Lecture 29 (Nov. 15, 2013)

Learning Goal: inverse trigonometric functions

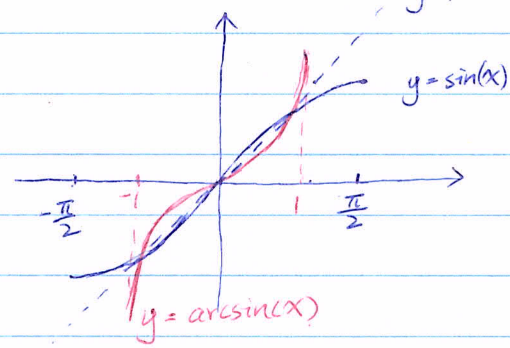
- Second order DE: $\frac{d^2y}{dt^2} + k^2 y = 0 \iff y(t) = A\sin(kt) + B\cos(kt)$, A, B - constants

Example 1: Find the solution to $\frac{d^2y}{dt^2} + y = 0$ with $y(0) = 3$ and $y'(0) = -2$

The solution is in the form $y(t) = A\sin(t) + B\cos(t)$

then $\left.\begin{array}{l} y(0) = A\sin(0) + B\cos(0) = B = 3 \\ y'(0) = A\cos(0) - B\sin(0) = A = -2 \end{array}\right\} \Rightarrow y(t) = -2\sin(t) + 3\cos(t)$

- Inverse Trigonometric Functions:

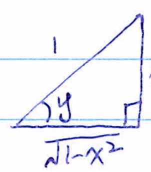


① Arcsine: $y = f(x) = \sin(x)$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$y = f^{-1}(x) = \arcsin(x)$ $\quad -1 \le x \le 1$

$= \sin^{-1}(x)$

Geometry aspect:



$\begin{cases} x = \sin(y) \\ y = \arcsin(x) \end{cases}$

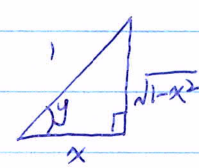
Recall that $x = e^{\ln x}$, $x = \ln e^x$

we have $x = \sin(\arcsin(x))$ for $-1 \le x \le 1$

$x = \arcsin(\sin(x))$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

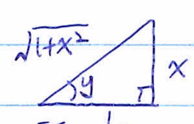
② Arccosine: $y = f(x) = \cos(x)$ $\quad 0 \le x \le \pi$

$y = f^{-1}(x) = \arccos(x)$, $\quad -1 \le x \le 1$



③ Arctangent: $y = f(x) = \tan(x)$, $\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$y = f^{-1}(x) = \arctan(x)$, $\quad x \in (-\infty, +\infty)$

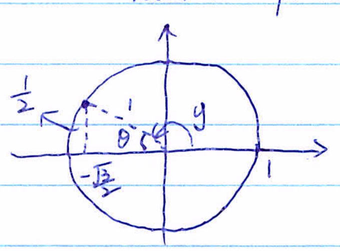


Example 1: $\arcsin(\frac{1}{2}) = \frac{\pi}{6}$ (based on $\sin(\frac{\pi}{6}) = \frac{1}{2}$ & $\sin(\frac{5\pi}{6}) = \frac{1}{2}$ but only $\frac{\pi}{6}$ in the domain)

$\arccos(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$ (same as finding y that $\cos(y) = -\frac{\sqrt{3}}{2}$

recall the particle moving around the unit circle

$\theta = \frac{\pi}{6}$, $y = \pi - \theta = \frac{5\pi}{6}$



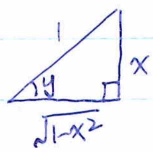
Example 2: $\arccos(\sin(-\frac{\pi}{3})) = \arccos(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$

$\cos(\arcsin(\frac{1}{2})) = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

Example 3: ① Simplify expression $\tan(\arcsin(x))$

② Simplify expression $\cos(\arctan(x))$

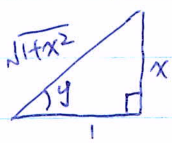
①



$y = \arcsin(x) \Leftrightarrow x = \sin(y)$

$\Rightarrow \tan(y) = \tan(\arcsin(x)) = \frac{x}{\sqrt{1-x^2}}$

②



$y = \arctan(x) \Leftrightarrow x = \tan(y)$

$\Rightarrow \cos(y) = \cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$