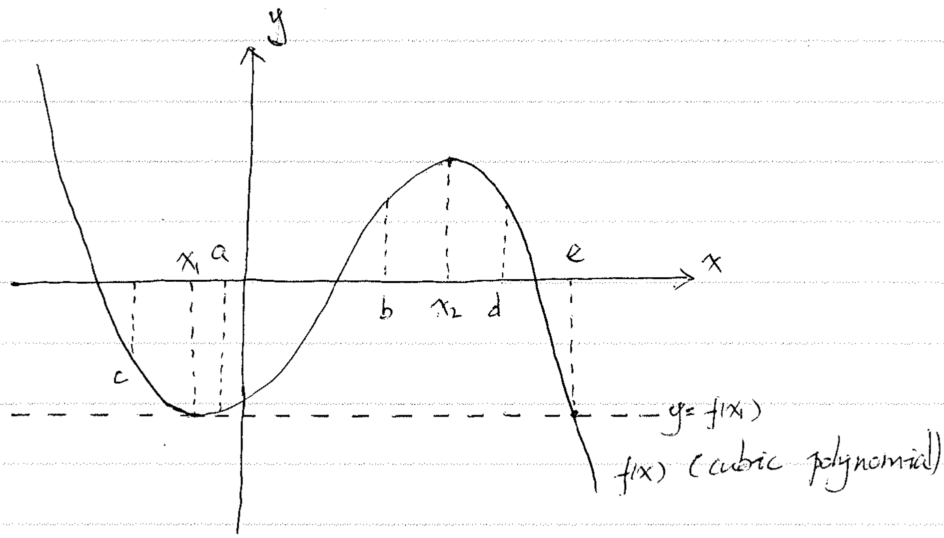
Lecture 12 (Oct. 02, 2013)

· Global Extrema:



(1) given $D = [a, b]$, $f(a)$ is the global min.

$f(b)$ is the global max

(2) given $D = [c, d]$, $f(x_1)$ is the global min.

$f(x_2)$ is the global max

(3) given $D = (a, b)$ no global extrema

(4) given $D = [c, e]$, both $f(x_1)$ and $f(e)$ are global minima

$f(x_2)$ is the global max

(5) given $D = \mathbb{R}$ $(x \in (-\infty, +\infty))$ $\lim_{x \to +\infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = +\infty$

no global extrema

Notice: (1) If the domain is a closed interval, global extrema can happen at local extrema or the endpoints of the interval

(2) If the domain is open / half-open interval, we need to check the behaviour of f(x) when x approaches open boundary (find the limit)

(3) Global extrema may not be unique

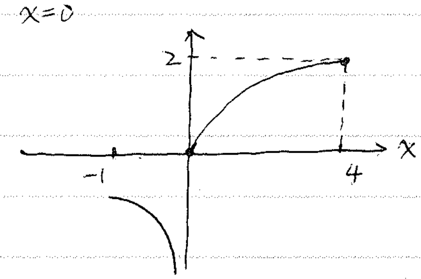
Example 1: Find the global extrema of $f(x) = \begin{cases} \frac{1}{x}, & -1 \le x < 0 \\ \sqrt{x}, & 0 \le x \le 4 \end{cases}$

$f'(x) = \begin{cases} -\frac{1}{x^2}, & -1 < x < 0 \\ \frac{1}{2\sqrt{x}}, & 0 < x < 4 \end{cases}$ $\Rightarrow f'(0)$ DNE $\Rightarrow$ critical point $x = 0$

Compare $f(-1) = -1$, and check the limit $\lim_{x \to 0^-} f(x) = -\infty$

$f(0) = 0$

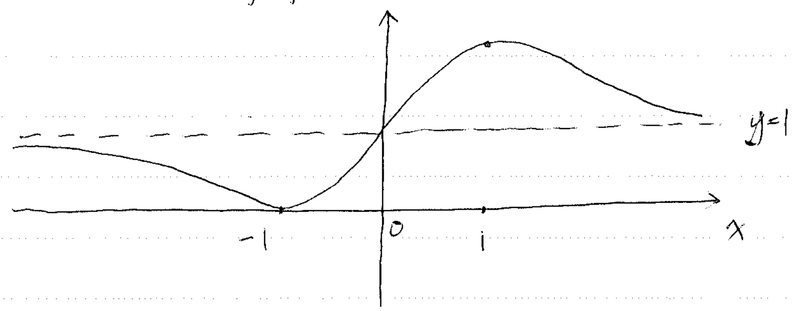
$f(4) = 2$



$\Rightarrow f(4) = 2$ is the global maximum and there's no global minimum.

Example 2: Find the global extrema of $f(x) = \frac{(x+1)^2}{x^2+1}$ given $D = \mathbb{R}$

Recall the curve of $f(x)$ (in Lecture 11)



critical points $x=-1$ and $x=1$

Compare $f(-1) = 0$ and $\lim_{x\to\infty} f(x) = 1$

$f(1) = 2$

then $f(-1)$ is the global min. and $f(1)$ is the global max

Notice: even though the extrema can't be at "$\pm\infty$", we still need to check the behaviour of $f(x)$ when $x \to \pm\infty$