Today

- Solving linear DEs: $y' = ay + b$.
- Note: office hour today is either in my office 12-1 pm or in MATX 1102 from 12:30-1:30 pm due to construction.
Solution method analogy

Document camera...
Solution method analogy

Solve $x^2-4x-12 = 0$ by substituting $y = x-2$.

Plug in $x = y+2$:

- $0 = (y+2)^2 - 4(y+2) - 12$
- $0 = y^2 + 4y + 4 - 4y - 8 - 12 = y^2 - 16$
- $y = 4, -4$.

Idea: Shift so the vertex of the parabola is at $y=0$ instead of at $x=2$.

Idea: For $y'=ay+b$, shift so that steady state is at $z=0$. 
Example: solve $y' = 2y + 10$
General case: solving $y' = ay + b$

$y' = ay + b$

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Shift $y(t)$ down by $-b/a$: $z(t) = y(t) - (-b/a)$
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- Shift $y(t)$ down by $-b/a$: $z(t) = y(t) - (-b/a)$
- The equation for $z(t)$ is $z' = az$. Solve it.
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- Want to solve $y$ eqn. Know solution to $z$ eqn.
- Velocities are exactly the same, just shifted.
- Shift $y(t)$ down by $-b/a$: $z(t) = y(t) - (-b/a)$
- The equation for $z(t)$ is $z' = az$. Solve it.
- Substitute back to get $y(t)$ which solves $y' = ay + b$. 
To solve $y' = ay + b$ with $y(0) = y_0$, define a new function

$$z(t) = y(t) - (-b/a) \quad \text{(subtract steady st.)}$$

What equation does $z(t)$ solve?

Note that

$$z'(t) = y'(t)$$

So we can replace $y'$ by $z'$ and $ay + b$ by $az$

New equation: $z' = az$. Solved by $z(t) = z_0 e^{at}$.

$y(t) = z_0 e^{at} - b/a$. What is $z_0$? Set $t = 0$ . . . $z_0 = y_0 + b/a$

Solution: $y(t) = (y_0 + b/a) e^{at} - b/a$. 

General case: solving $y' = ay + b$
Where do these equations come from?
An application...
A drug delivered by IV accumulates at a constant rate $k_{IV}$. The body metabolizes the drug proportional to the amount of the drug.

\begin{align*}
(A) \quad d'(t) &= k_{IV} - k_m \, d(t) \\
(B) \quad d'(t) &= (k_{IV} - k_m) \, d(t) \\
(C) \quad d'(t) &= k_{IV} \, d(t) - k_m \\
(D) \quad d'(t) &= -k_{IV} + k_m \, d(t)
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\[(D) \quad d'(t) = -k_{IV} + k_m \, d(t)\]
\[ d'(t) = k_{IV} - k_m d(t) \text{ with IC } d(0)=0 \]

You measure the mass of drug in the patient’s body as a function of time, \( d(t) \), and plot it. Use the graph to determine the constant \( k_{IV} \).

(A) \( k_{IV} = 1 \)
(B) \( k_{IV} = 2 \)
(C) \( k_{IV} = 3 \)
(D) \( k_{IV} = 6 \)
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\[ D_{ss} = \frac{k_{IV}}{k_m} = 3 \]
A drug delivered by IV accumulates at a constant rate $k_{IV}$. The body metabolizes the drug proportional to the amount of the drug.

\[ d'(t) = k_{IV} - k_m d(t), \quad d(0) = 0. \]

(A) $d(t) = \frac{k_{IV}}{k_m} - \frac{k_{IV}}{k_m} e^{k_m t}$

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(A) \( d(t) = \frac{k_{IV}}{k_m} - \frac{k_{IV}}{k_m} e^{k_m t} \) ← exp growth (unstable s.s.)

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(D) \[ d(t) = \frac{k_{IV}}{k_m} - e^{-k_m t} \quad \leftarrow d(0) = \frac{k_{IV}}{k_m} - 1 \quad \times \]

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(B) $d(t) = \frac{k_{IV}}{k_m} - \frac{k_{IV}}{k_m} e^{-k_m t}$ ← how quickly does it get there? where is it going?

(C) $d(t) = \frac{k_{IV}}{k_m} - e^{k_m t}$ ← exp growth (unstable s.s.)

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