Today

- Additional office hours Tuesday 1:30-3:30 pm.
- Absolute extrema.
- More on inflection points.
- Sketching using derivative information.

Absolute extrema

- A continuous function on a closed interval [a,b] takes on its highest (lowest) value either at a local maximum (minimum) or at an end point ($x=a$ or $x=b$). Call this an absolute maximum (minimum).
- When looking for absolute extrema, check critical points AND end points!

Where does $f(x)=x^3-x^2$ take on its absolute minimum on the interval [-1,2]?

(A) $x=-1$

(B) $x=0$

(C) $x=2/3$

(D) $x=2$

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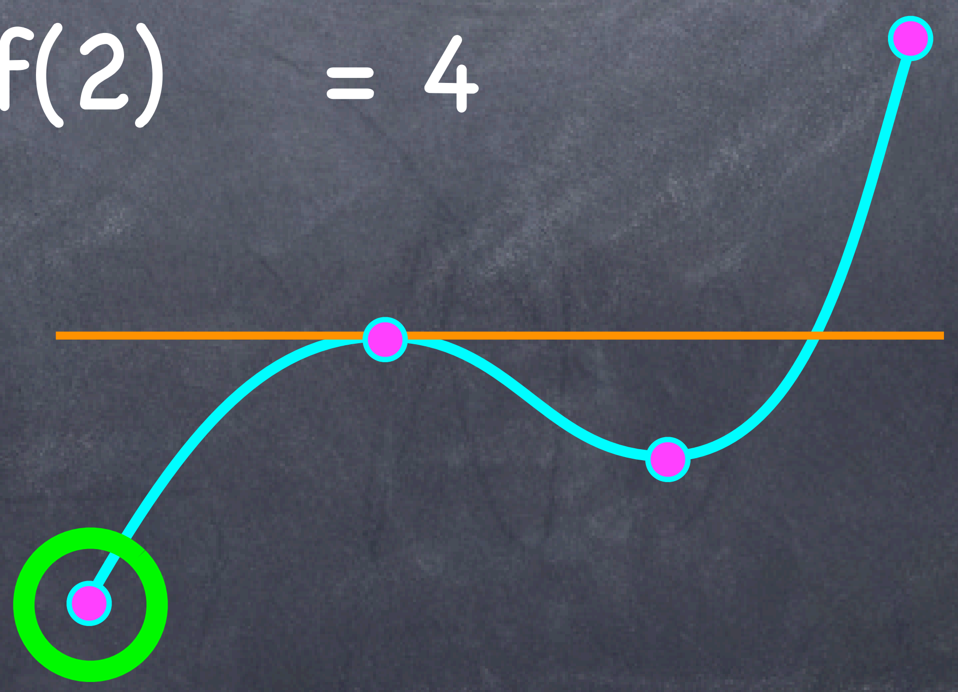
$f(-1) = -2$

$f(0) = 0$

$f(2/3) = -4/27$

$f(2) = 4$



Back to $f(x) = 3x^4-4x^3$

- $f'(x) = 12\ (x^3 - x^2) = 0$ --> $x=0$, $x=1$.
- $f''(x) = 12\ (3x^2 - 2x)$.
- $f''(0) = 0$ --> inflection point? maybe!!!
- $f''(1) = 1 > 0$
  - --> $f'(x)$ is increasing near $x=1$.
  - --> slope of $f(x)$ is increasing near $x=1$.
  - --> $f(x)$ has a minimum at $x=1$.

Is x=0 an inflection point?

(A) Yes because $f''(0)=0$.

(B) Yes because $f''(0)=0$ and $f'''(0)<0$.

(C) No because $f''(-1)=60$ and $f''(1)=12$.

(D) Yes because $f''(-1)=60$ and $f''(1/2)=-3$.

Is x=0 an inflection point?

(A) Yes because $f''(0)=0$.

(B) Yes be… .

(C) No be… 2.

(D) Yes because $f''(-1)=60$ and $f''(1/2)=-3$.

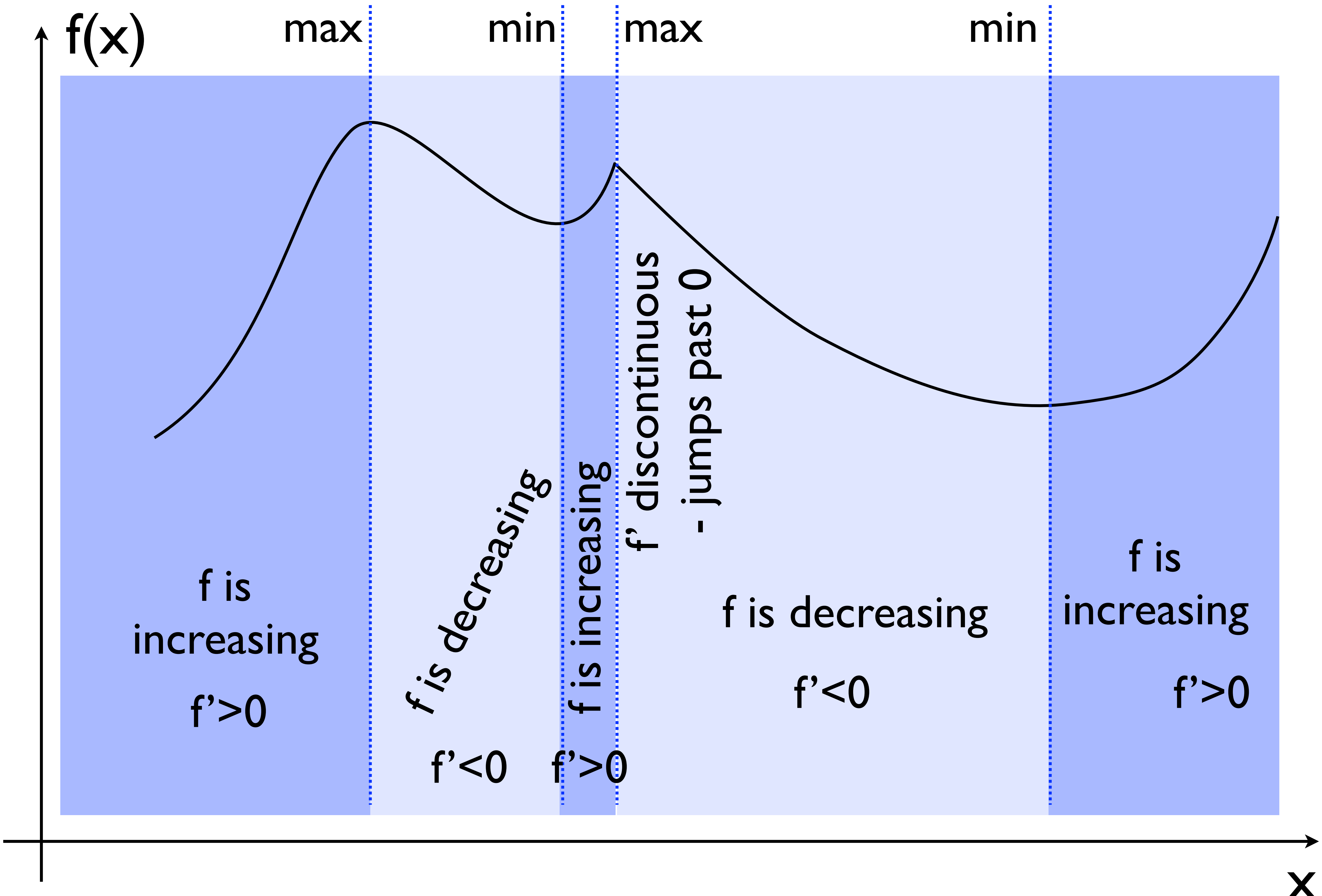
DANGER - f″ might also change sign at a vertical asymptote or a point at which f′ or f″ DNE.

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
$f''(x)$	+	0	-	0	+

$f'''(0)<0$

Using f, f′ and f″ to graph f

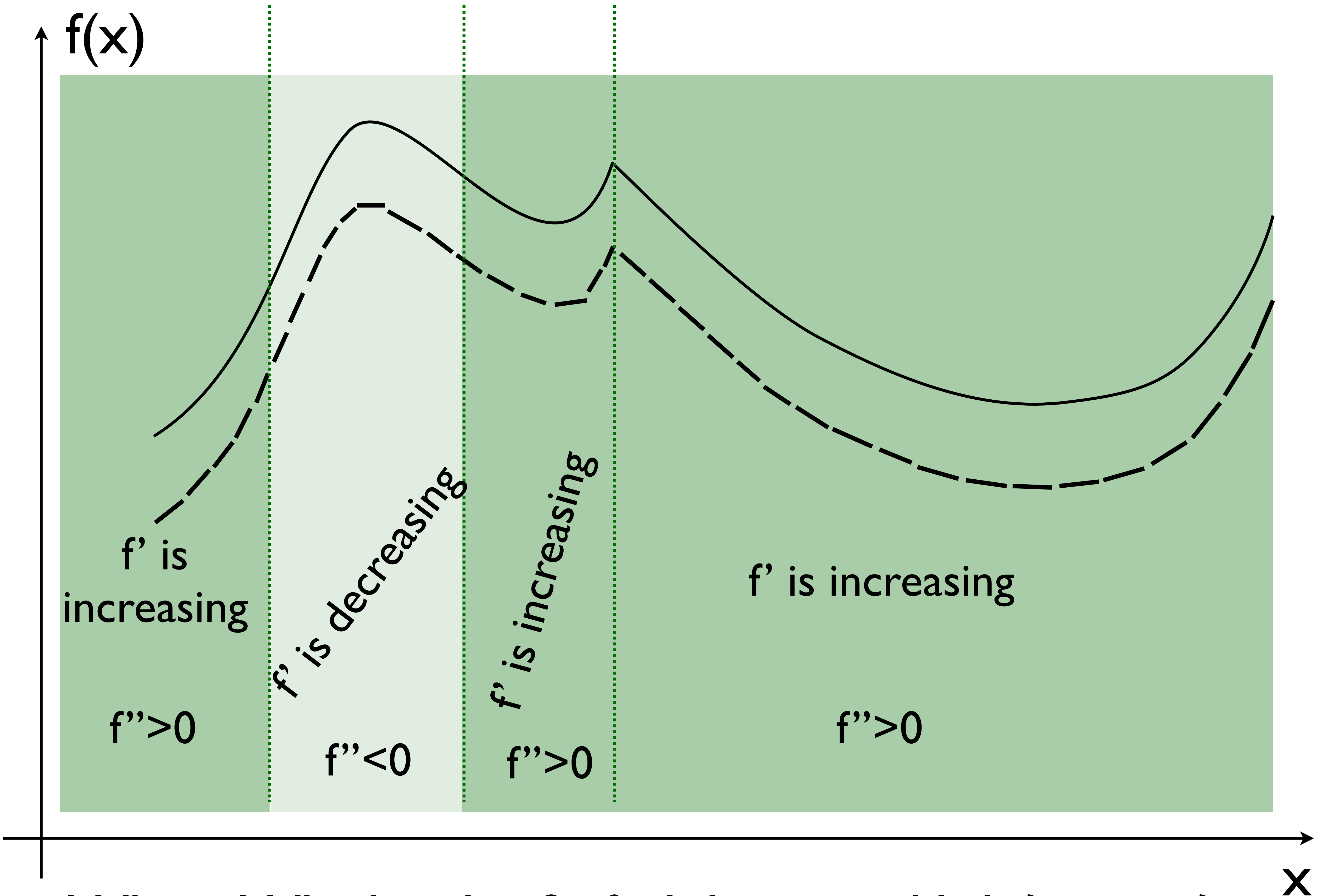
Annotating the graph of f(x) with f'(x) info



What does that mean for the derivative?

What is the function doing?

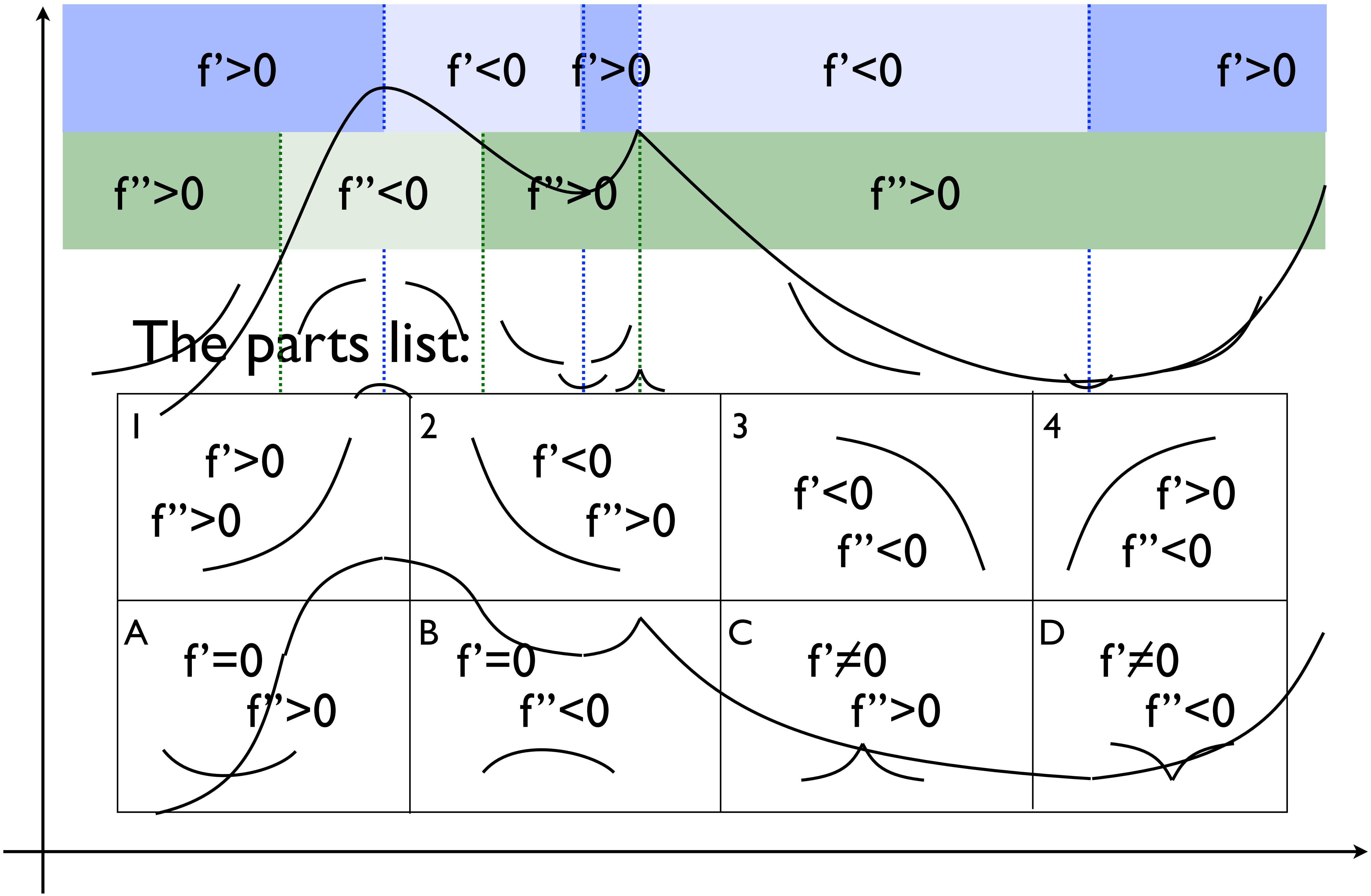
Annotating the graph of f(x) with f''(x) info



What does this mean for the second derivative?

What is the first derivative doing?

What you have to do to graph it.



Sketch the graph of

$$f''(x) = 12(3x^2-2x)$$

x	(−∞,0)	0	(0,4/3)	4/3	(4/3,∞)
f(x)	+	0	−	0	+

x	(−∞,0)	0	(0,1)	1	(1,∞)
f′(x)	−	0	−	0	+

x	(−∞,0)	0	(0,2/3)	2/3	(2/3,∞)
f″(x)	+	0	−	0	+

The whole table

x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,1)$	1	$(1,4/3)$	4/3	$(4/3,\infty)$
$f(x)$	+	0	–	–	–	–	–	0	+
$f'(x)$	–	0	–	–	–	0	+	+	+
$f''(x)$	+	0	–	0	+	+	+	+	+

Not a min/max

minimum

$-4x^3$

inflection inflection point

Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

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(C) $f'''(0) = 0$ so no.

Can't tell if $f''(x)$ is inc. or dec. near $x=0$!

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

Not sure about (C)? Try this for $f(x)=x^5$.