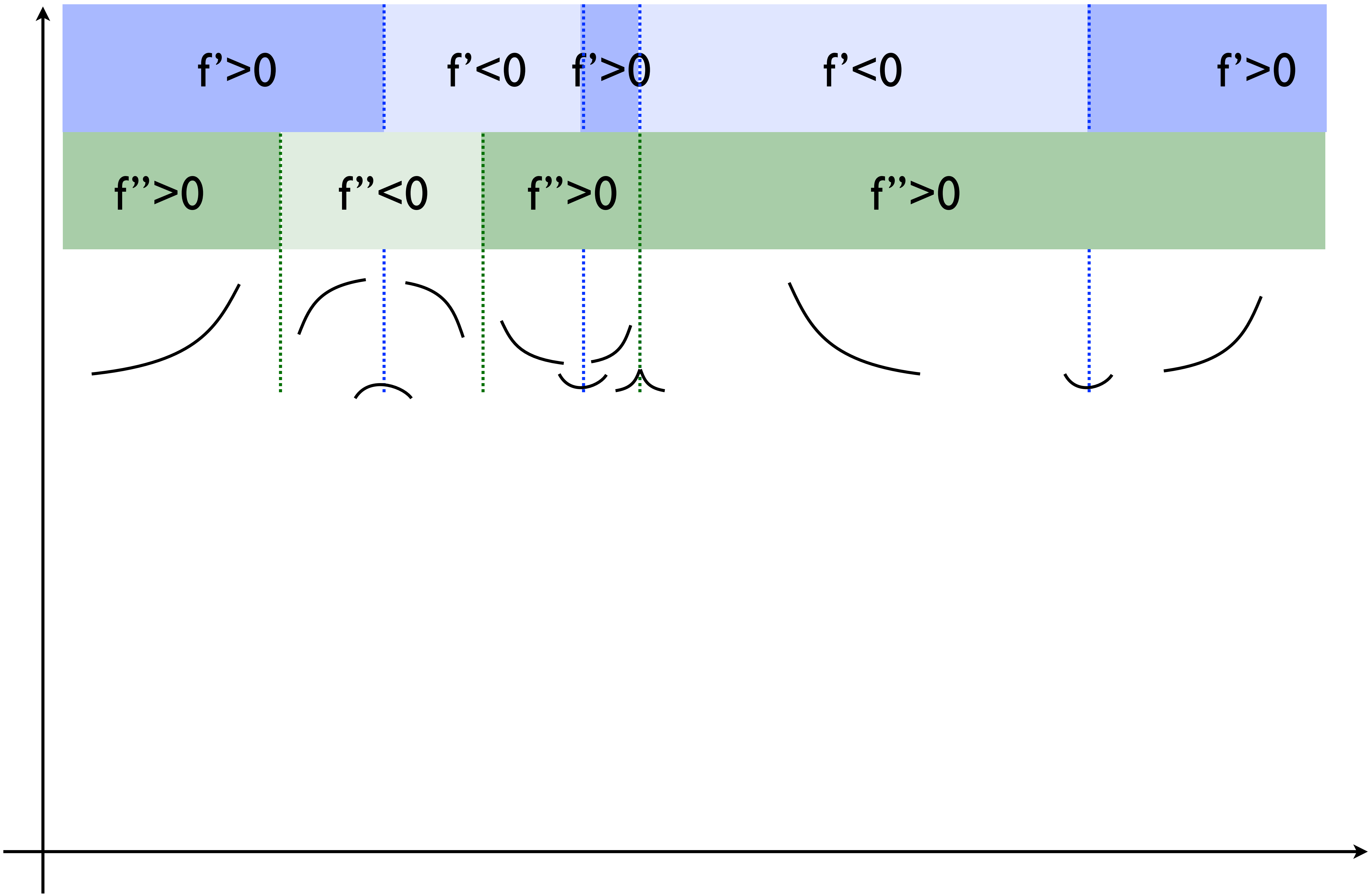

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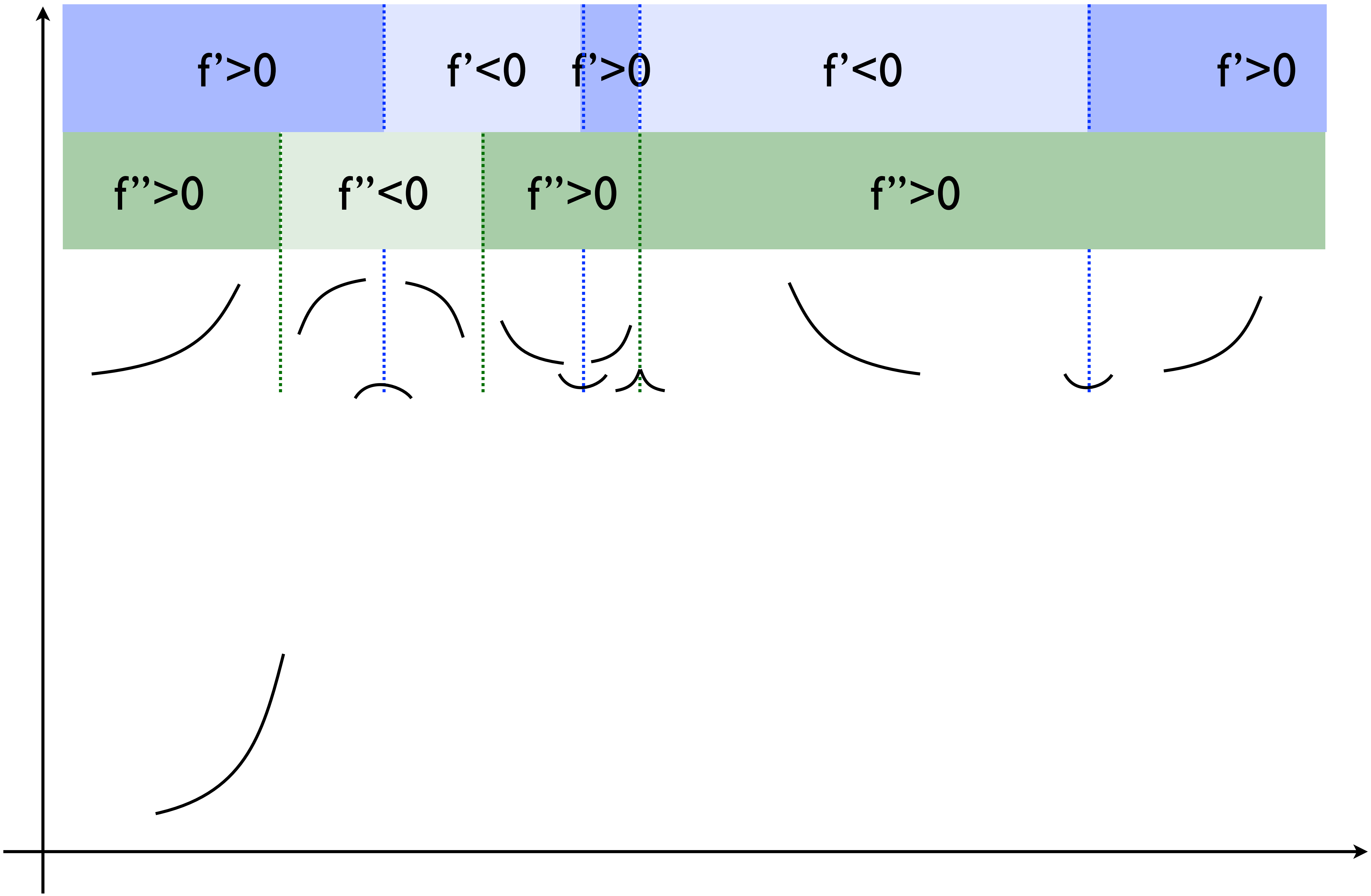

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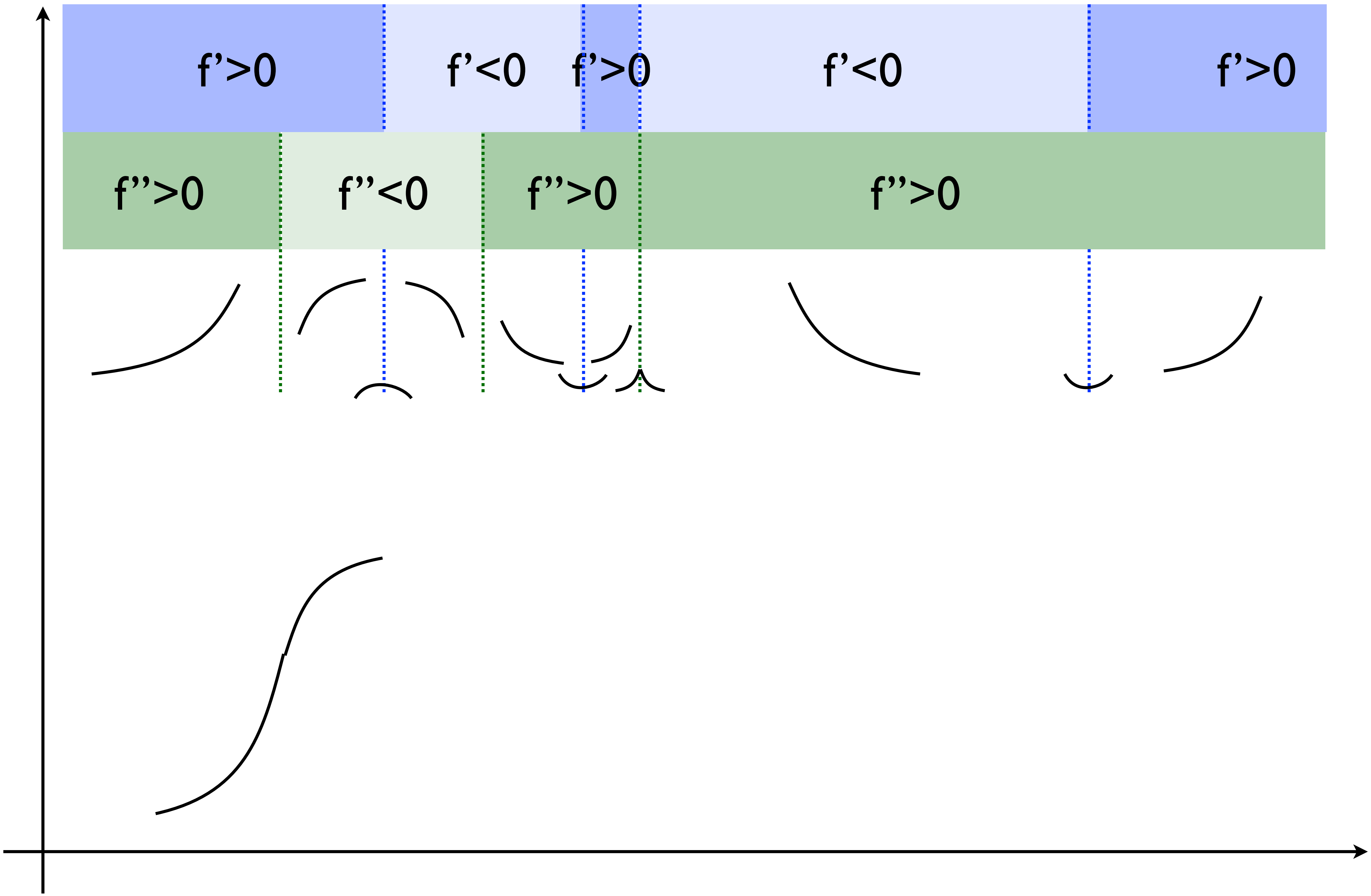

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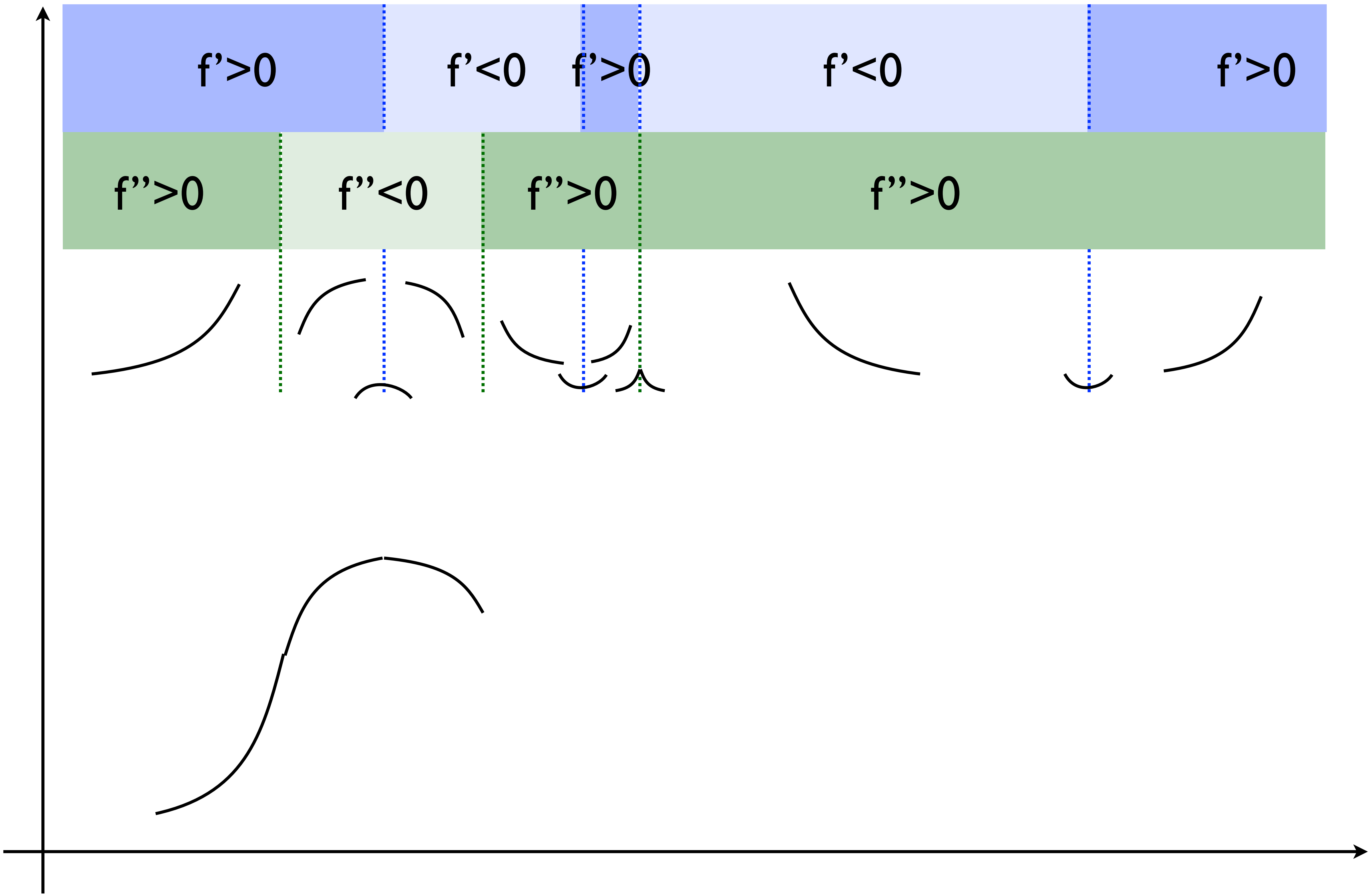

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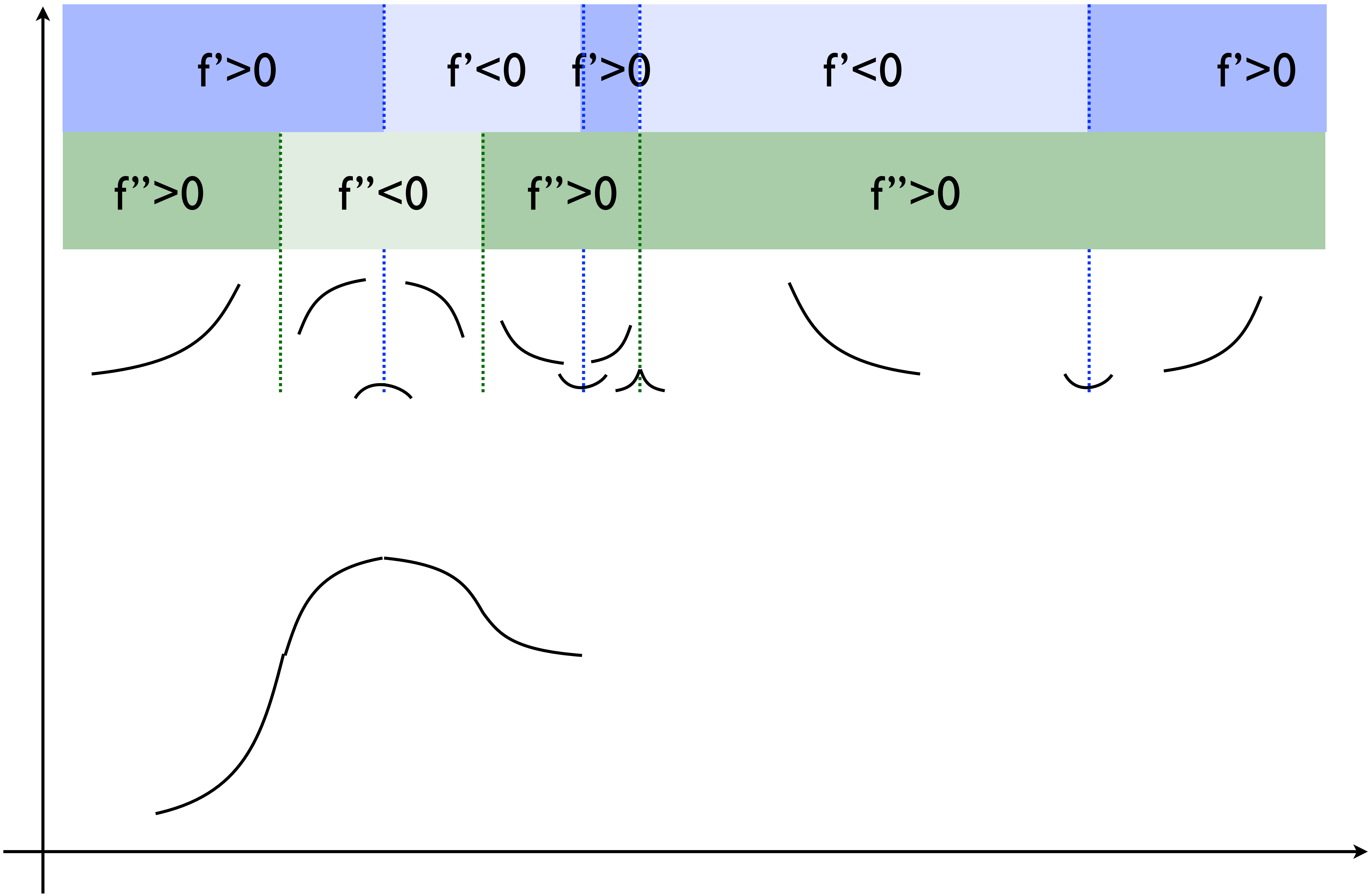

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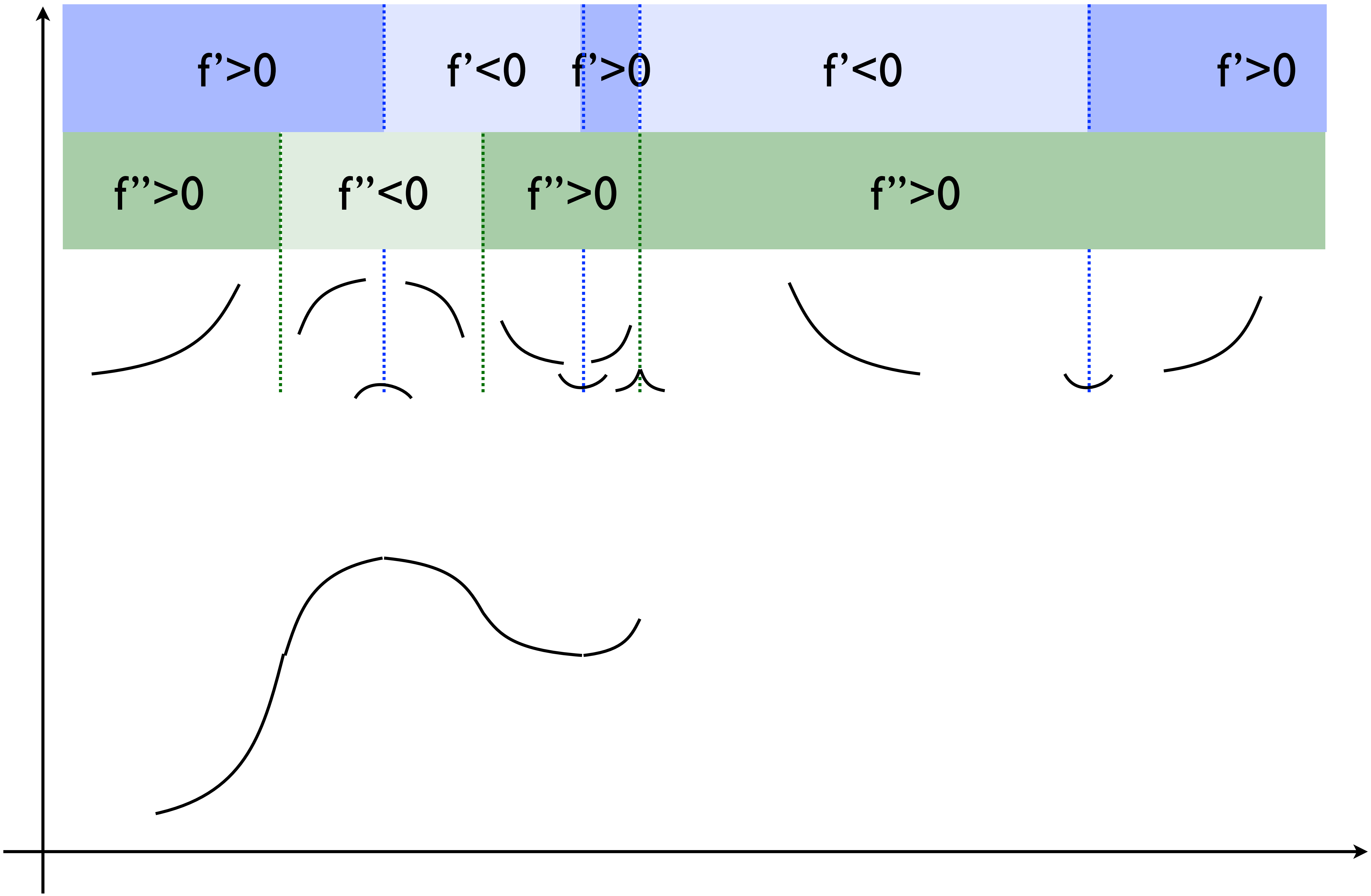

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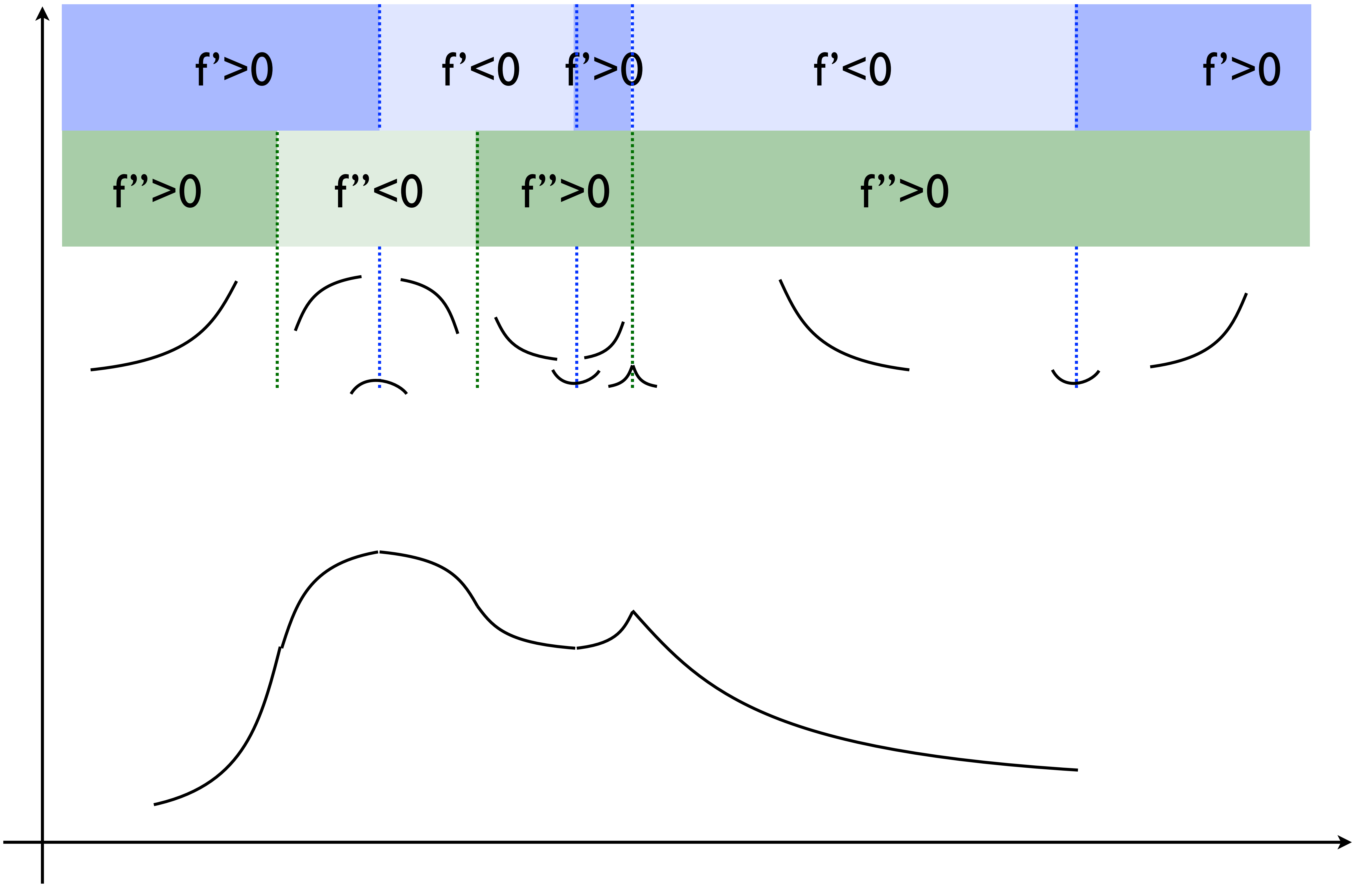

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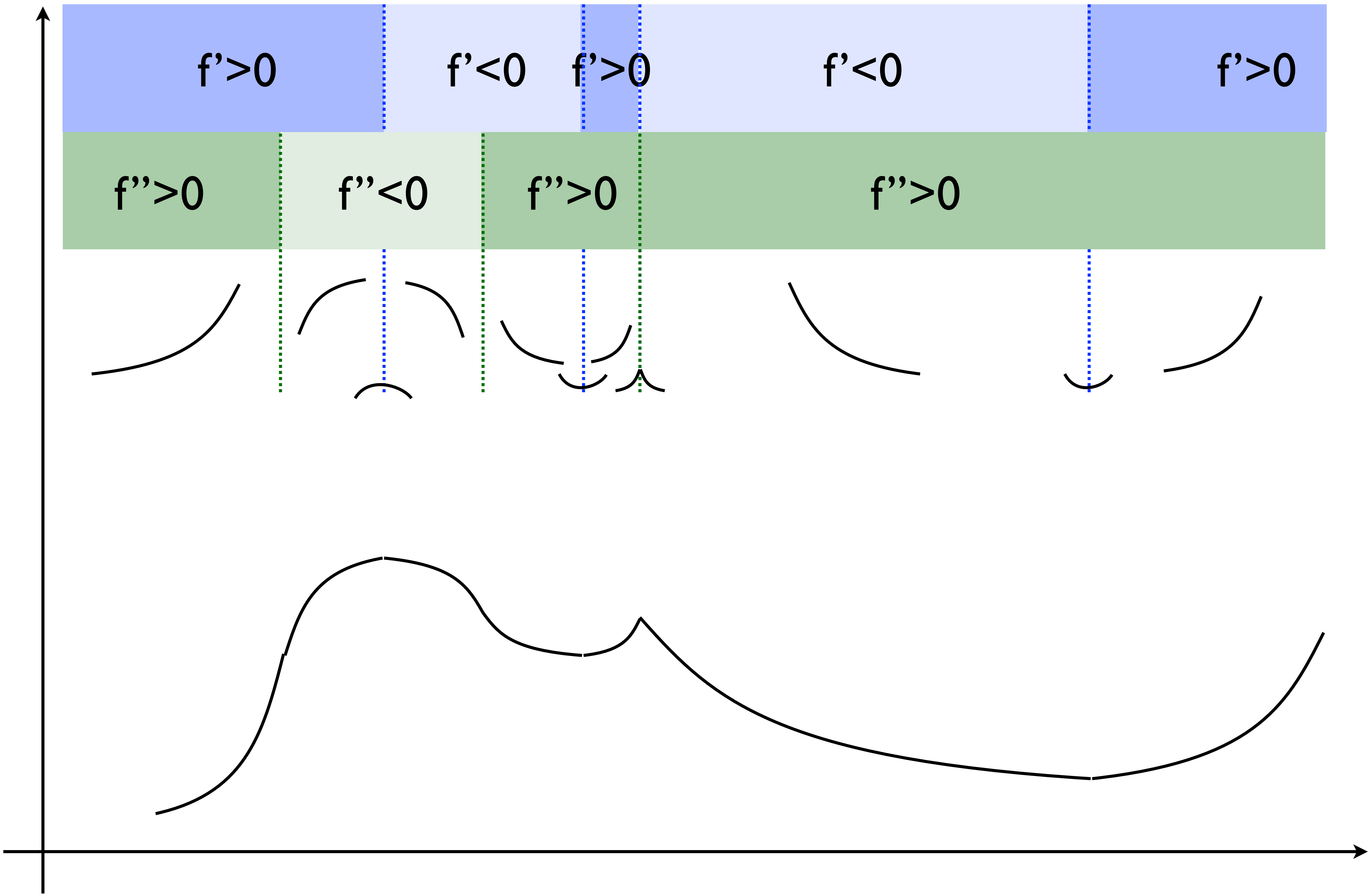

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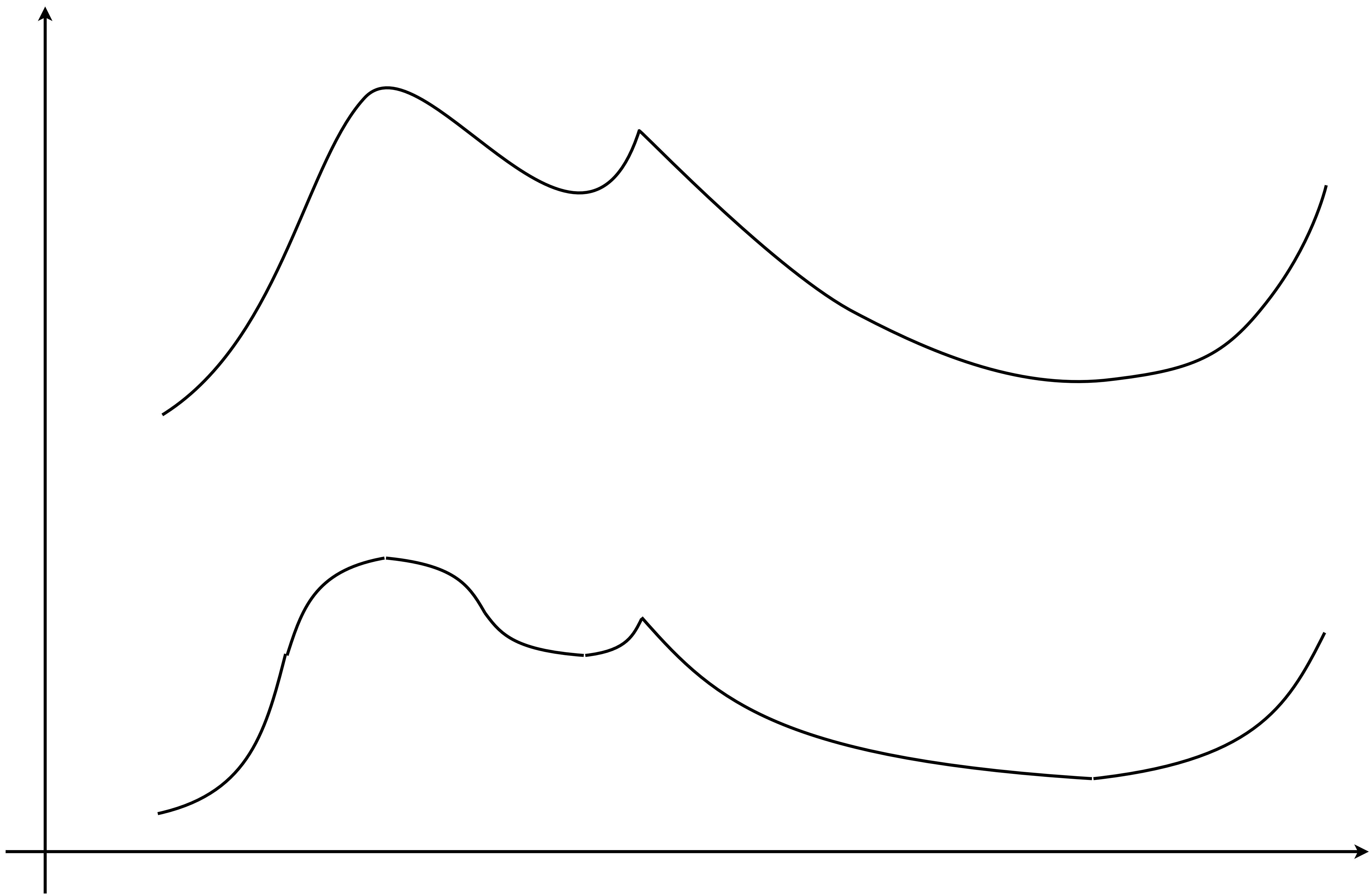

Sketch the graph of

$$f(x) = 3x^4 - 4x^3$$

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x					
f(x)					

Sketch the graph of

$f(x) = 3x^4 - 4x^3$

x		0		4/3		
f(x)		0		0		

Sketch the graph of $f(x) = 3x^4 - 4x^3$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)		0		0	

Sketch the graph of $f(x) = 3x^4 - 4x^3$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

$$f'(x) = 12(x^3-x^2)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x					
f′(x)					

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	(-∞,0)	0	(0,4/3)	4/3	(4/3,∞)
f(x)	+	0	-	0	+

x		0		1	
f′(x)		0		0	

Sk … of

$$f'(x) = 12(x^3-x^2)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
f′(x)		0		0	

Sketch the graph of

$$f'(x) = 12(x^3 - x^2)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
f′(x)	-	0	-	0	+

Sketch the graph of

$$f''(x) = 12(3x^2-2x)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
f′(x)	-	0	-	0	+

x					
f″(x)					

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$$f''(x) = 12(3x^2-2x)$$

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f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
f′(x)	-	0	-	0	+

x		0		2/3	
f″(x)		0		0	

Sketch the graph of

$$f''(x) = 12(3x^2-2x)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
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x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
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Sketch the graph of

$$f''(x) = 12(3x^2-2x)$$

x	$(-\infty,0)$	0	$(0,4/3)$	4/3	$(4/3,\infty)$
$f(x)$	+	0	-	0	+

x	$(-\infty,0)$	0	$(0,1)$	1	$(1,\infty)$
$f'(x)$	-	0	-	0	+

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
$f''(x)$	+	0	-	0	+

The whole table

x	(-∞,0)	0	(0,2/3)	2/3	(2/3,1)	1	(1,4/3)	4/3	(4/3,∞)
f(x)	+	0	-	-	-	-	-	0	+
f′(x)	-	0	-	-	-	0	+	+	+
f″(x)	+	0	-	0	+	+	+	+	+

$$f(x) = 3x^4-4x^3$$

The whole table

x	(-∞,0)	0	(0,2/3)	2/3	(2/3,1)	1	(1,4/3)	4/3	(4/3,∞)
f(x)	+	0	-	-	-	-	-	0	+
f′(x)	-	0	-	-	-	0	+	+	+
f″(x)	+	0	-	0	+	+	+	+	+

Not a min/max

$$f(x) = 3x^4-4x^3$$

The whole table

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,1)$	1	$(1,4/3)$	4/3	$(4/3,\infty)$
$f(x)$	+	0	–	–	–	–	–	0	+
$f'(x)$	–	0	–	–	–	0	+	+	+
$f''(x)$	+	0	–	0	+	+	+	+	+

Not a min/max

minimum

$$f(x) = 3x^4 - 4x^3$$

The whole table

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,1)$	1	$(1,4/3)$	4/3	$(4/3,\infty)$
f(x)	+	0	-	-	-	-	-	0	+
f′(x)	-	0	-	-	-	0	+	+	+
f″(x)	+	0	-	0	+	+	+	+	+

inflection point

$$f(x) = 3x^4-4x^3$$

The whole table

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,1)$	1	$(1,4/3)$	4/3	$(4/3,\infty)$
$f(x)$	+	0	−	−	−	−	−	0	+
$f'(x)$	−	0	−	−	−	0	+	+	+
$f''(x)$	+	0	−	0	+	+	+	+	+

inflection point

$-4x^3$

The whole table

x	(-∞,0)	0	(0,2/3)	2/3	(2/3,1)	1	(1,4/3)	4/3	(4/3,∞)
f(x)	+	0	-	-	-	-	-	0	+
f′(x)	-	0	-	-	-	0	+	+	+
f″(x)	+	0	-	0	+	+	+	+	+

$$f(x) = 3x^4-4x^3$$

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

Can't tell if $f''(x)$ is inc. or dec. near $x=0$!

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

Not sure about (C)? Try this for $f(x)=x^5$.