Today

- Chain rule
- Related rates examples
- Announcement: CLASS, the Conference for Learning and Student Success, is taking place this Saturday (Oct. 18). See my.science.ubc.ca where CLASS is one of the three highlighted programs on the home page.

Composition of functions

If $f(x) = 2x+3$ and $g(x) = -4x+2$,

(A) $h(x) = f(g(x)) = -8x+7$

(B) $h(x) = f(g(x)) = -8x-10$

(C) $h(x) = f(g(x)) = -8x^2-8x+6$

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Notation for composition:

$f \circ g(x)=f(g(x))$

Composition of functions

If $h(x) = f(g(x))$, then

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(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$

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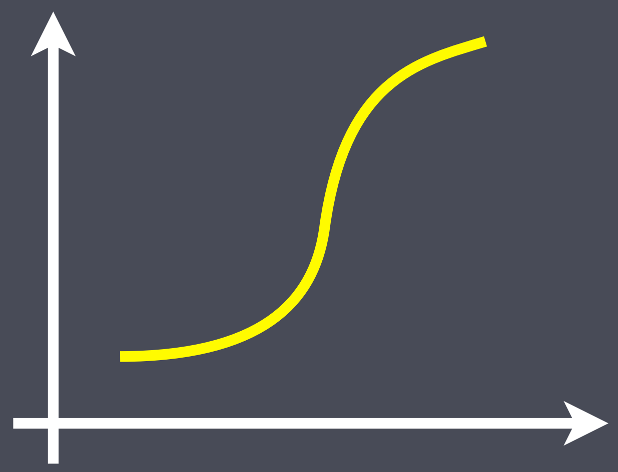

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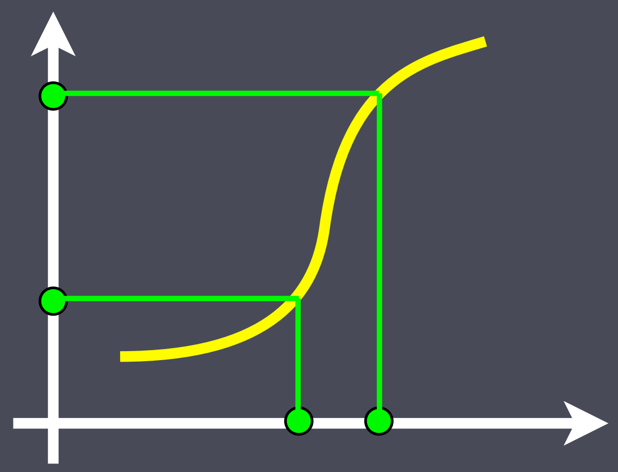

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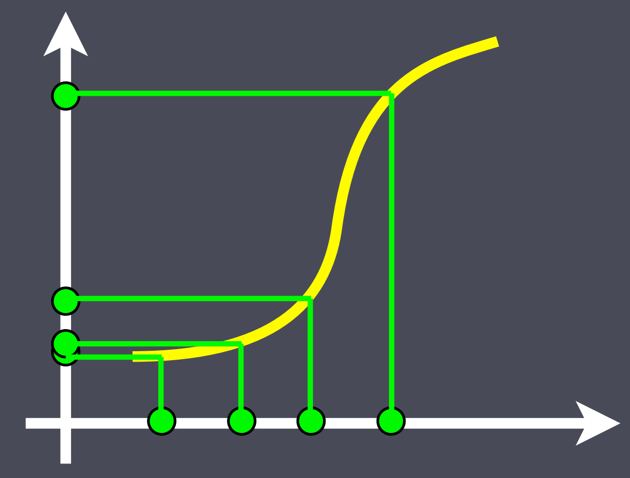

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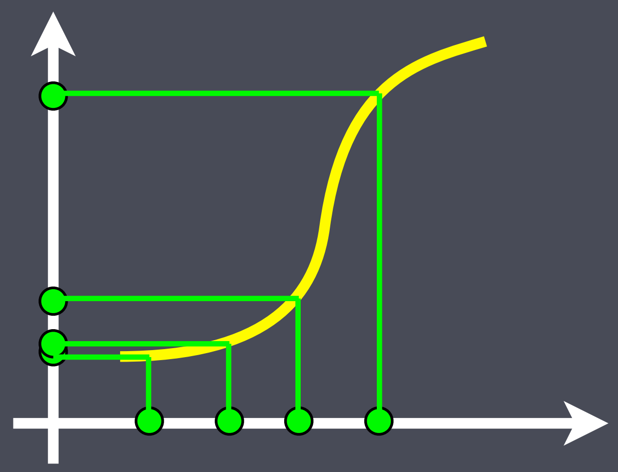

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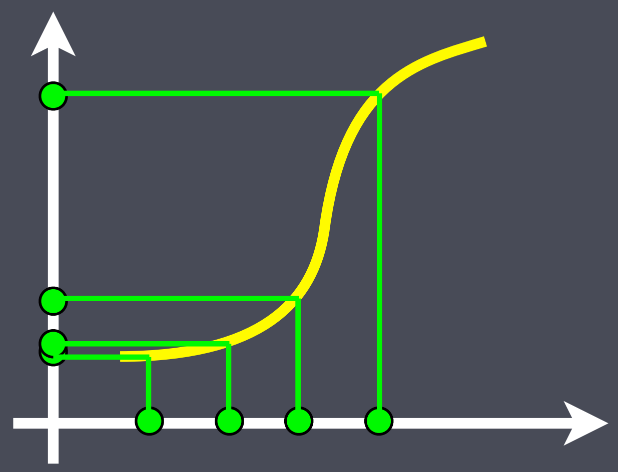
- Where you are matters!



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- When you do one and then the other, you multiply their effects.
- Where you are on the function matters - multiply the stretch factor of g near x: g'(x), by the stretch factor of f near g(x): f'(x).

Gas costs $1.25/litre. Your car consumes 7 litres/100 km. You've driven 130 km. How much does it cost to drive one more km?

(A) $1.25 \cdot (7/100) \cdot 130$

(B) $1.25 \cdot (7/100)$

(C) 125/7

(D) 7/125

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- These are straight lines so the slopes are independent of L, x.

Related rates

- When two quantities (e.g. Q_1 and Q_2) are related to each other, if one changes in time so will the other.
- Knowing the relationship between Q_1 and Q_2 gives you the relationship between Q_1' and Q_2'.

The radius of a spherical tumor grows at a constant rate, k. Determine the rate of growth of the volume of the tumor when the radius is one centimeter.

Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2\ k$

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Now we can plug in r=1.

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

Which is the relevant equation relating the quantities when the water is at height h (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

(B) $V = 1/3\ \pi\ (R^2/H^2)\ h$

(C) $V = 1/3\ \pi\ (R^2/H^2)\ h^3$

(D) $V = 1/3\ \pi\ r^2 h$