

# Today...

- Reminders- OSH 2 Monday! WW2 Thurs 7AM!
- Hole-in-graph examples - derivative at a point.
- Limit properties.
- Examples - continuous, infinite, indeterminate.
- Ensuring continuity (like OSH 2 #1).
- Limits at infinity (asymptotes).



# Examples in which $f'(2)$ does not exist

$$f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$$

On the board...





# Limit properties

1.

$$\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

2.

$$\lim_{x \rightarrow a} (cf(x)) = c \lim_{x \rightarrow a} f(x)$$

3.

$$\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \left( \lim_{x \rightarrow a} f(x) \right) \cdot \left( \lim_{x \rightarrow a} g(x) \right)$$

4. Provided that  $\lim_{x \rightarrow a} g(x) \neq 0$ , we also have that

$$\lim_{x \rightarrow a} \left( \frac{f(x)}{g(x)} \right) = \left( \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \right).$$



# Continuous functions

- Some examples:
  - Polynomials
  - Exponentials
  - $\sin$ ,  $\cos$ ,  $\tan$  (cont. at all points in its domain)



$$\lim_{x \rightarrow 2} (x^3 - 2x + 1)$$

(A) 1

(B) 2

(C) 5

(D)  $\infty$

Continuous at 2.



$$\lim_{x \rightarrow 2} \frac{1}{x^2 - 2}$$

- (A) 1
- (B) 2
- (C) 5
- (D)  $\infty$

Continuous at 2.



$$\lim_{x \rightarrow 2} \frac{1}{(x - 2)^2}$$

(A) 0

(B) 4

(C)  $\infty$

(D)  $-\infty$

(E) Does not exist

$$f'(2) = \lim_{x \rightarrow 2} x + 4 = 6$$

$$f'(2) = \lim_{x \rightarrow 2} \frac{1}{(x - 2)^2} = \infty$$

Infinite limit or singular at 2.



$$\lim_{x \rightarrow 2} \frac{1}{x - 2}$$

(A) 0

(B) 4

(C)  $\infty$

(D)  $-\infty$

(E) Does not exist

Infinite limit or singular at 2.



$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

(C)  $\infty$

(D)  $-\infty$

(E) Does not exist

Indeterminate form "0/0".



$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C)  $\infty$

(D)  $-\infty$

(E) Does not exist

Indeterminate form "0/0".



$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

(B) 4

(C)  $\infty$

(D)  $-\infty$

(E) Does not exist

Indeterminate form "0/0".



# Summary

1. Continuous.
2. Rational, nonzero denominator.
3. Rational, zero denominator, nonzero numerator:
  - asymptote, same direction ( $\rightarrow +/\infty$ ).
  - asymptote, opposite directions (DNE) – emphasize diff  $\lim = 2$  and  $\lim = \infty$ .
4. Rational, zero denominator, zero numerator:
  - numer. saves the day (hole-in-graph).
  - numer. goes overkill saving the day (0).
  - numer. unable to save the day (see 3. for cases.)



# Summary

1.  $\lim (x \rightarrow 2)$  of  $x^3 - 2x + 1$ .
2.  $\lim (x \rightarrow 2)$  of  $x/(x^2 - 1)$ .
3. Rational, zero denominator, nonzero numerator:
  - $\lim (x \rightarrow 2) 1/(x-2)^2 = \text{infinity}$ .
  - $\lim (x \rightarrow 2) 1/(x-2)$  does not exist.
4. Rational, zero denominator, zero numerator:
  - $\lim (x \rightarrow 2) (x^2 - 4)/(x-2) = 4$ .
  - $\lim (x \rightarrow 2) (x^2 - 4x + 4)/(x-2) = 0$ .
  - $\lim (x \rightarrow 2) (x-2)/(x^2 - 4x + 4)$  does not exist.



# Ensuring continuity

• For what value of  $a$  is the following function continuous at all points  $x$ ?

(A)  $a=2$

(B)  $a=-2$

(C)  $a=0$

(D)  $a=1$

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$