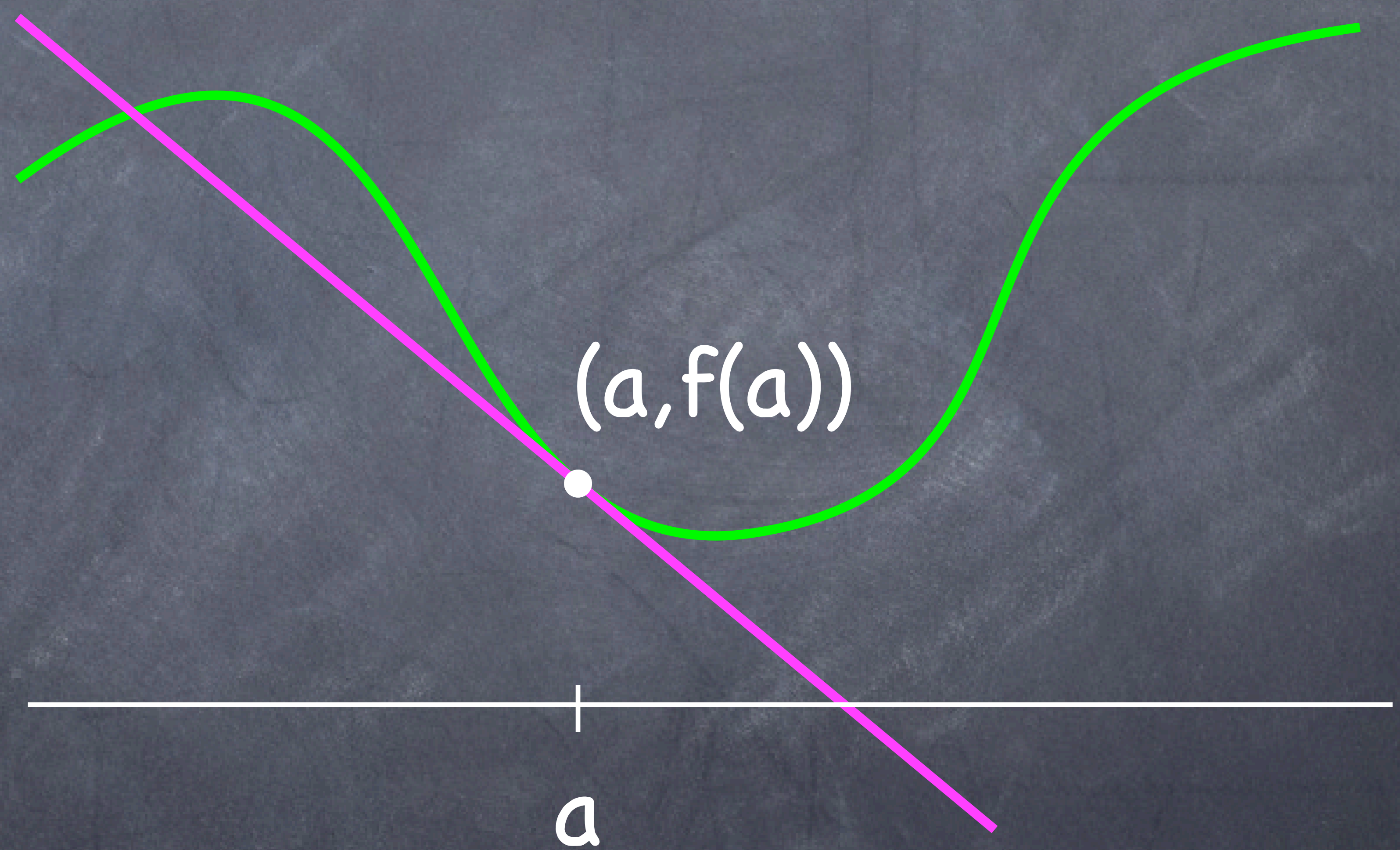
Today

- Tangent lines
- Linear approximation
- Reminders:
  - OSH 3 on Monday
  - Midterm 1 on Tuesday @ 6pm
  - Assignment 4a - due Tuesday @ 7am
  - Assignment 4b - due Friday @ 5 pm

Find the tangent line to f(x) at (a,f(a)).



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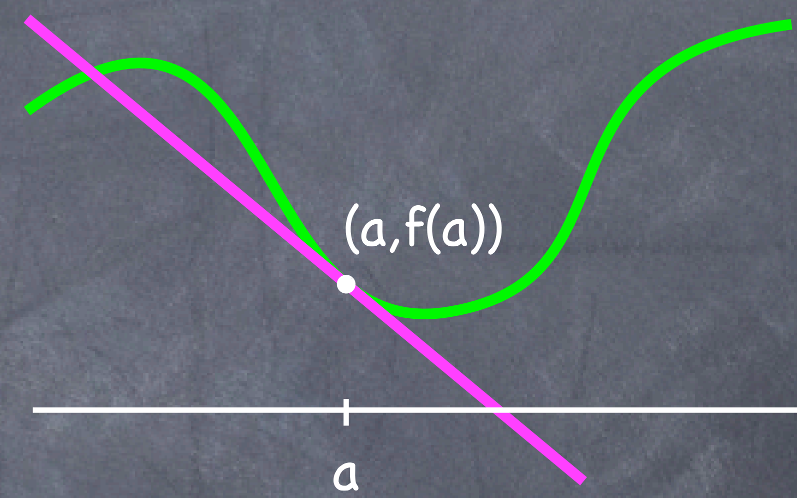

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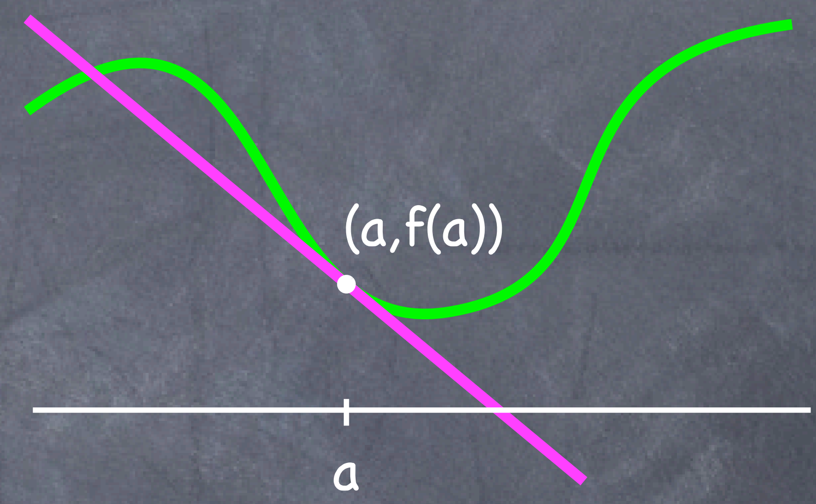

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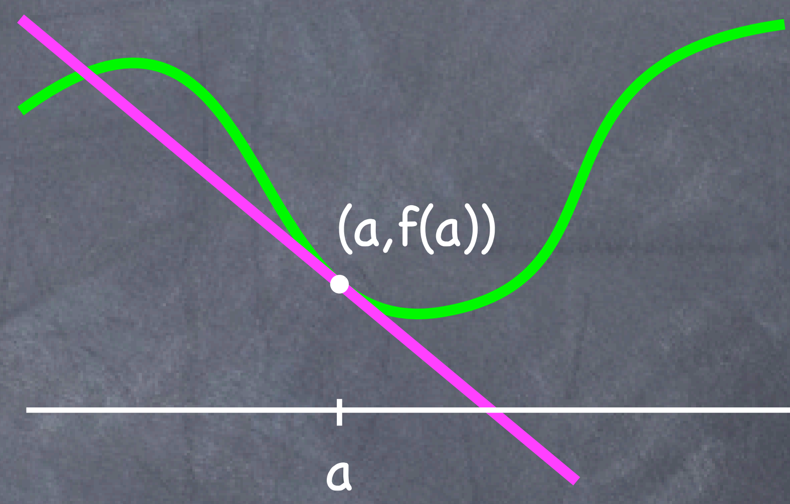

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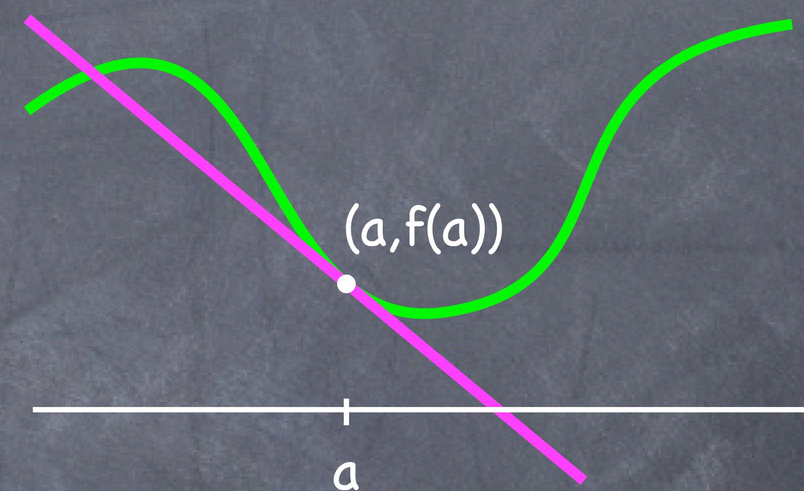

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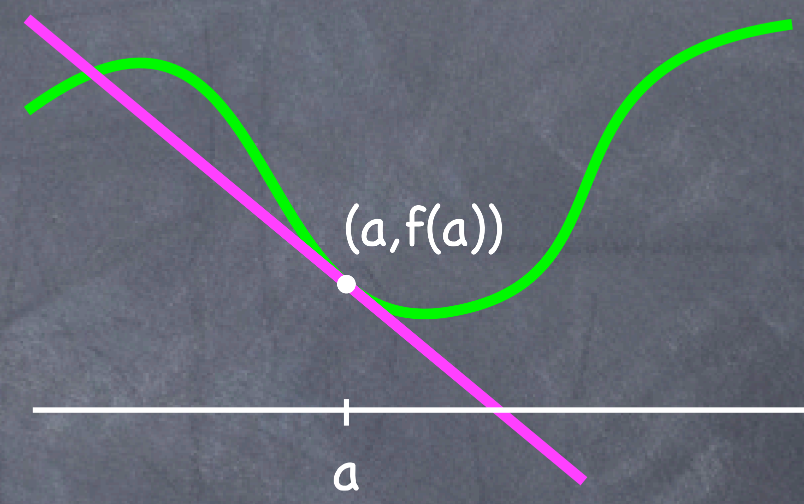

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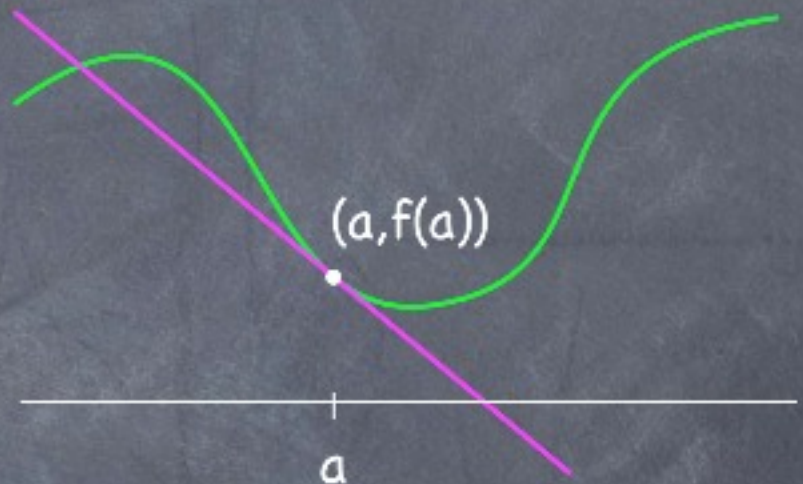

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If $x = a$ then $y = f(a)$, so the line goes through $(a, f(a))$. It also has slope $f'(a)$.

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(B) $y = x$

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$a=0$,

$f(a)=0$,

$f'(a) = 1$.

From midterm 1, 2013

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(A) $b = a - \frac{f(a)}{f'(a)}$,

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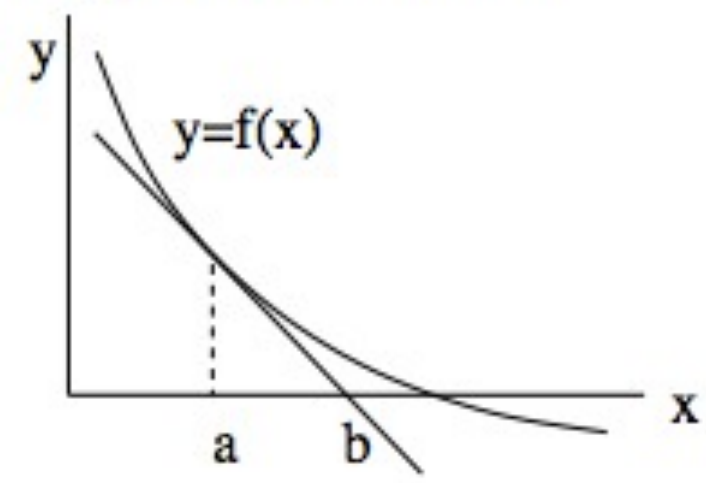

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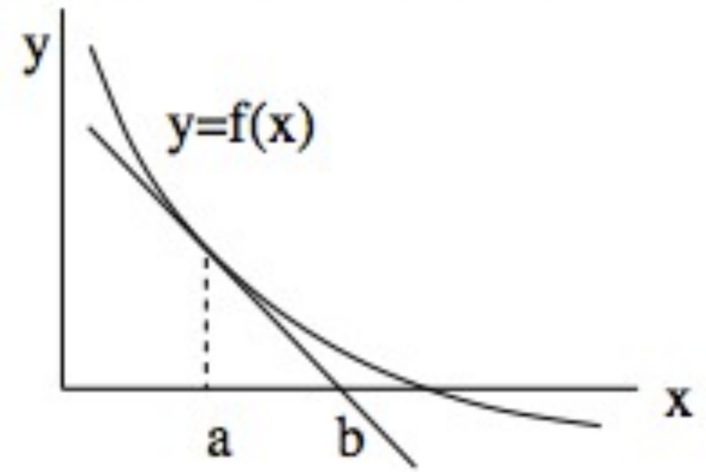

- y = f′(a) (x–a) + f(a)

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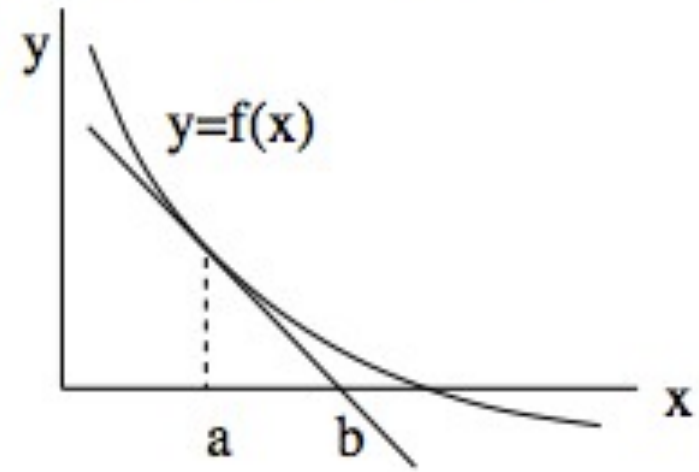

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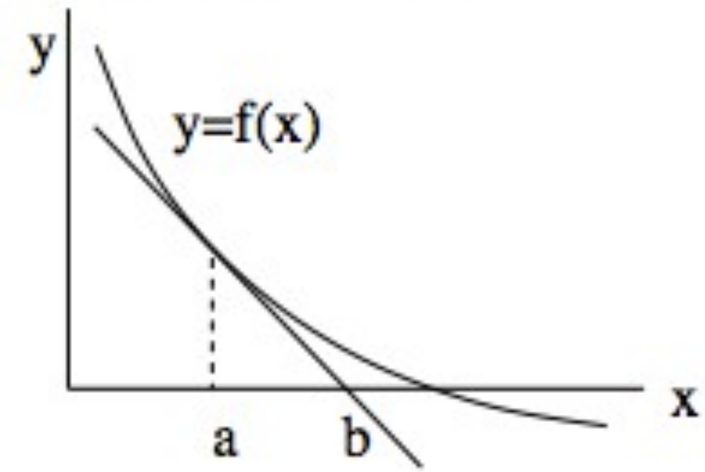

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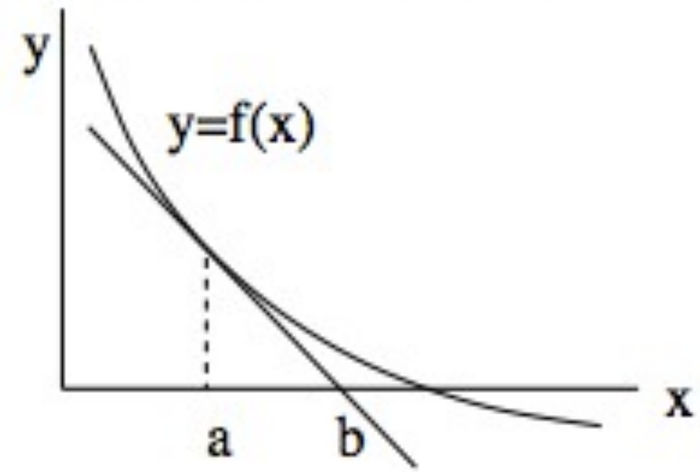

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- Some piece of information is missing - could be from any of these.

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(B) $y = 38x + 44$

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$3a^2 + 4a - 1 = -1$

$3a^2 + 4a = 0$

$a = 0, -4/3$

$y = -x + 2$

Example - even harder

- Find tangent line to $f(x)=x^2$ that goes through (1,-1). Note: $f(1) \neq -1$!!

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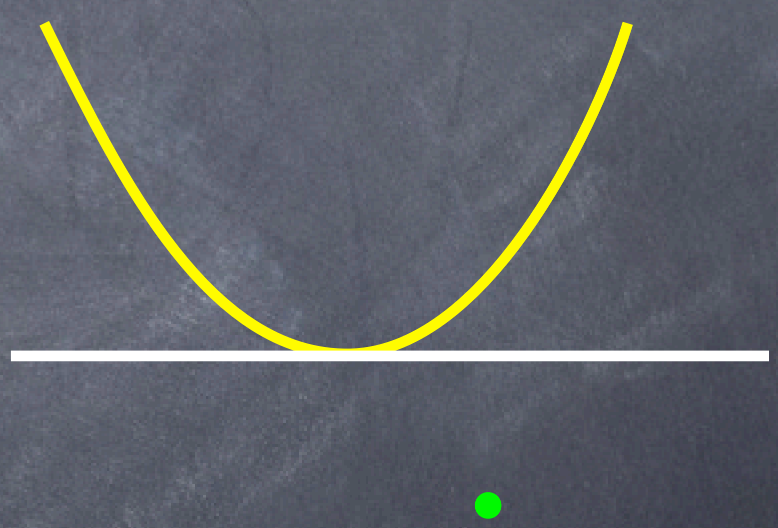

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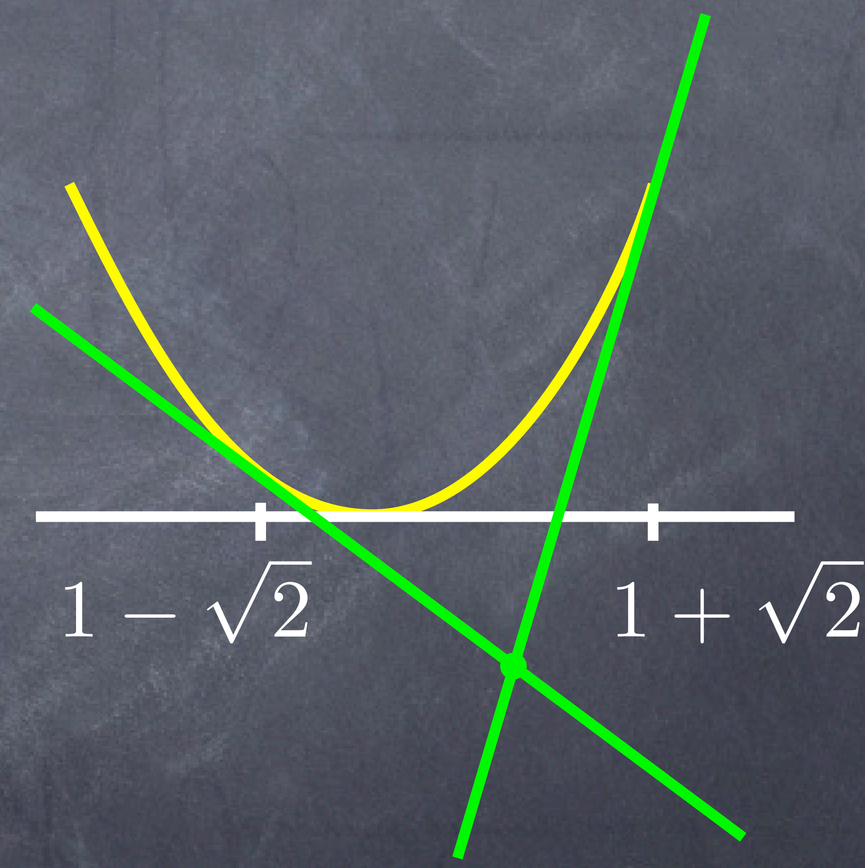

Work through it on your own!