Today

- Survey - check.
- Annual sunlight (continued)
- Derivatives of sin(x), cos(x)
- Related rates with trig (if time)
- Reminders:
    - Teaching evals

Annual variation in daylight per day in Vancouver (Jan 1 --> t=0)

(A) $$L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t-172)\right)$$

(B) $$L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t+80)\right)$$

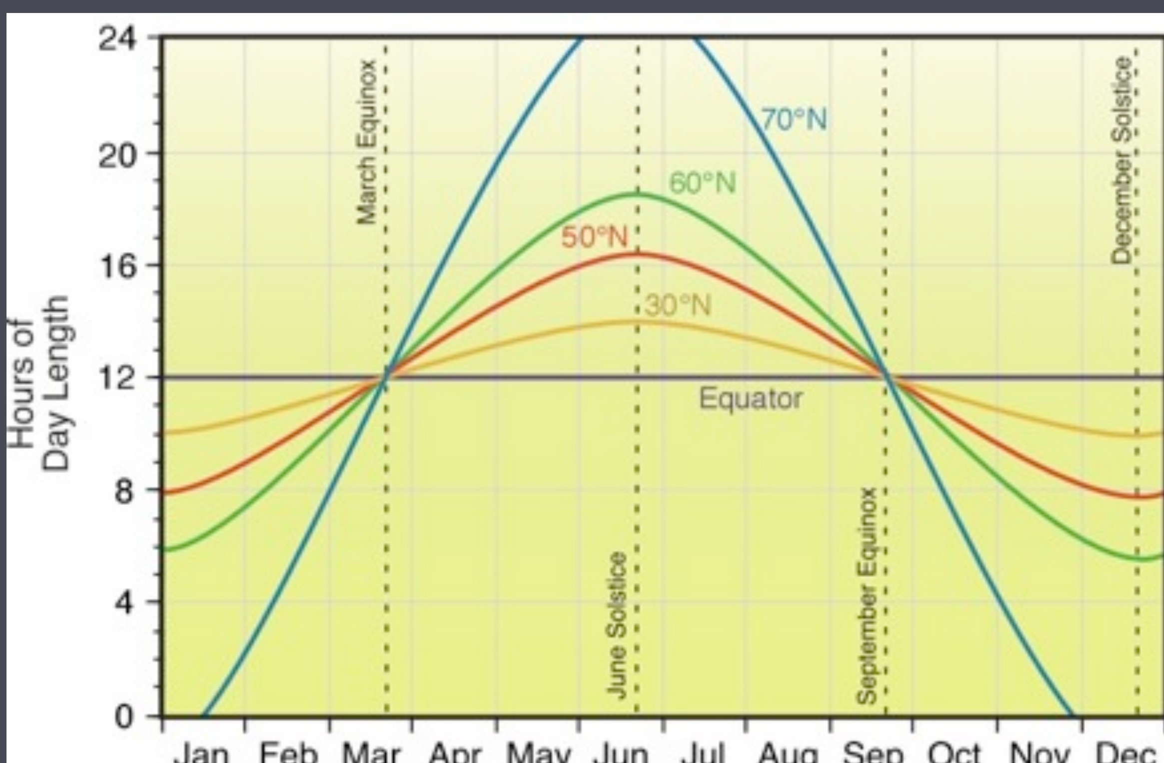
(C) $$L(t) = 12 + 4\cos\left(\frac{2\pi}{365}(t-172)\right)$$

(D) $$L(t) = 12 - 4\sin\left(\frac{2\pi}{365}(t-80)\right)$$

Note: t=172 is June 21; t=80 is March 21.

Annual variation in daylight per day in Vancouver (Jan 1 --> t=0)

(A) $$L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t-172)\right)$$



$$+ 4\sin\left(\frac{2\pi}{365}(t+80)\right)$$

$$+ 4\cos\left(\frac{2\pi}{365}(t-172)\right)$$

$$- 4\sin\left(\frac{2\pi}{365}(t-80)\right)$$

Note: t=172 is June 21; t=80 is March 21.

Derivative of f(x)=sin(x)

- $f'(x) = \lim_{h\to 0} (f(x+h) - f(x))/ h$

$= \lim_{h\to 0} (\sin(x+h) - \sin(x)) /h$

$= \lim_{h\to 0} (\sin(x)\cos(h)+\cos(x)\sin(h) -\sin(x)) /h$

$= \lim_{h\to 0} (\sin(x) (\cos(h)-1)/h + \cos(x)\sin(h) /h)$

$= \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

$+ \cos(x) \lim_{h\to 0} \sin(h) /h$

See what h=0.0001 gives...

$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

Note: this last step requires a bunch of work to show.

More details on that last step (not shown in class)

- $f'(x) = \sin(x) \lim_{h\to 0} (\cos(h)-1)/h$

  $+ \cos(x) \lim_{h\to 0} \sin(h)\ /h$

First, we simplify $\lim_{h\to 0} (\cos(h)-1)/h$

$= \lim_{h\to 0} (\cos^2(h)-1)/((\cos(h)+1)h)$

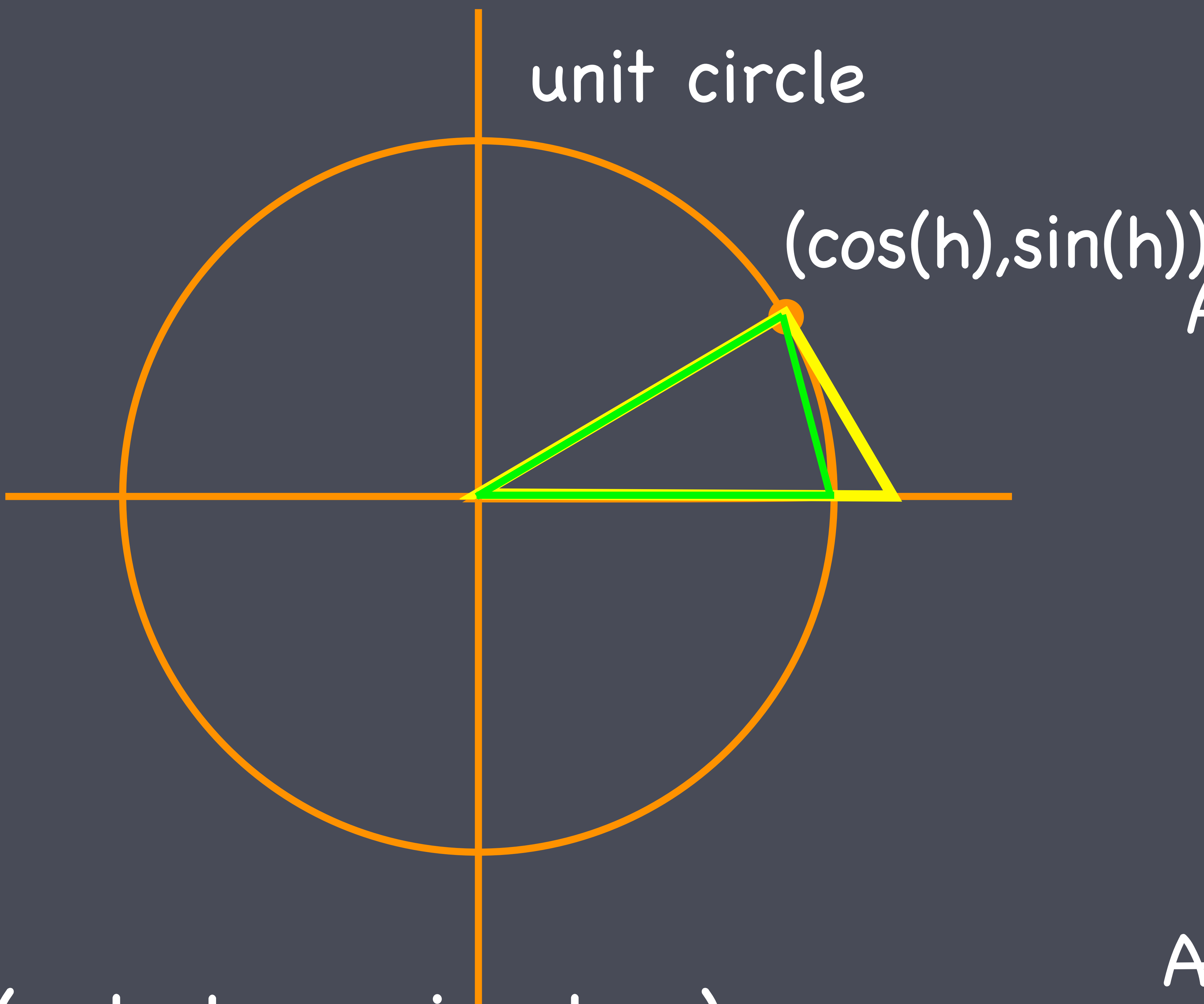
$= \lim_{h\to 0} (-\sin^2(h))/((\cos(h)+1)h)$

$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \sin(h)/h$

$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \lim_{h\to 0} \sin(h)/h$

$= \quad 0 \quad \times \quad 1$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



Area=sin(h)/2

Area=h/2

Area=tan(h)/2

(not shown in class)

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

$$\sin(h) < h < \tan(h)$$

$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

$$1 < h/\sin(h) < 1/\cos(h)$$

$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$: $\downarrow$ $\downarrow$ $\downarrow$

1 1 1

sin(h)/h is stuck between!

(not shown in class)

Derivative of $g(x)=\cos(x)$. Rewrite $\cos(x)$ as…

(A) $g(x) = \cos(x) = \sin(x-\pi/2)$

(B) $g(x) = \cos(x) = \sin(x+\pi/2)$

(C) $g(x) = \cos(x) = \sin(x+\pi)$

(D) $g(x) = \cos(x) = \sin(x-\pi)$

(E) $g(x) = \cos(x) = \sin(x+3\pi/2)$

Derivative of $g(x)=\cos(x)$. Rewrite $\cos(x)$ as...

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Derivative of $g(x)=\sin(x+\pi/2)$

(A) $g'(x) = \cos(x+\pi/2) = \sin(x)$

(B) $g'(x) = \cos(x+\pi/2) = -\sin(x)$

(C) $g'(x) = \cos(x+\pi/2) = \sin(x-\pi/2)$

(D) $g'(x) = \cos(x+\pi/2) = \sin(x+\pi/2)$

(E) $g'(x) = \cos(x+\pi/2) = \sin(x-3\pi/2)$

Derivative of $g(x)=\sin(x+\pi/2)$

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