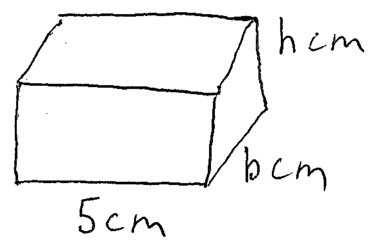
Name: Solution Quiz Score:____/20

Student Number:____________________

Answer questions in the space provided. Show your work.

1. A company distributes salt. It wants to package the salt in a box with dimensions 5 cm $\times\ b$ cm $\times\ h$ cm. The side of length 5 cm is fixed for efficient stacking in crates. Each package should contain 500 cm^3 of salt. The company wants to minimize the cost of producing the package, which is done by minimizing the surface area of the package.

   (a) (1 point) Sketch the box, labeling side lengths.



   (b) (2 points) What is the objective function that the company wants to minimize, in terms of b and h?

S (surface area) $= 2\cdot 5\cdot b + 2\cdot b\cdot h + 2\cdot 5\cdot h$

$= 10b + 2bh + 10h$

   (c) (2 points) What is the constraint on the objective function, in terms of b and h?

Volume $= 500\ cm^3$

$\Rightarrow 5bh = 500$

(d) (6 points) What b minimizes the constrained objective function?

From part (c) $5bh = 500 \Rightarrow h = \frac{100}{b}$

S, from part (b), becomes $S = 10b + 2 \cdot b \cdot \frac{100}{b} + 10 \cdot \frac{10}{b}$

$= 10b + 200 + \frac{1000}{b}$

$\frac{dS}{db} = 10 - \frac{1000}{b^2}$,

critical points occur where $\frac{dS}{db} = 0 \Rightarrow b^2 = 100 \Rightarrow b = 10$.

$\frac{d^2S}{db^2} = \frac{2 \cdot 1000}{b^3} > 0$ for $b > 0 \Rightarrow b = 10$ is a global minimum. $\boxed{b = 10}$

Note: $S(b)$ is similar in form to $R(x)$ in OSH 4

(e) (1 point) What h minimizes the constrained objective function?

$h = \frac{100}{b} \Rightarrow h = 10$ when $b = 10$.

$\boxed{h = 10}$

(f) (1 point) What is the minimal surface area of the package?

$S = 10b + 2bh + 10h \Rightarrow$ when $b = 10$ and $h = 10$

$S = 10 \cdot 10 + 2 \cdot 10 \cdot 10 + 10 \cdot 10 = 400 \text{ cm}^2$

$\boxed{S = 400 \text{ cm}^2}$

2. (7 points) A cylindrical cell lengthens at a rate of 3 μm/hr, while maintaining a constant volume of 32π μm^3 by constricting radially (the cell becomes longer and narrower in time, keeping the same volume). How is the radius of the cell changing in time when the cell is 2 μm long? [If you don't know the volume of a cylinder, the instructor will give you the formula but you will lose one point.]

Volume of a cylinder $= \pi r^2 l$, where r is the radius and l is the length.

Volume is constant in time $\Rightarrow \pi r^2 l = 32\pi \, um^3$.

r and l are functions of time

$\Rightarrow \pi \, r(t)^2 l(t) = 32\pi$

$$\frac{d}{dt}\left[\pi r(t)^2 l(t)\right] = \frac{d}{dt}(32\pi)$$

$$\Rightarrow \pi\left(r(t)^2 \frac{dl}{dt} + 2r\frac{dr}{dt} l(t)\right) = 0$$

$$\Rightarrow \frac{dr}{dt} = \frac{-r^2}{2rl}\frac{dl}{dt} = \frac{-r}{2l}\frac{dl}{dt}.$$

When $l = 2um$, $\frac{dl}{dt} = 3um/hr$ and $r^2 = \frac{\pi 32 \, um^3}{\pi \cdot 2 \, um} \Rightarrow r = 4$

Thus, $\frac{dr}{dt} = \frac{-4}{2\cdot 2} \cdot \frac{3um}{hr} = -3 \, um/hr$.

$$\boxed{\frac{dr}{dt} = -3 \, um/hr}$$