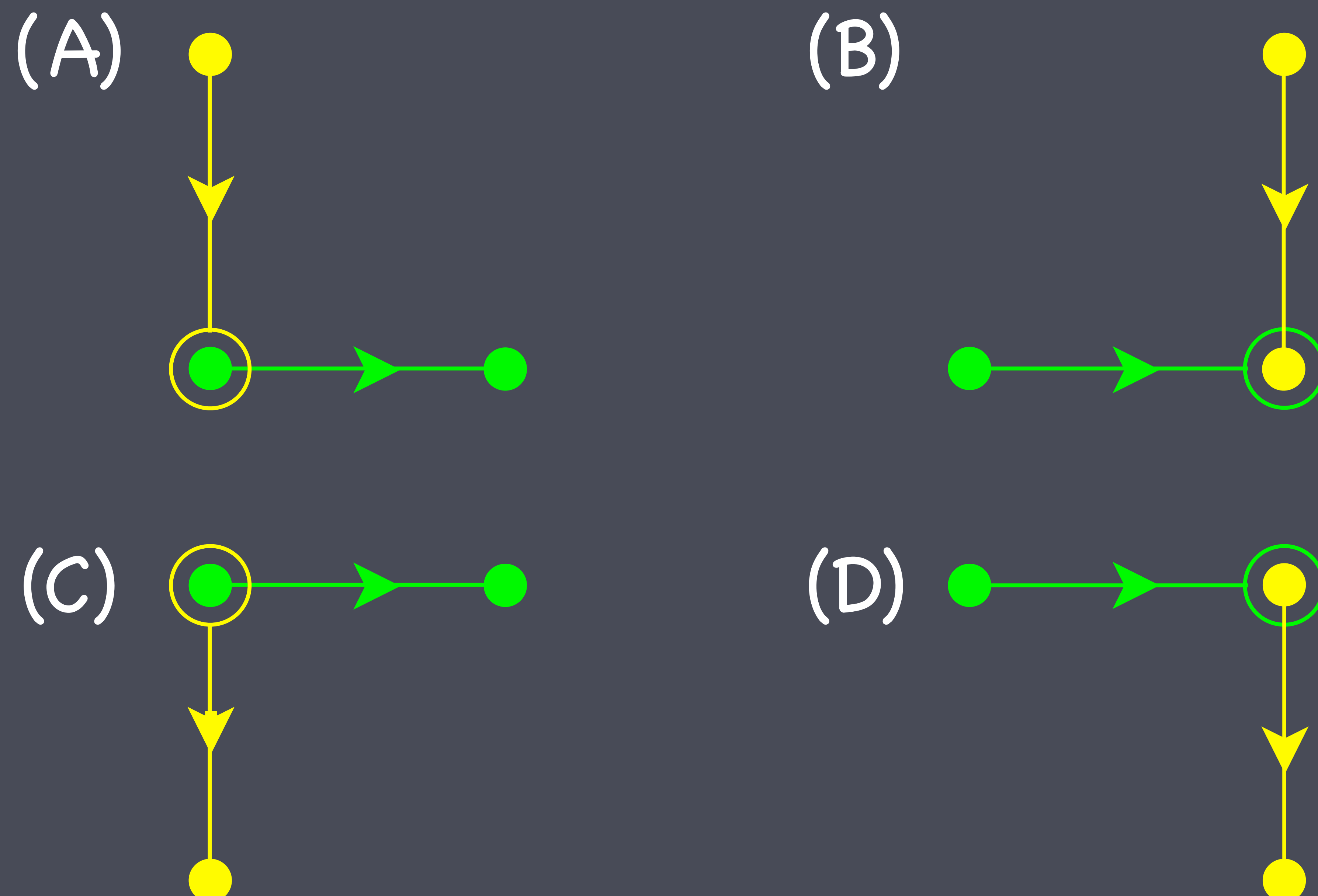
Today

- An optimizatoion example
- Residuals, SSR, Least squares
- Reminders:
  - No class on Monday (Thanksgiving)
  - OSH 4 due Wednesday
  - Regular PLQs.
- Wednesday - bring laptop/tablet for spreadsheet practice

A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 5 km/h. Another boat has been heading due east at 5 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

Which sketch is relevant?



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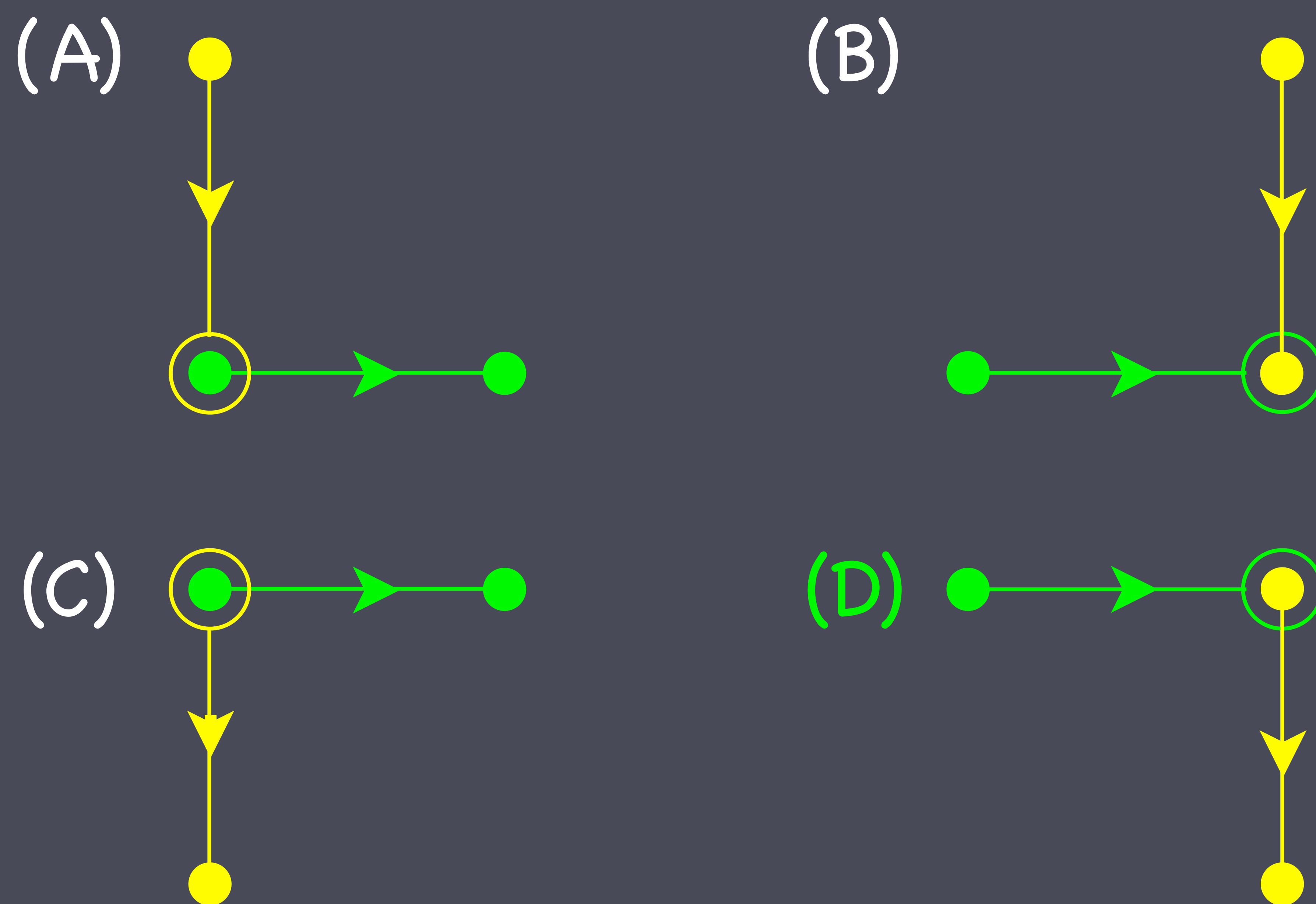

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Two quantities relevant to solving this problem are:

(A) $x = 5/60\ t$, $y = 5/60\ (60-t)$.

(B) $x = 5(t-2)$, $y=5(3-t)$.

(C) $x = 5-2$, $y=5+3$.

(D) $x = 5t-2$, $y=5t-3$.

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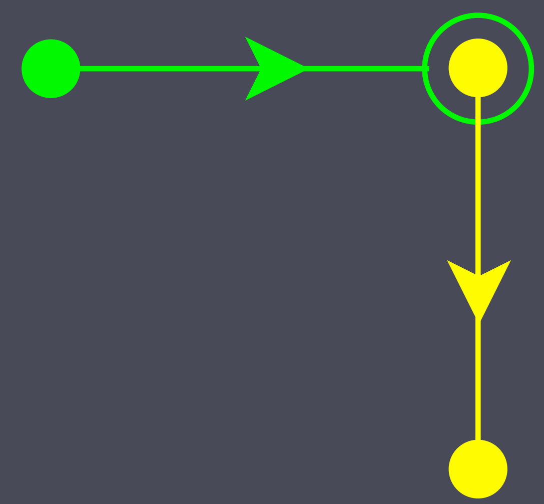

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Objective function to be minimized:

(A) $f(t) = 25|t| + 25|60-t|$

(B) $f(t) = 5/60 \text{ sqrt}(2t^2)$

(C) $f(t) = t^2 + (60-t)^2$

(D) $f(t) = \text{sqrt}(25(t-2)^2 + 25(3-t)^2)$

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Figure it out - this is a homework problem, after all.

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- When minimizing the function f(t), if the derivatives are easier to calculate, we can minimize the function _____ instead.

  (A) $g(t) = f(t)^2$

  (B) $h(t) = 1/f(t)$

  (C) $k(t) = f(t)^3$

  (D) You have to minimize f(t).

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- When minimizing the function f(t), if the derivatives are easier to calculate, we can minimize the function _____ instead.

(A) $g(t) = f(t)^2$ <--- if $f(t) \geq 0$.

(B) $h(t) = 1/f(t)$ <--- no, maximize (if $f(t) \neq 0$).

(C) $k(t) = f(t)^3$ <--- yes

(D) You have to minimize f(t).

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Expectation: The boats will be closest together...

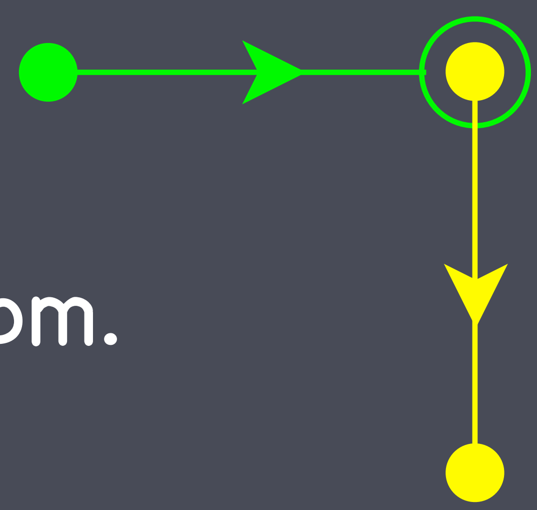
(A) at 2 pm.

(B) at 3 pm.

(C) sometime between 2 pm and 3 pm.

(D) before 2 pm.

(E) after 2 pm.



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Constraint:

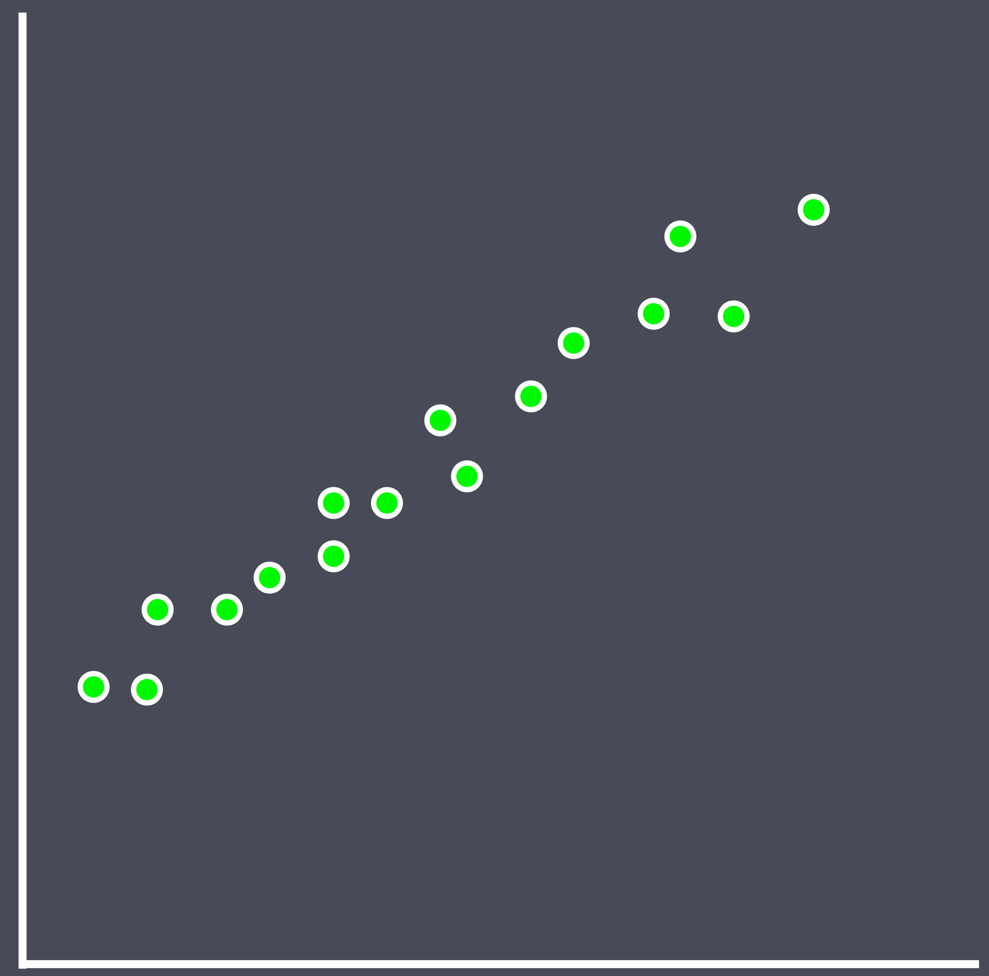
(A) The minimum distance must occur between 2 pm and 3 pm.

(B) $x(t)^2+y(t)^2 = t^2/6$.

(C) $x(t) = 60-y(t)$.

(D) There isn't really a constraint for this problem.

Least squares model fitting



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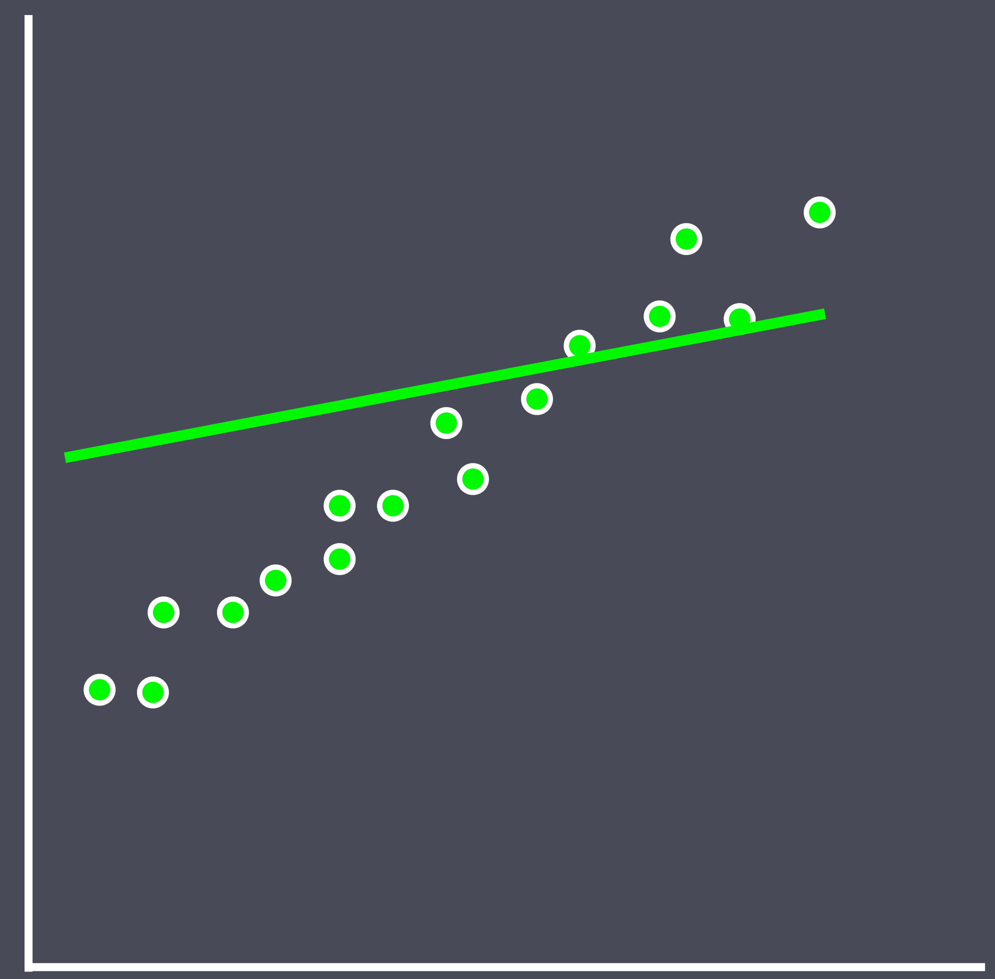
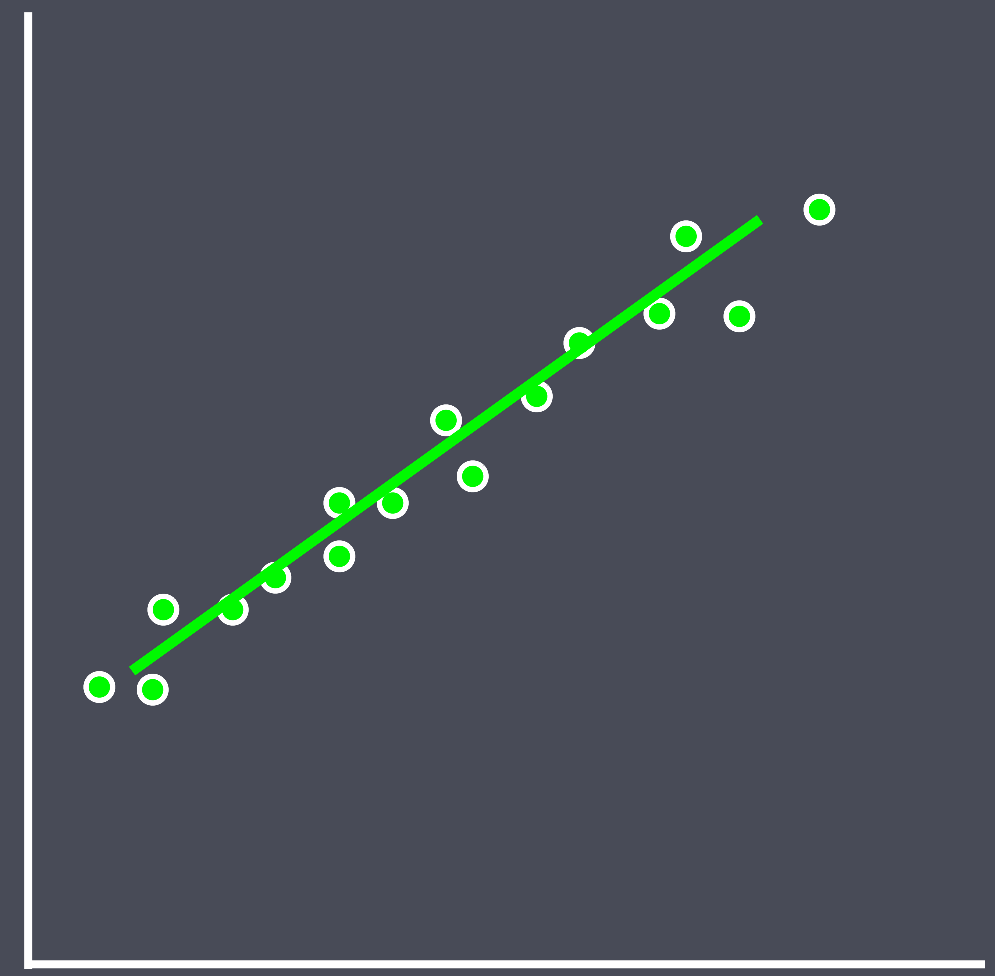


How do we find the best line
to fit through the data?

Least squares model fitting



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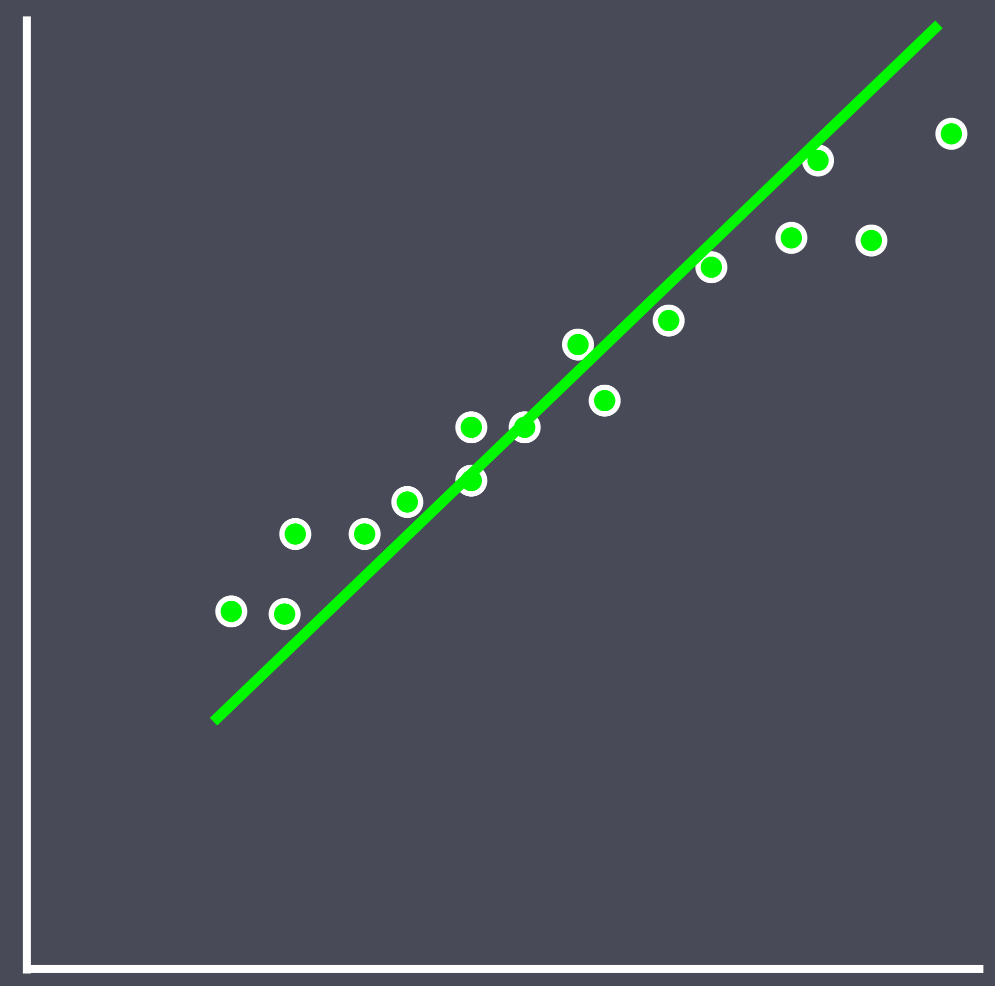

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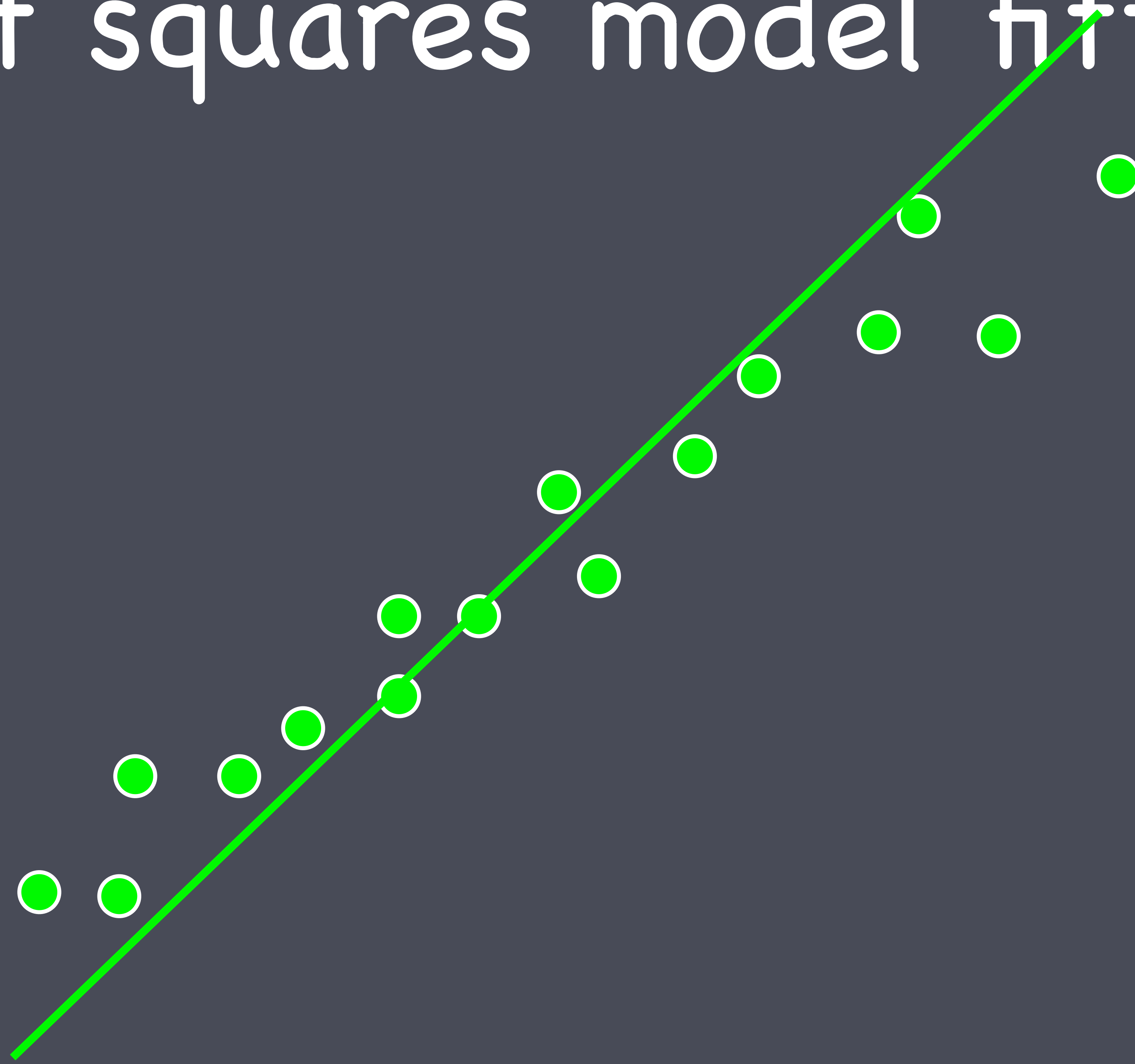

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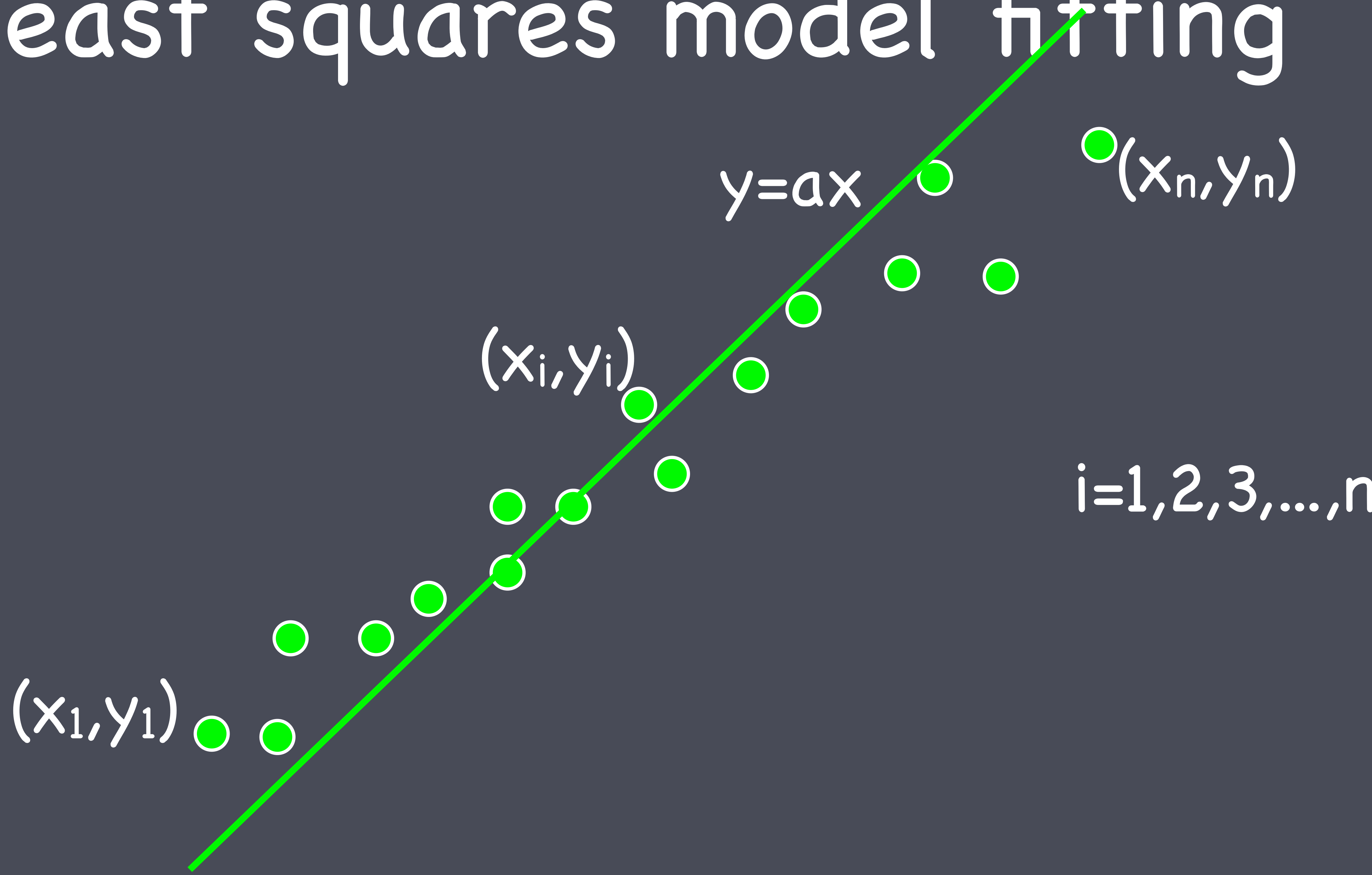

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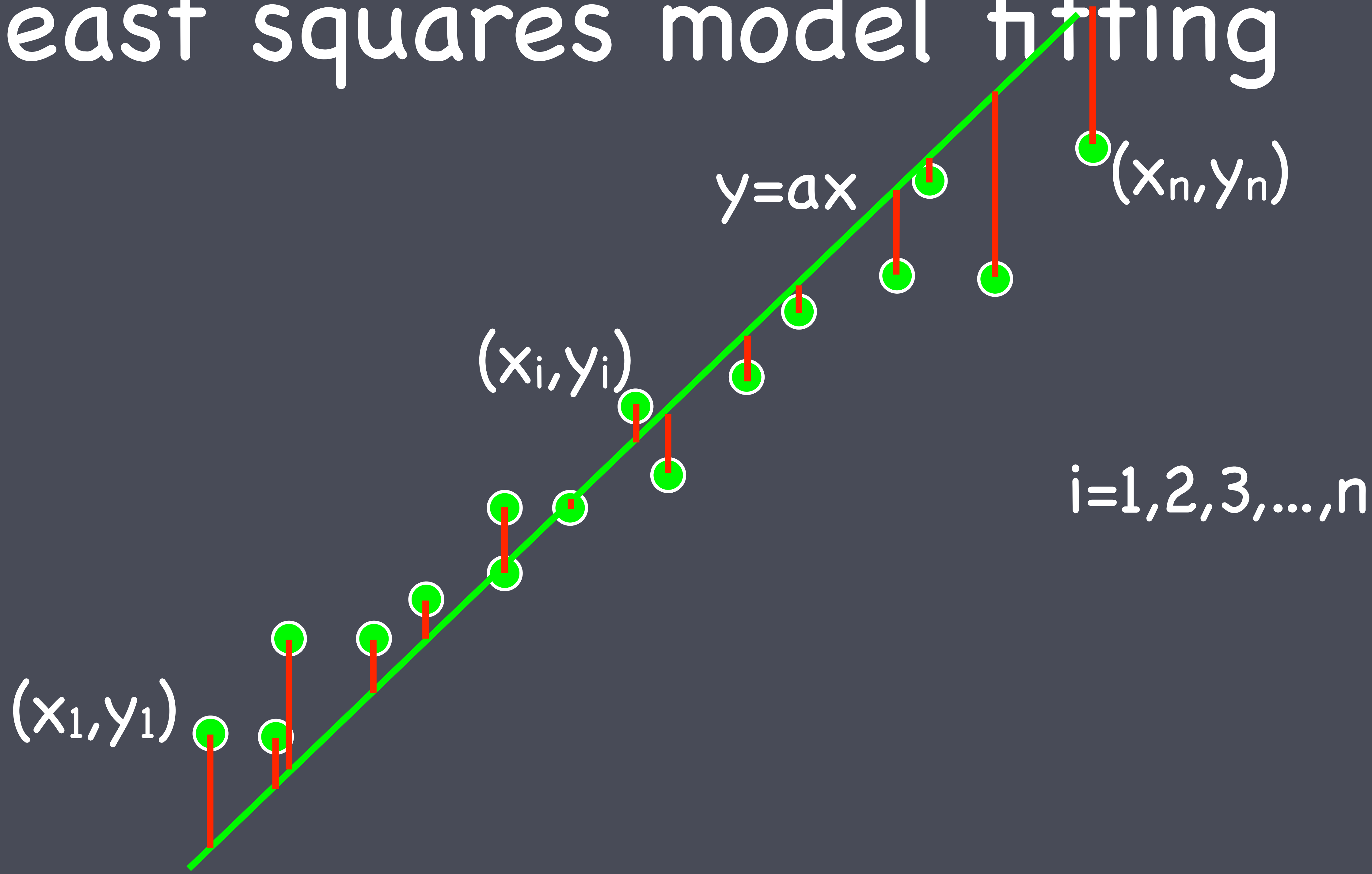

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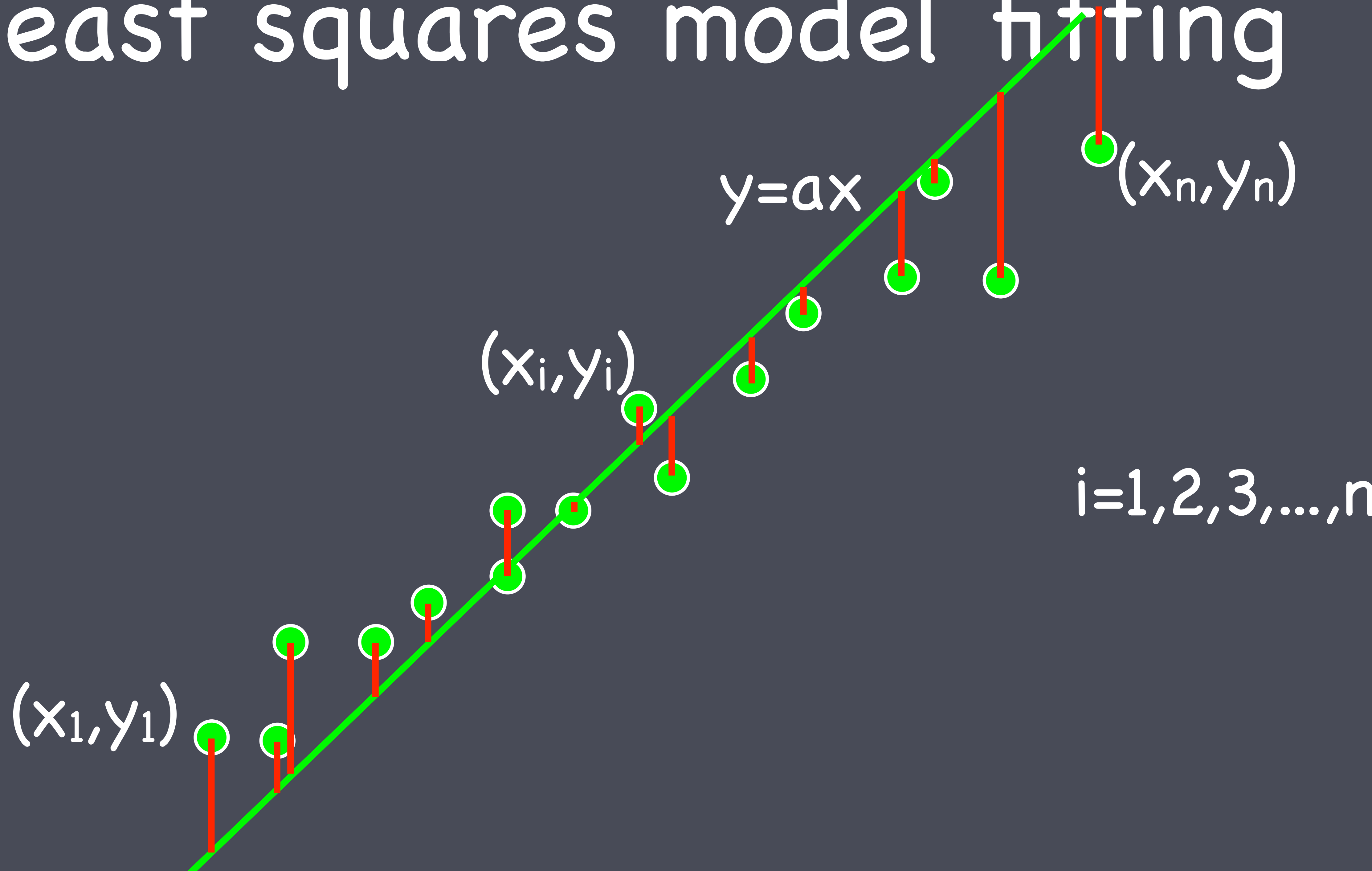


Each red bar is called a residual. We want all the residuals to be as small as possible.

The residuals are...

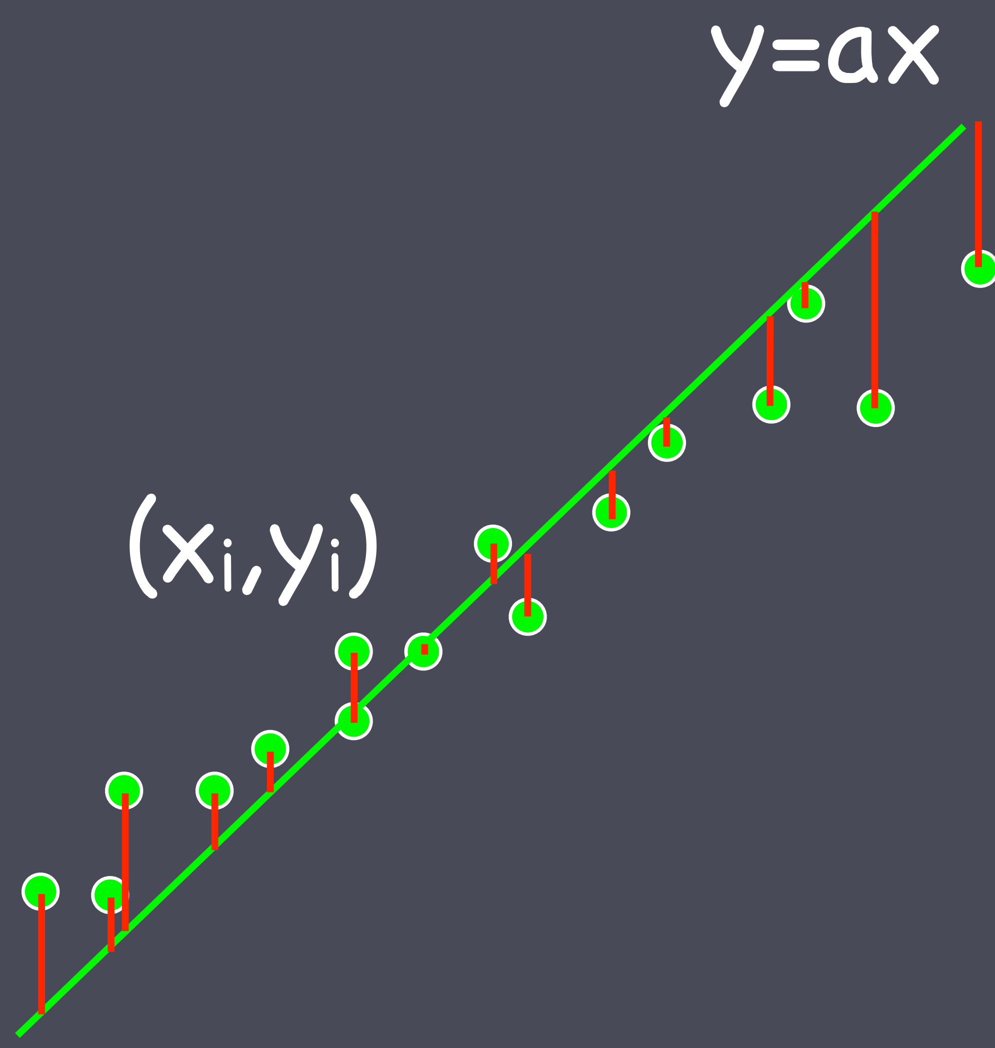
(A) $r_i = y_i^2 + x_i^2$

(B) $r_i = a^2(y_i^2 + x_i^2)$

(C) $r_i = y_i - ax_i$

(D) $r_i = y_i - x_i$

(E) $r_i = x_i - y_i$



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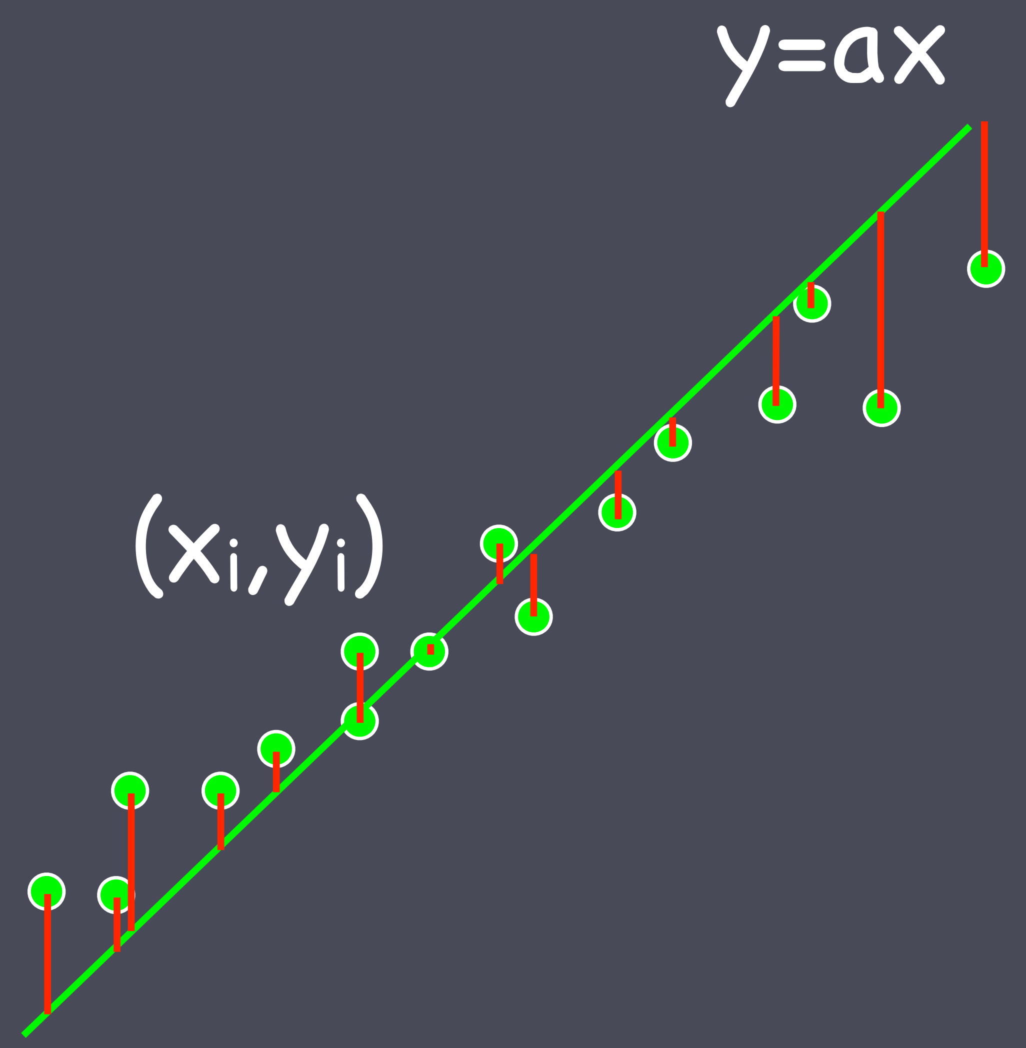

To minimize the residuals, we define the objective function…

(A) $f(a) = |y_1 - ax_1| + |y_2 - ax_2| + \ldots + |y_n - ax_n|$

(B) $f(a) = (y_1 - ax_1)^2 + (y_2 - ax_2)^2 + \ldots + (y_n - ax_n)^2$

(C) $f(a) = (y_1 - ax_1)(y_2 - ax_2)\ldots(y_n - ax_n)$

(D) $f(a) = (ay_1 - x_1)^2 + (ay_2 - x_2)^2 + \ldots + (ay_n - x_n)^2$

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(B) is called the "sum of squared residuals".
(A) is also reasonable but not as "good"
(take a stats class to find out more).

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

Define f(a):

(A) $SSR(a) = |5-4a| + |7-6a|$

(B) $SSR(a) = (4-5a)^2 + (6-7a)^2$

(C) $SSR(a) = (5-4a)^2 + (7-6a)^2$

(D) $SSR(a) = (5-4-a)^2 + (7-6-a)^2$

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Recall: $f(a) = (y_1-ax_1)^2 + (y_2-ax_2)^2$

Find a so that y=ax fits (4,5), (6,7) in the "least squares" sense.

Find the a that minimizes SSR(a):

(A) $a = 7/6$

(B) $a = 5/4$

(C) $a = (7/6 + 5/4) / 2$

(D) $a = 31/26$

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$$= 5^2 - 2 \cdot 4 \cdot 5a + 4^2 a^2$$

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$$= (4\cdot 5 + 6\cdot 7) / (4^2 + 6^2) = 62/52$$

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$$= (4 \cdot 5 + 6 \cdot 7) / (4^2 + 6^2) = 62/52$$

$$= (x_1 \cdot y_1 + x_2 \cdot y_2) / (x_1^2 + x_2^2)$$