Newton’s Law of Cooling and Euler’s Method

Cole Zmurchok
Math 102 Section 106

November 16, 2016
Today...

1. Newton’s Law of Cooling (a complete example)
2. Euler’s Method Introduction
3. Euler’s Method Worksheet

▶ Reminder: Office Hours today from 3-4 pm in Math Annex 1118, and Thursday 3-4 in LSK 300B

▶ Quiz on Friday: Lectures 8.2 to 11.1 (up to “Solving Differential Equations”)
Last time

The solution to the initial value problem (differential equation together with an initial condition)

\[ \frac{dF}{dt} = a - bF, \quad F(0) = F_0. \]

is

\[ F(t) = \frac{a}{b} + \left(F_0 - \frac{a}{b}\right) e^{-bt} \]

- This equation has two important parameters: steady-state \( \frac{a}{b} \) and the characteristic time \( \frac{1}{b} \)
Solving $\frac{dF}{dt} = a - bF$

I understand how to algebraically solve $\frac{dF}{dt} = a - bF$ using the shift $y = F - \frac{a}{b}$.

A. Yes!
B. Sort of.
C. I don’t know.
D. Not really.
E. No!
Solving \( \frac{dF}{dt} = a - bF \)

I understand that by using the shift \( y = F - \frac{a}{b} \) we move the entire slope field downwards.

A. Yes!
B. Sort of.
C. I don’t know.
D. Not really.
E. No!
1. Newton’s Law of Cooling Example:

Yesterday, the air temperature was 10°C. I left my coffee at 100°C on the table while I ate lunch outside. At what time is the coffee’s temperature 30°C.
Where are we?

- We’re able to write down the solutions to some differential equations:
  \[
  \frac{dC}{dt} = -kC, \quad \frac{dF}{dt} = a - bF
  \]

- yet for other equations we are not able (or haven’t learned the right solution method yet) to write down solutions:
  \[
  \frac{dN}{dt} = rN \left( 1 - \frac{N}{K} \right)
  \]
Euler’s Method

Euler’s Method (pronounced Oil-err) is a method to numerically approximate the solution to a differential equation

\[
\frac{dy}{dt} = f(y), \quad y(0) = y_0
\]
Euler’s Method

Q1. True or false?

\[ \frac{dy}{dt} \approx \frac{y(t + h) - y(t)}{h} \]

A. True

B. False
Euler’s Method

\[ f(y) = \frac{dy}{dt} \approx \frac{y(t + h) - y(t)}{h} \]

Q2. Which of the following is correct?

A. \( y(t + h) = y(t) - hf(y(t)) \)

B. \( y(t + h) = y(t) + hf(y(t)) \)

C. \( y(t + h) = y(t) - hf(y(t + h)) \)

D. \( y(t + h) = y(t) + hf(y(t + h)) \)
Euler’s Method

\[ \frac{dy}{dt} = f(y) \quad \Rightarrow \quad y(t + h) = y(t) + hf(y(t)) \]

Q3. Suppose you know the solution at some point in time, \( y_n \). Which of the following is correct?

A. \( y_{n+1} = y_n - hf(y_n) \)
B. \( y_{n+1} = y_n + hf(y_n) \)
C. \( y_{n+1} = y_n - hf(y_{n+1}) \)
D. \( y_{n+1} = y_n - hf(y_{n+1}) \)
Euler’s Method

To approximate the solution to

\[
\frac{dy}{dt} = f(y), \quad y(0) = y_0
\]

generate an approximation using Euler’s Method:

\[
y_{n+1} = y_n + hf(y_n), \quad \text{with } y_0 = y(0)
\]

Euler’s Method Demo and Worksheet
Summary

- We can use Euler’s Method to numerically approximate the solution to a differential equation.
- The idea for Euler’s method is to use a secant line as an approximation for the derivative over discrete time steps.
- Euler’s method is not exact. There is an error between the solution and the error.
Answers

1. A
2. B
3. B
1. If Euler’s method is used to solve the differential equation

\[ \frac{dy}{dt} = y(4 - y), \quad y(0) = 2, \]

with step size \( \Delta t = h = 0.1 \), find the value of \( y_1 \) obtained (i.e., the approximate solution at the first time step).