Today

- Qualitative analysis of DEs continued.
- Drawing the phase line.
- Determining long term behaviour.
- Sketching solutions from the phase line.
Qualitative analysis

Finding a formula for a solution to a DE is ideal but what if you can't?

Qualitative analysis - extract information about the general solution without solving.

- Steady states
- Slope fields
- Stability of steady states
- Plotting $y'$ versus $y$ (state space/phase line)
\[ x' = x(1 - x) \]

Slope field.

At any \( t \), don't know \( x \) yet so plot all possible \( x' \) values.

Now draw \( x(t) \) when \( x = \frac{1}{2} \) for all \( t \)?

- (A) 0
- (B) \( \frac{1}{4} \)
- (C) \( \frac{1}{2} \)
- (D) 1

Solution curves must be tangent to slope field everywhere.
\[ y' = -y(y-1)(y+1) \]

What are the steady states of this equation?

Draw the slope field for this equation.

Include the steepest slope element in each interval between steady states and two others (roughly).
Velocity versus position

Velocity (x’) vs. position (x)

\[ x' = f(x) = x(1-x) \]

Stable steady state – all nearby solutions approach

Unstable steady state – not stable
Determine stability

\[ x' = x(1 - x) \]

- If you start at \( x(0) = -0.01 \), the solution
  - (A) increases
  - (B) decreases
Determine stability

\[ x' = x(1 - x) \]

If you start at \( x(0) = 0.01 \), the solution

(A) increases

(B) decreases
Determine stability

\[ x' = x(1 - x) \]

If you start at \( x(0) = 0.99 \), the solution

(A) increases

(B) decreases
Determine stability

\[ x' = x(1 - x) \]

If you start at \( x(0) = 1.01 \), the solution

(A) increases

(B) decreases
Determine stability

\[ x' = x(1 - x) \]

(A) Both \( x(t) = 0 \) and \( x(t) = 1 \) are stable steady states.

(B) \( x(t) = 0 \) is stable and \( x(t) = 1 \) is unstable.

(C) \( x(t) = 0 \) is unstable and \( x(t) = 1 \) is stable.

(D) Both \( x(t) = 0 \) and \( x(t) = 1 \) are unstable steady states.
Determine stability

\[ \frac{dx}{dt} = x(1 - x) \]

(A) Both \( x(t) = 0 \) and \( x(t) = 1 \) are stable steady states.

(B) \( x(t) = 0 \) is stable and \( x(t) = 1 \) is unstable.

(C) \( x(t) = 0 \) is unstable and \( x(t) = 1 \) is stable.

(D) Both \( x(t) = 0 \) and \( x(t) = 1 \) are unstable steady states.

Stable - solid dot. Unstable - empty dot.