Today

- $\ln(x)$ as inverse function for e^x.
- Derivative of $\ln(x)$.
- Derivative of a^x.
- Converting between a^x and e^{kx}.
- Note that topics covered on Midterm 1 might reappear on Midterm 2 in the context of exponential and logarithmic functions.
  (definition of derivative, product/quotient rules, tangent lines, linear approximation, Newton's method)

A note about units

A note about units

- e is a “pure” number without units, called dimensionless.

A note about units

- e is a "pure" number without units, called dimensionless.
- This means e^a for any a is also dimensionless.

A note about units

- e is a "pure" number without units, called dimensionless.
- This means e^a for any a is also dimensionless.
- Furthermore, the exponent, a, must also be dimensionless.

A note about units

- e is a "pure" number without units, called dimensionless.
- This means e^a for any a is also dimensionless.
- Furthermore, the exponent, a, must also be dimensionless.
- If $y(t) = y_0 e^{-kt}$, and t is time in seconds, what must be the units of k?

What is the definition of the inverse function of $f(x)$?

(A) The function $g(x)$ for which $g(f(x))=x$.

(B) The mirror image of graph of $f(x)$ in the line $y=x$.

(C) $1/f(x)$

(D) $-f(x)$

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x.

(B) The mirror image of graph of f(x) in the line y=x.

(C) 1/f(x)

(D) -f(x)

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x. $g(x) = f^{-1}(x)$

(B) The mirror image of graph of f(x) in the line y=x.

(C) 1/f(x)

(D) -f(x)

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x. $g(x) = f^{-1}(x)$

(B) The mirror image of graph of f(x) in the line y=x. <---might not be a function.

(C) 1/f(x)

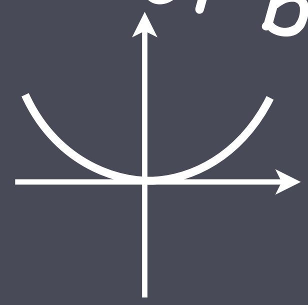
(D) -f(x)

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x. $g(x) = f^{-1}(x)$

(B) The mirror image of graph of f(x) in the line y=x.

<---might not be a function.



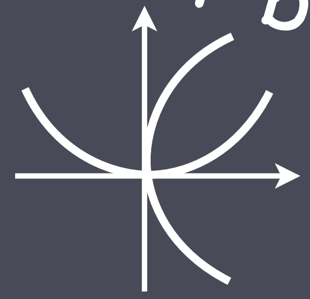
(C) 1/f(x)

(D) -f(x)

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x. $g(x) = f^{-1}(x)$

(B) The mirror image of graph of f(x) in the line y=x. <---might not be a function.



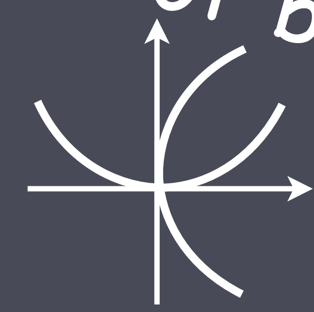
(C) 1/f(x)

(D) -f(x)

What is the definition of the inverse function of f(x)?

(A) The function g(x) for which g(f(x))=x. $g(x) = f^{-1}(x)$

(B) The mirror image of graph of f(x) in the line y=x. <---might not be a function.



(C) 1/f(x)

(D) -f(x)

$f^{-1}(x)$ is the function that goes backwards through f(x). If you plug the output of f(x) into $f^{-1}(x)$, you will get back to x.

Let $f(x)=e^x$. Define $\ln(x)$ to be $f^{-1}(x)$.

Which of the following is false?

(A) If $a=e^b$ and $c=e^d$ then $\ln(a/c) = b-d$.

(B) If $a=e^b$ and $c=e^d$ then $\ln(a-c) = b/d$.

(C) If $c=a^d$ then $\ln(c) = d \ln(a)$.

(D) If $a=e^b$ and $c=a^d$ then $\ln(c) = bd$.

(E) If $a=e^b$ and $c=e^d$ then $\ln(ac) = b+d$.

Let $f(x)=e^x$. Define $\ln(x)$ to be $f^{-1}(x)$.

Which of the following is false?

(A) If $a=e^b$ and $c=e^d$ then $\ln(a/c) = b-d$.

(B) If $a=e^b$ and $c=e^d$ then $\ln(a-c) = b/d$.

(C) If $c=a^d$ then $\ln(c) = d\ \ln(a)$.

(D) If $a=e^b$ and $c=a^d$ then $\ln(c) = bd$.

(E) If $a=e^b$ and $c=e^d$ then $\ln(ac) = b+d$.

Let $f(x)=e^x$. Define $\ln(x)$ to be $f^{-1}(x)$.

Which of the following is false?

(A) If $a=e^b$ and $c=e^d$ then $\ln(a/c) = b-d$.

(B) If $a=e^b$ and $c=e^d$ then $\ln(a-c) = b/d$.

(C) If $c=a^d$ then $\ln(c) = d \ln(a)$.

(D) If $a=e^b$ and $c=a^d$ then $\ln(c) = bd$.

(E) If $a=e^b$ and $c=e^d$ then $\ln(ac) = b+d$.

$\ln(x)$ = natural logarithm of x = $\log_e(x)$)

Derivative of ln(x)

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} =$

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} =$

(A) 1

(B) x

(C) 1/x

(D) e

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} =$

(A) 1

(B) x

(C) 1/x

(D) e

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

  (A) $e^{y'} = 1$

  (B) $e^y y' = 1$

  (C) $e^y = x'$

  (D) $ye^{y-1} = 1$

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

  (A) $e^{y'} = 1$

  (B) $e^y y' = 1$

  (C) $e^y = x'$

  (D) $ye^{y-1} = 1$

- Solve for y':

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

  (A) $e^{y'} = 1$

  (B) $e^y y' = 1$

  (C) $e^y = x'$

  (D) $ye^{y-1} = 1$
- Solve for y': $y' = e^{-y}$

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

  (A) $e^{y'} = 1$

  (B) $e^y y' = 1$

  (C) $e^y = x'$

  (D) $ye^{y-1} = 1$

- Solve for y': $y' = e^{-y} = 1/x$

Derivative of ln(x)

- If $y = \ln(x)$ then $e^y = e^{\ln(x)} = f(f^{-1}(x)) = x$.
- Implicit differentiation:

  (A) $e^{y'} = 1$

  (B) $e^y y' = 1$

  (C) $e^y = x'$

  (D) $ye^{y-1} = 1$

$g(x) = \ln(x)$

$\quad \text{-->} \; g'(x) = 1/x$

- Solve for y': $\quad y' = e^{-y} = 1/x$

$f(x)=a^x$. $f'(x)=C_a a^x$. C_a=??

- Recall that we got stuck on this derivative.
- Time to get unstuck…

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}.$

(B) $f'(x) = \ln(2)e^{\ln(2)x}.$

(C) $f'(x) = \ln(2)\cdot 1/2 \cdot e^{\ln(2)x}.$

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}.$

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}$.

(B) $f'(x) = \ln(2)e^{\ln(2)x}$.

(C) $f'(x) = \ln(2)\cdot 1/2\cdot e^{\ln(2)x}$.

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

$$f(x) = e^{\ln(2)x}.$$

(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

$$f(x) = e^{\ln(2)x}.$$

(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

From the last two clicker Qs…

From the last two clicker Qs…

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.

From the last two clicker Qs…

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.

From the last two clicker Qs…

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.

From the last two clicker Qs...

- $f(x) = e^{\ln(2)x}$ --> $f'(x) = \ln(2)e^{\ln(2)x}$.
- $f(x) = e^{\ln(2)x}$ --> $f(x) = 2^x$.
- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.
- In general, $f(x) = a^x$ --> $f'(x) = a^x \ln(a)$.

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$a^x = (e^k)^x$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$

$$a = e^k$$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$

$$a = e^k$$

$$\ln(a) = \ln(e^k)$$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$

$$a = e^k$$

$$\ln(a) = \ln(e^k)$$

$$\ln(a) = k\ \ln(e)$$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$
$$a = e^k$$
$$\ln(a) = \ln(e^k)$$
$$\ln(a) = k \ln(e)$$
$$\ln(a) = k$$

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

(E) $k=\ln(-a)$

$$a^x = (e^k)^x$$
$$a = e^k$$
$$\ln(a) = \ln(e^k)$$
$$\ln(a) = k\ \ln(e)$$
$$\ln(a) = k$$

$f(x) = a^x = e^{\ln(a)x}$

--> $f'(x) = a^x \ln(a)$.

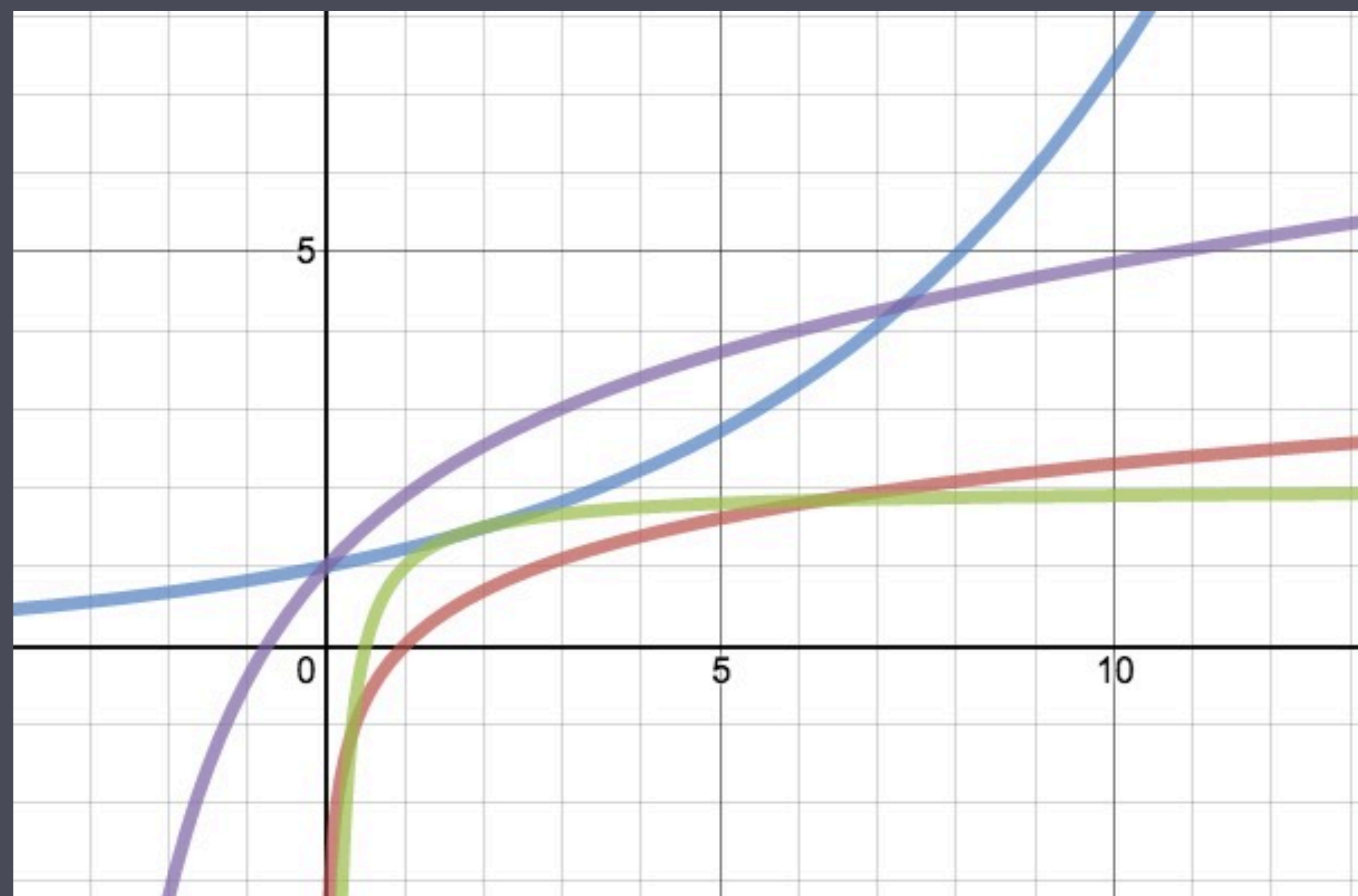
Which of following is the graph of ln(x)?

(A)

(B)

(C)

(D)



Log-log and semi-log plots

- A log-log plot is a plot on which you plot log(y) versus log(x) instead of y versus x.
- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

Log-log and semi-log plots

- A log-log plot is a plot on which you plot log(y) versus log(x) instead of y versus x.
- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

<---this is what OSH 5 asks you to use

Semi-log plot of exponential function

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.
- $V = A + kx$ where $A=\ln(a)$.

Semi-log plot of exponential function

- Suppose $y = ae^{kx}$. a and k are constants.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ae^{kx}) = \ln(a) + kx$.
- $V = A + kx$ where $A=\ln(a)$.
- On a semi-log plot, $y= ae^{kx}$ looks linear.

Log-log plot of power function

Log-log plot of power function

- Suppose $y = ax^p$.

Log-log plot of power function

- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.

Log-log plot of power function

- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ax^p) = \ln(a) + p \ln(x)$.

Log-log plot of power function

- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ax^p) = \ln(a) + p \ln(x)$.
- $V = A + pU$ where $A=\ln(a)$, $U=\ln(x)$.

Log-log plot of power function

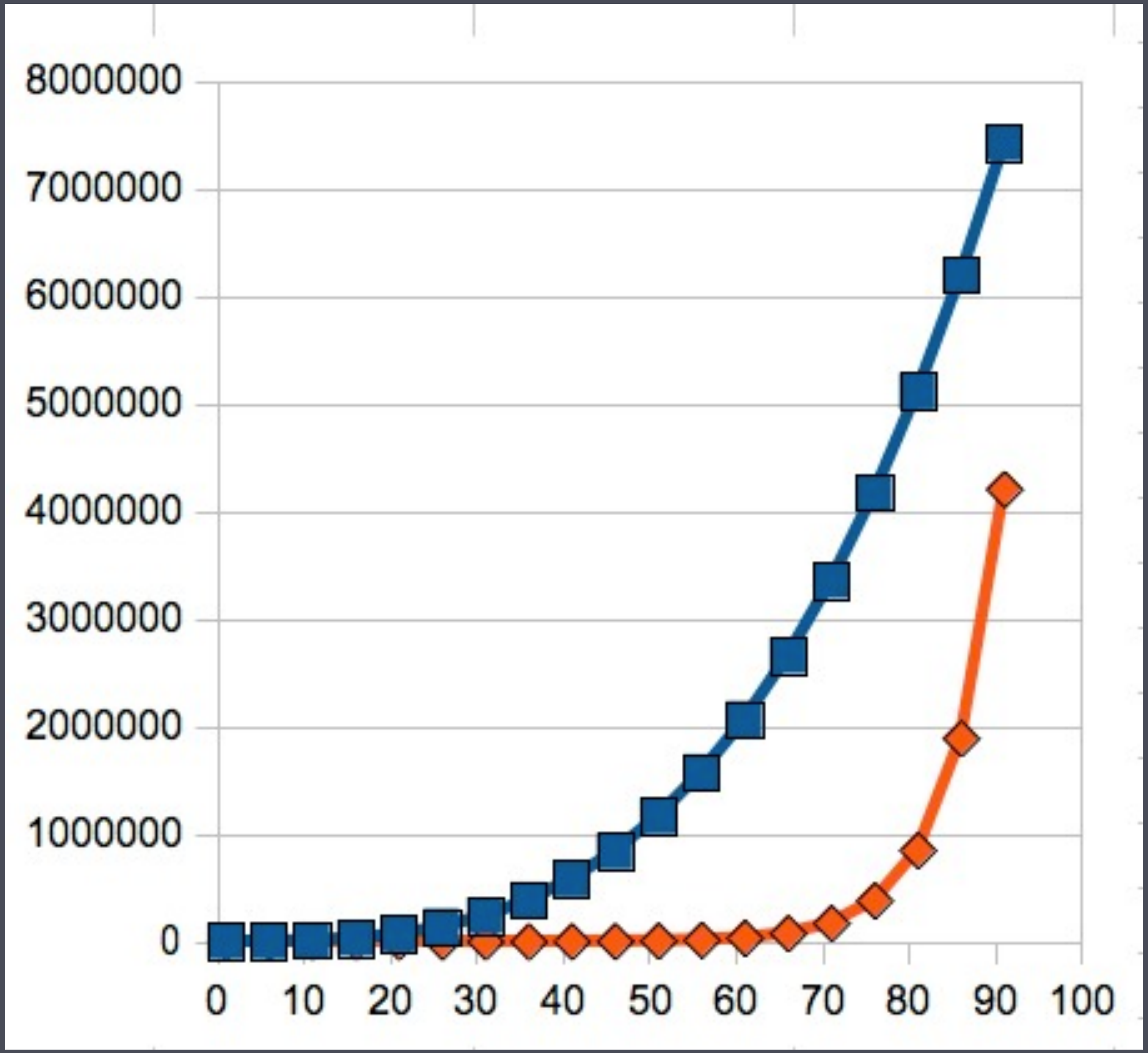
- Suppose $y = ax^p$.
- Define new variable $V=\ln(y)$.
- $V = \ln(y) = \ln(ax^p) = \ln(a) + p\ \ln(x)$.
- $V = A + pU$ where $A=\ln(a)$, $U=\ln(x)$.
- On a log-log plot, $y=ax^p$ looks linear.

Regular, log-log and semi-log plots

Two data sets.

Power function?

Exponential function?



Regular x-y plot.

Plot $Y_i = \ln(y_i)$ versus x_i.

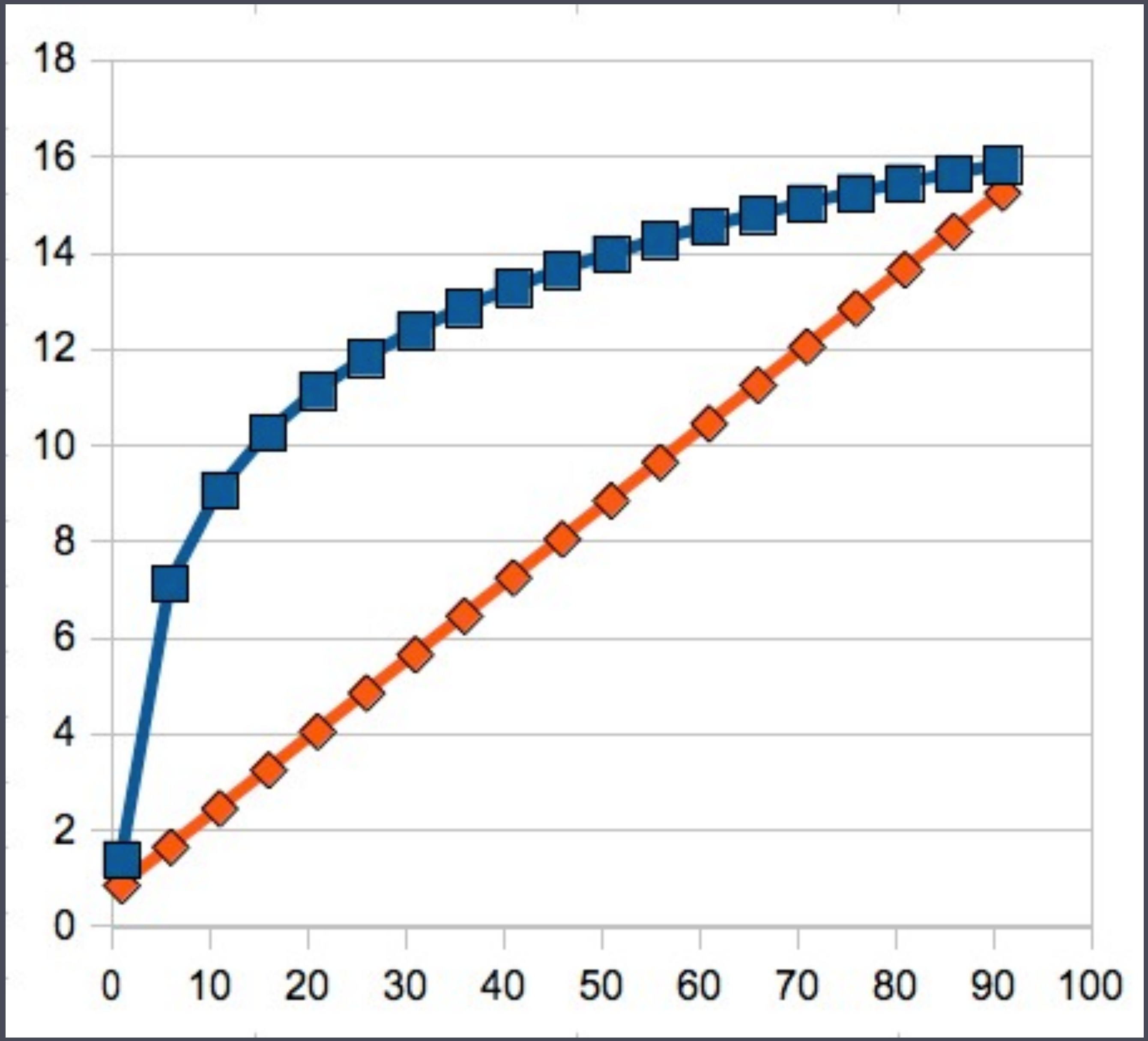
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



Semi-log plot.

Plot $Y_i = \ln(y_i)$ versus x_i.

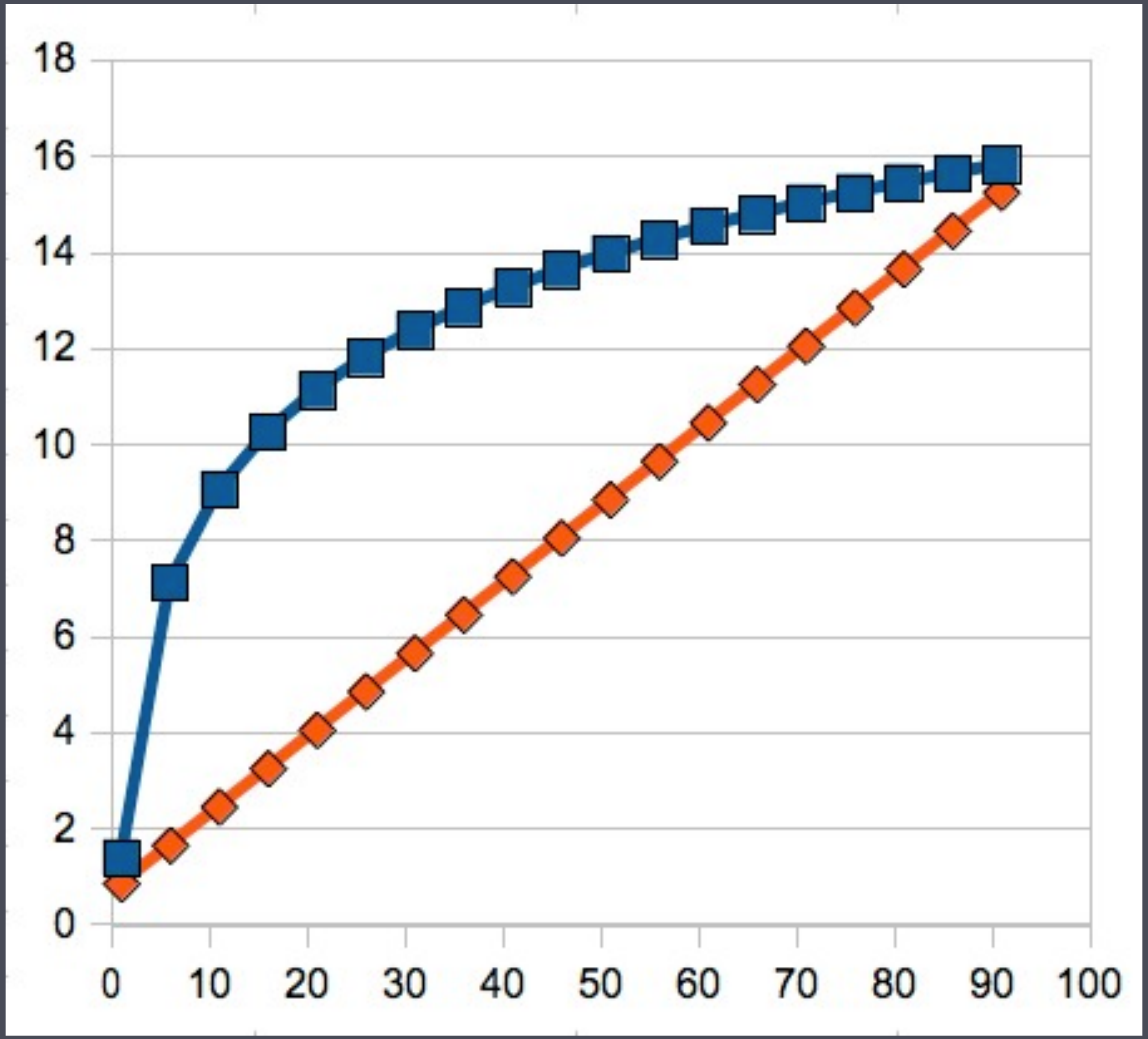
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



Semi-log plot.

Plot $Y_i = \ln(y_i)$ versus $X_i = \ln(x_i)$.

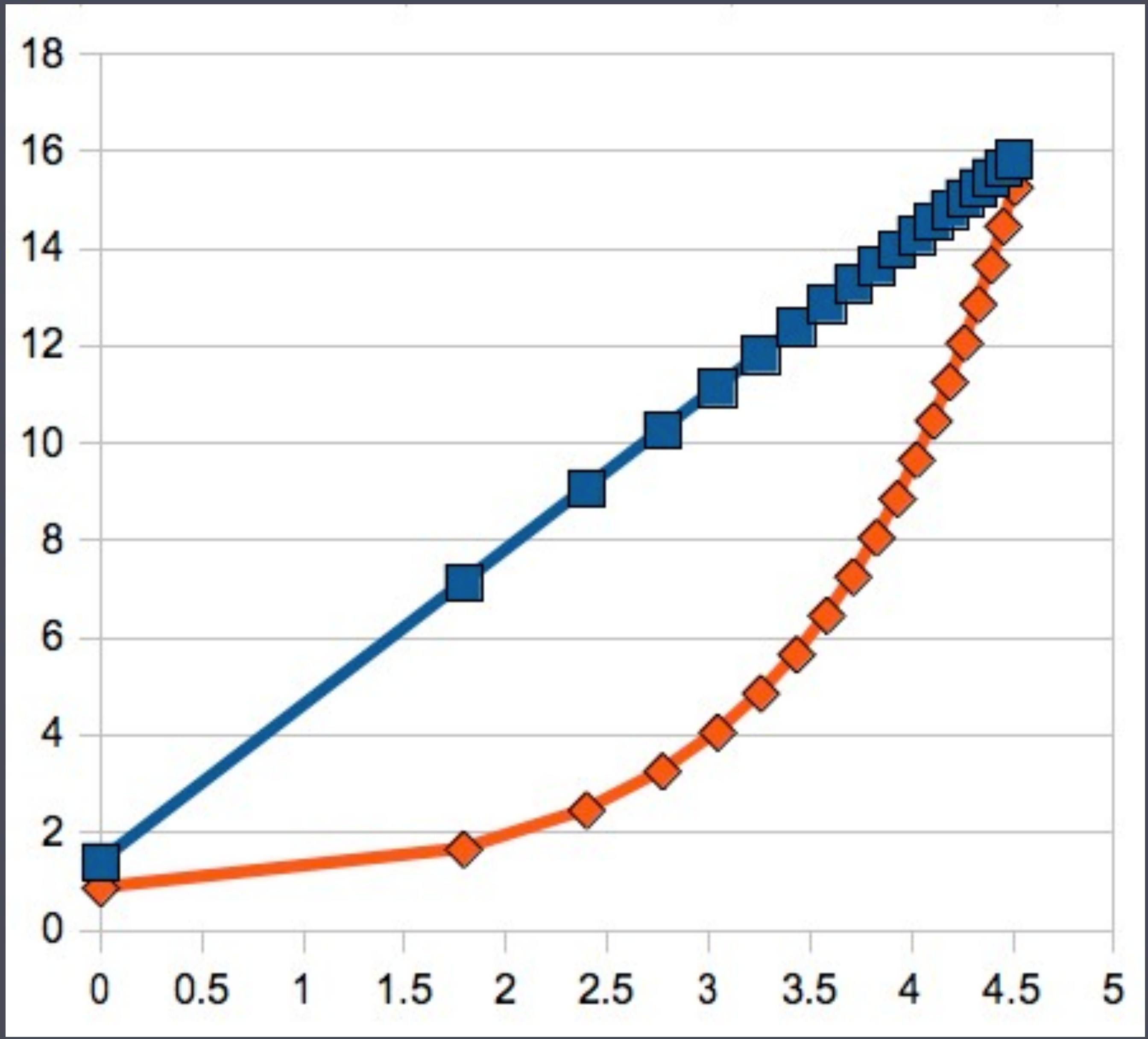
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



Log-log plot.

Plot $Y_i = \ln(y_i)$ versus $X_i = \ln(x_i)$.

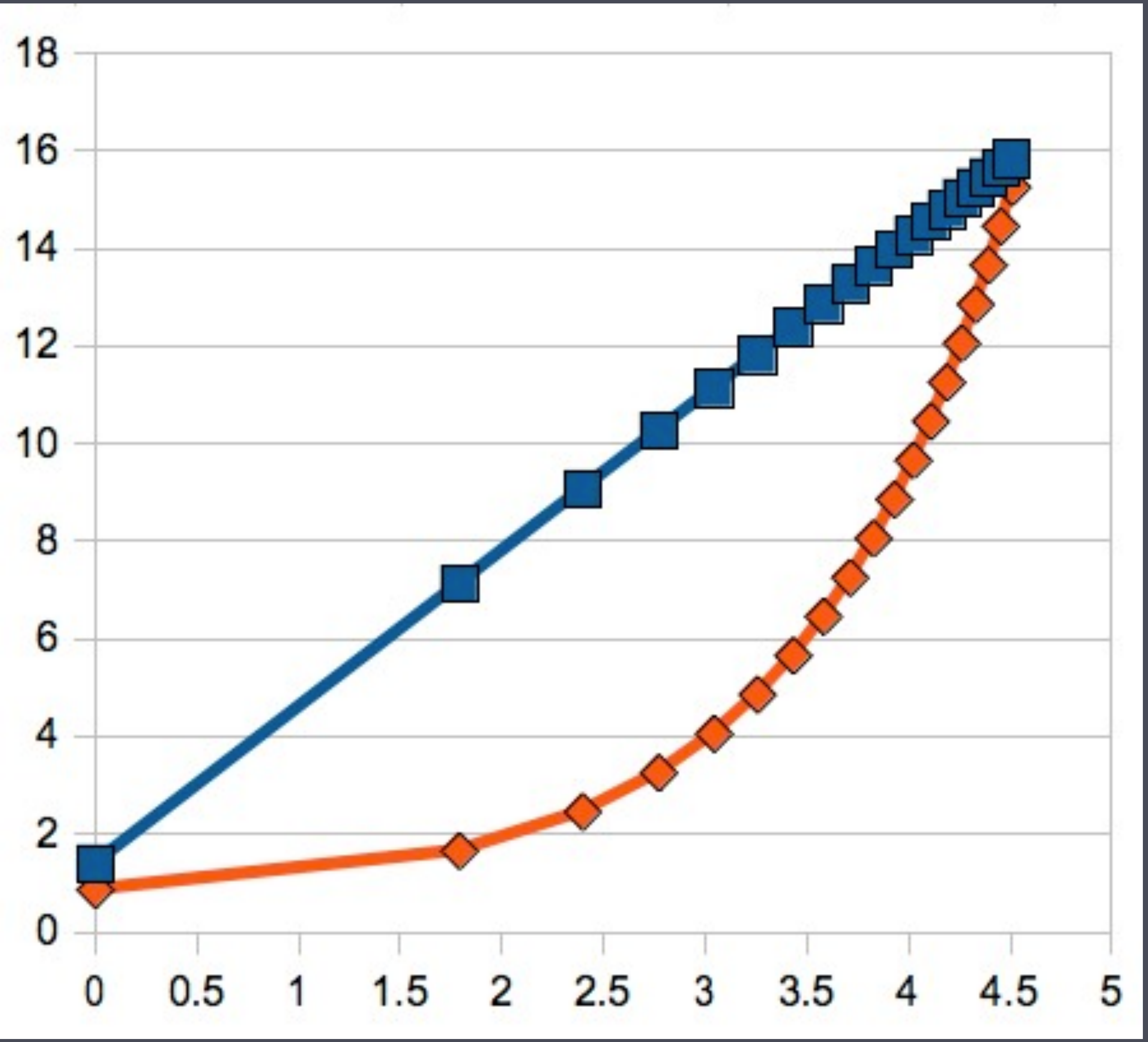
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



Log-log plot.