Today

- Qualitative analysis of DEs continued.
  - Drawing the phase line.
  - Determining long term behaviour.
  - Sketching solutions from the phase line.

Qualitative analysis

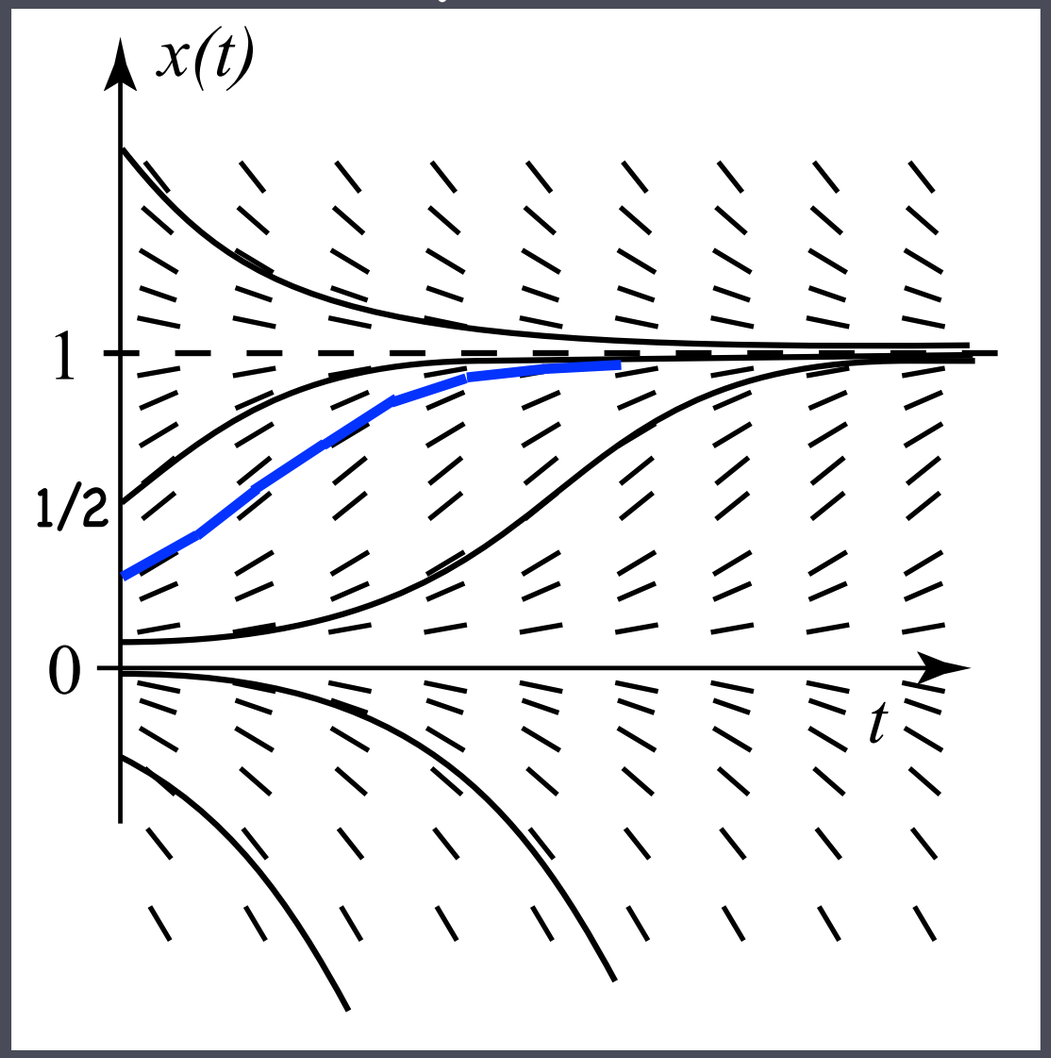
- Finding a formula for a solution to a DE is ideal but what if you can't?
- Qualitative analysis - extract information about the general solution without solving.
    - Steady states
    - Slope fields
    - Stability of steady states
    - Plotting y' versus y (state space/phase line)

$$x' = x(1 - x)$$

velocity

position

Slope field



- Slope field.
- At any t, don't know x yet so plot all possible x′ values
- Now draw x(t) for all t.
- Solution curves must be tangent to slope field everywhere.

When x(t) = 1/2 for all t, x′?

(A) 0

(B) 1/4

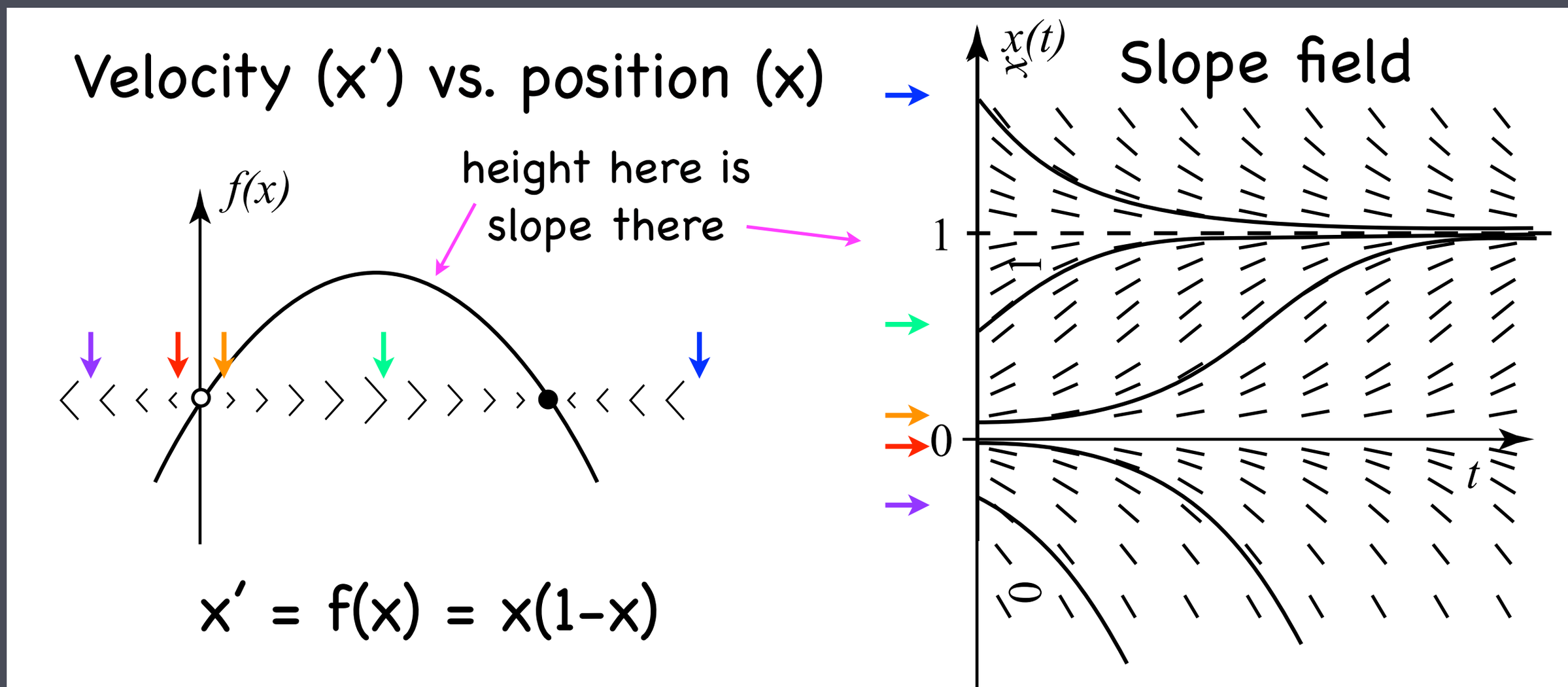
(C) 1/2

(D) 1

$$y' = -y(y-1)(y+1)$$

- What are the steady states of this equation?
- Draw the slope field for this equation.
    - Include the steepest slope element in each interval between steady states and two others (roughly).

Velocity versus position

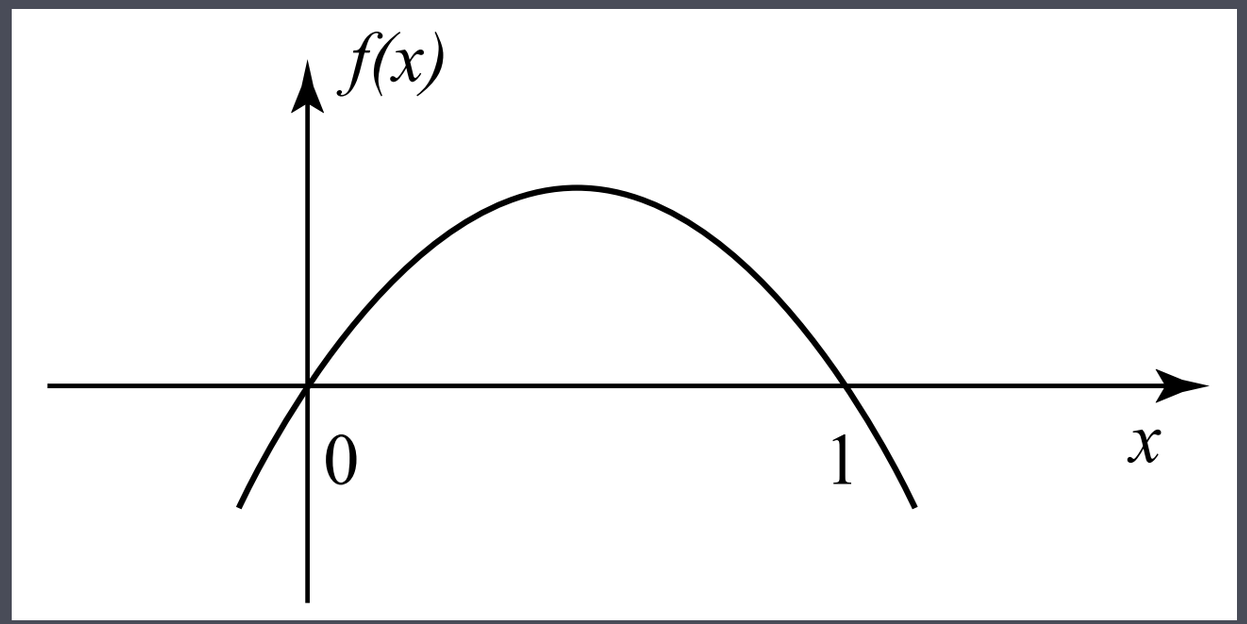


Stable steady state- all nearby solutions approach

Unstable steady state - not stable

Determine stability

$$x' = x(1-x)$$



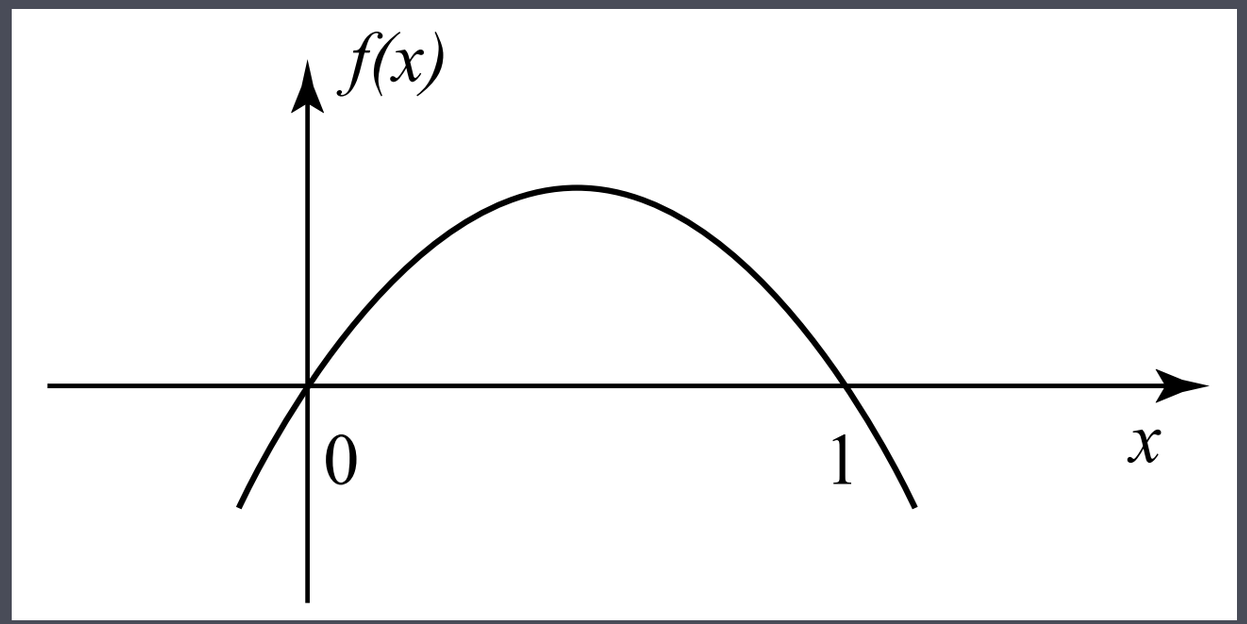
- If you start at x(0) = -0.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



- If you start at x(0) = 0.01, the solution

(A) increases

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Determine stability

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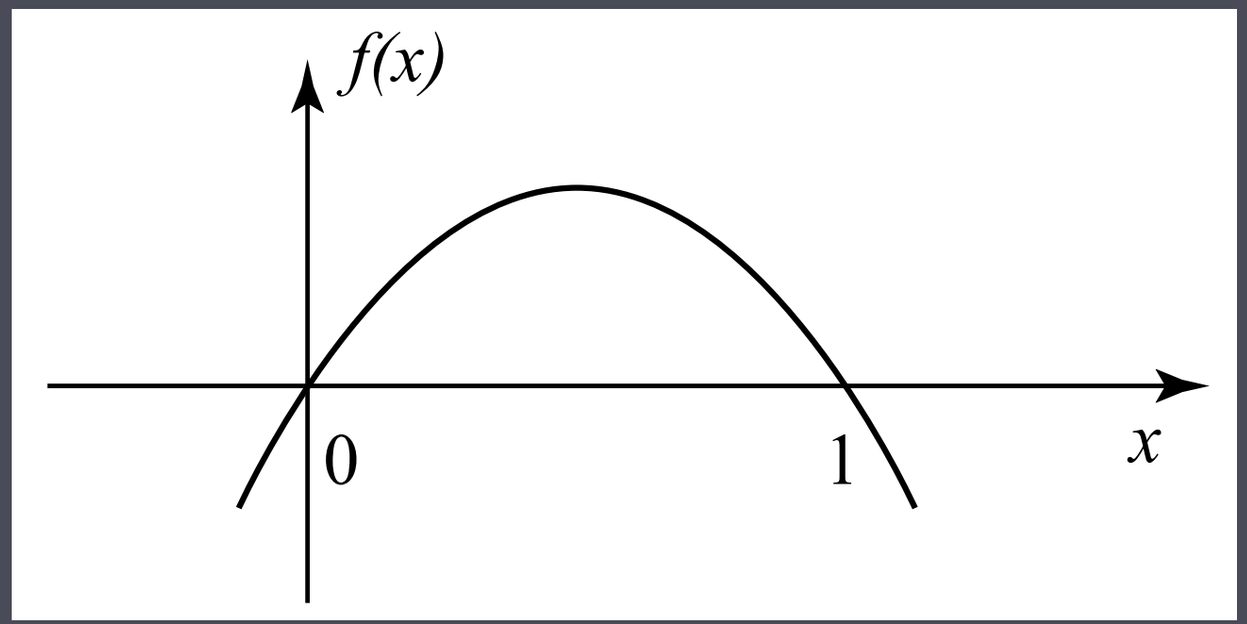


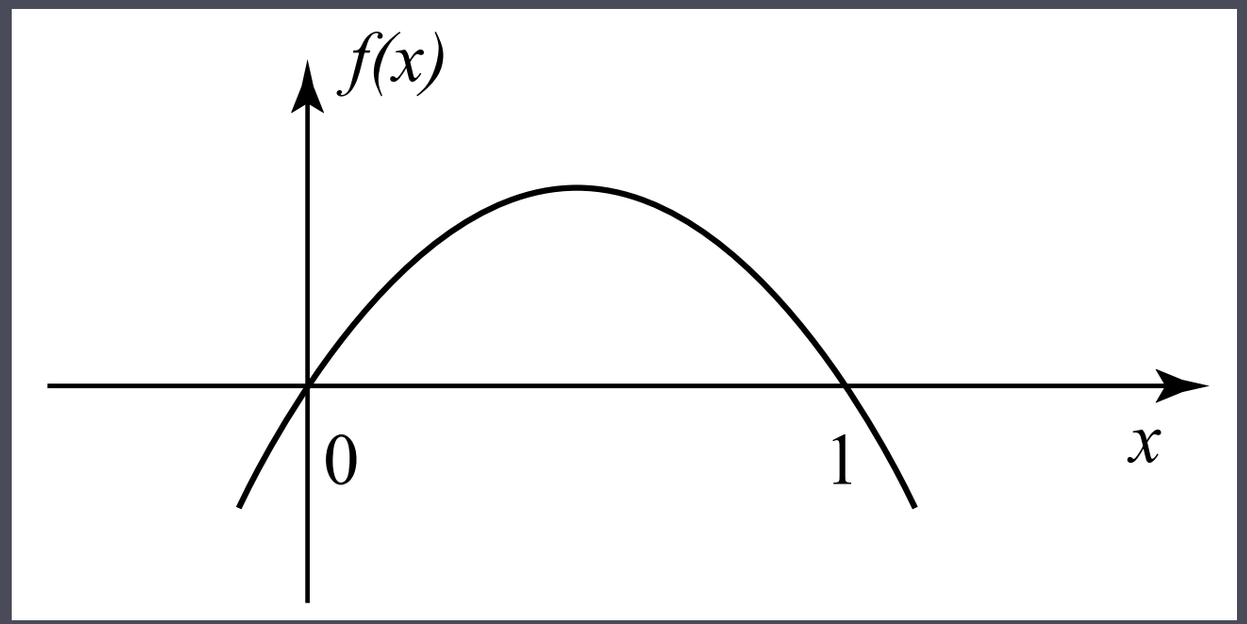
- If you start at x(0) = 0.99, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



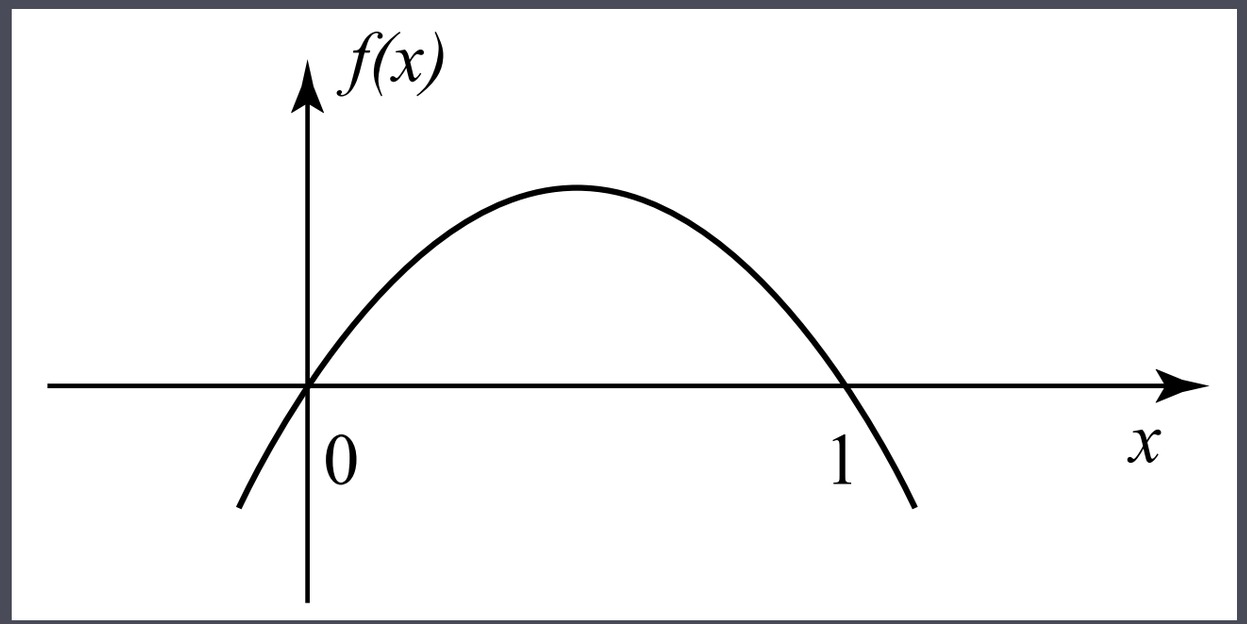
- If you start at x(0) = 1.01, the solution

(A) increases

(B) decreases

Determine stability

$$x' = x(1-x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

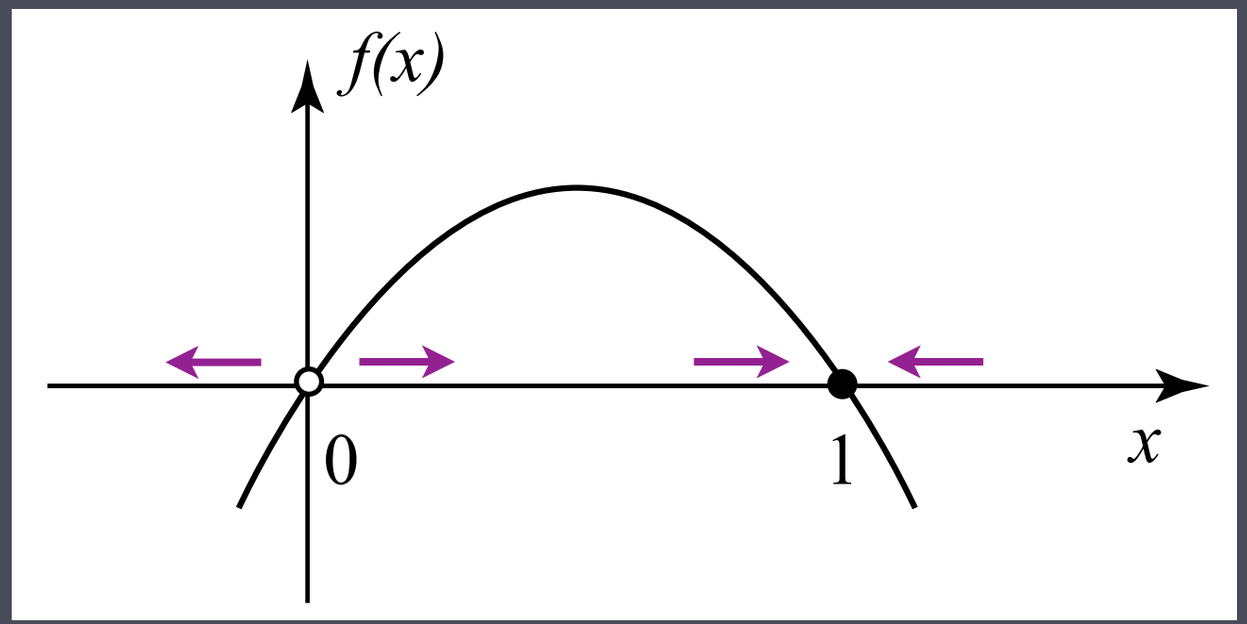
(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Determine stability

$$x' = x(1-x)$$



(A) Both x(t)=0 and x(t)=1 are stable steady states.

(B) x(t)=0 is stable and x(t)=1 is unstable.

(C) x(t)=0 is unstable and x(t)=1 is stable.

(D) Both x(t)=0 and x(t)=1 are unstable steady states.

Stable - solid dot. Unstable - empty dot.