Today

- Solving linear DEs: $y'=ay+b$.
- Note: office hour today is either in my office 12-1 pm or in MATX 1102 from 12:30-1:30 pm due to construction.

Solution method analogy

Document camera...

Solution method analogy

- Solve $x^2-4x-12 = 0$ by substituting $y = x-2$.
- Plug in $x = y+2$:
  - $0 = (y+2)^2 - 4(y+2) - 12$
  - $0 = y^2 + 4y + 4 - 4y - 8 - 12 = y^2 - 16$
  - $y = 4, -4$.
- Idea: Shift so the vertex of the parabola is at $y=0$ instead of at $x=2$.
- Idea: For $y'=ay+b$, shift so that steady state is at $z=0$.

Example: solve $y'=2y+10$

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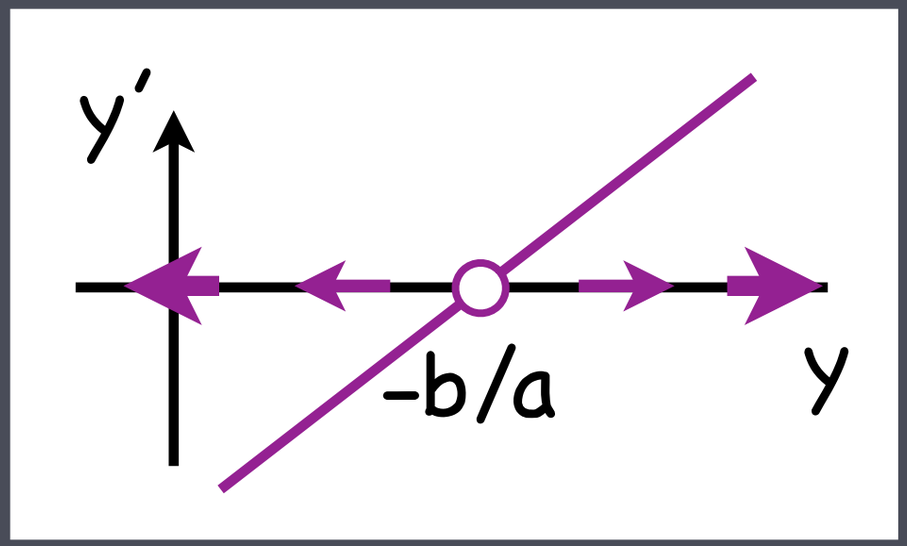
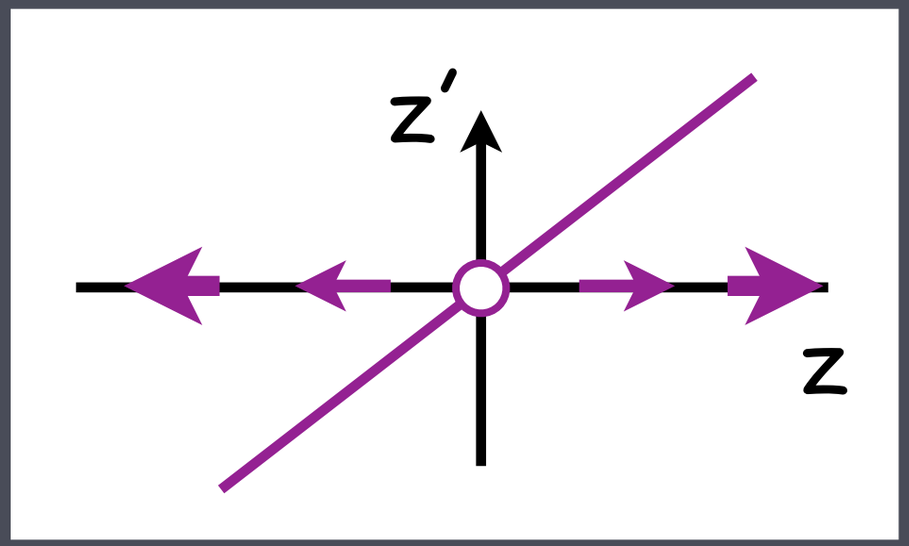
General case: solving y′=ay+b

$y' = ay + b$



$z' = az$



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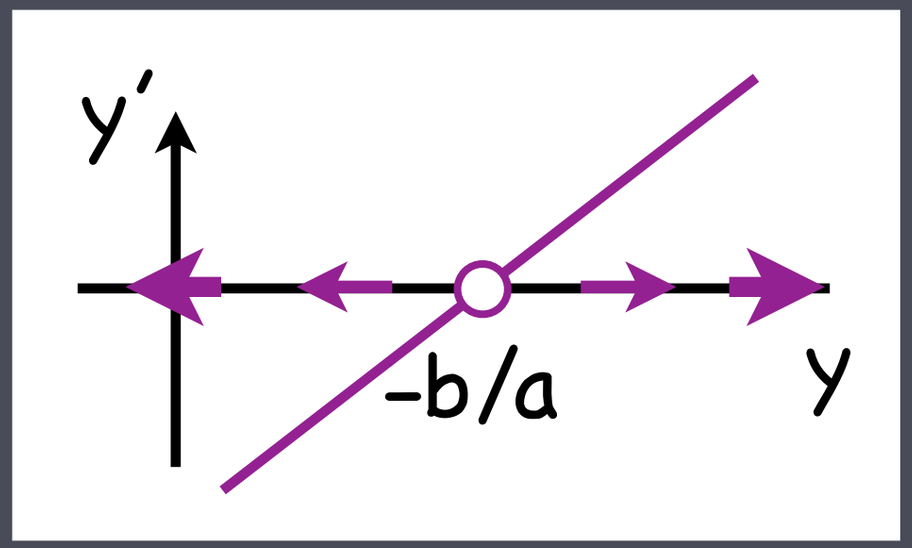

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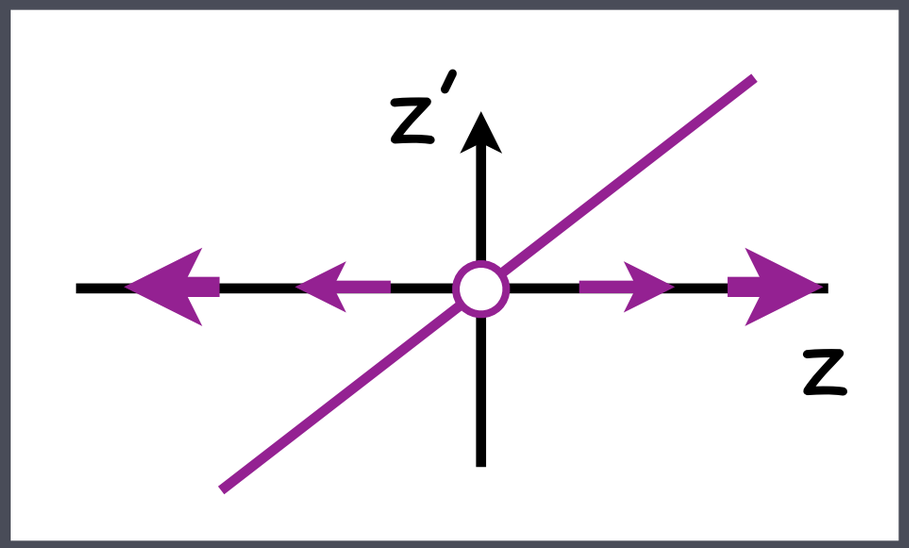

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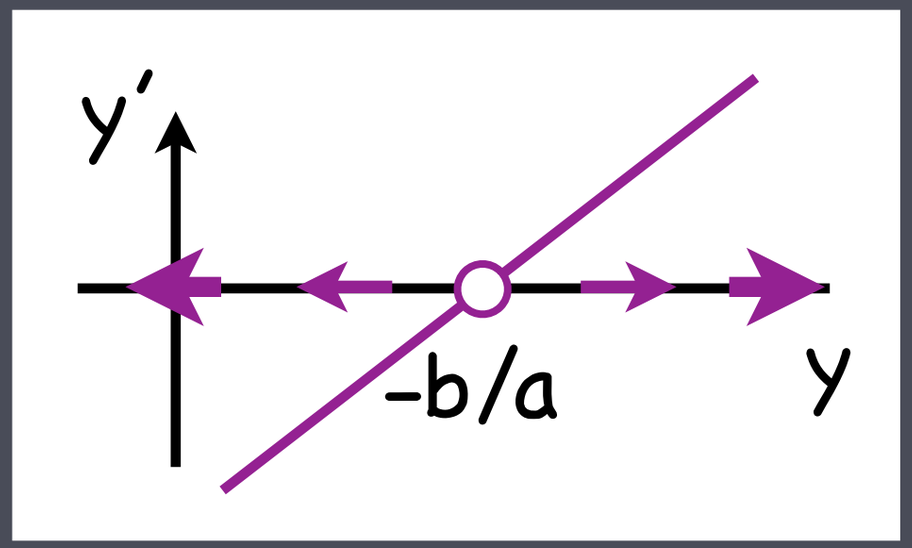

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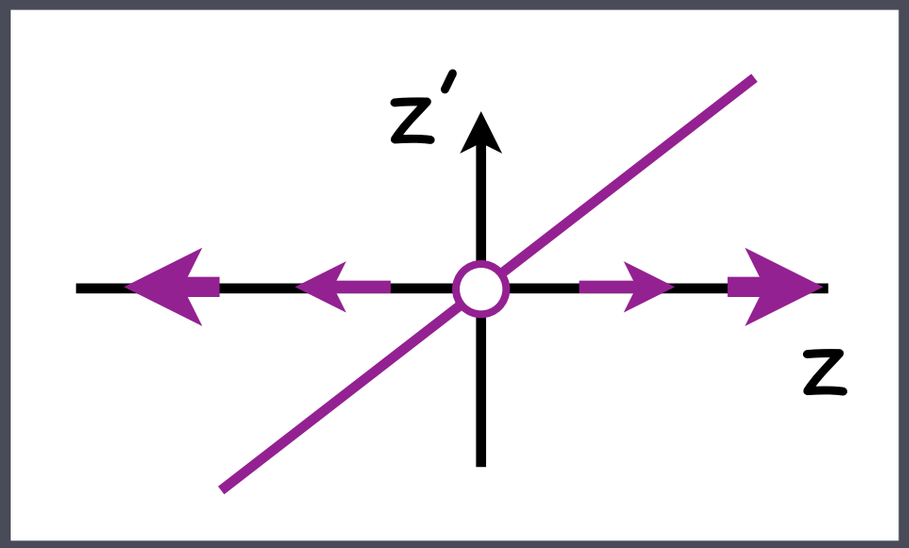

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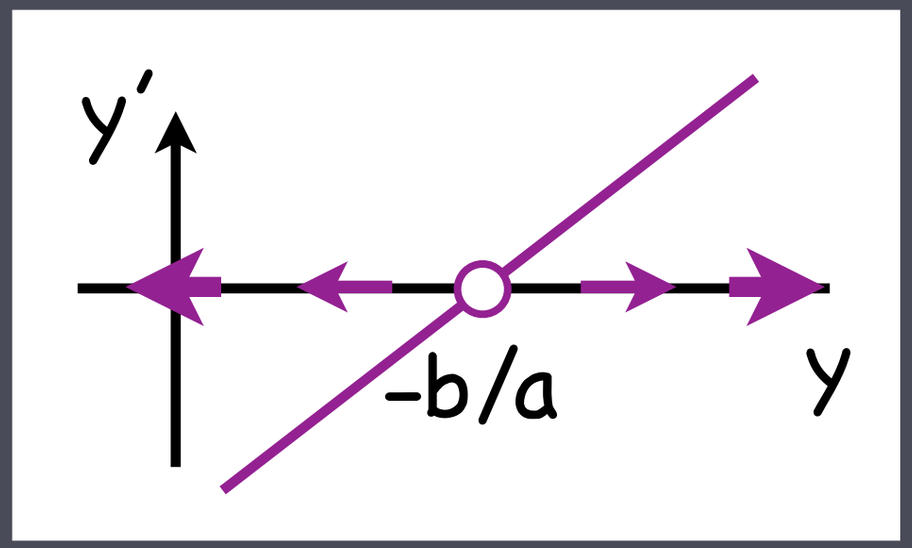

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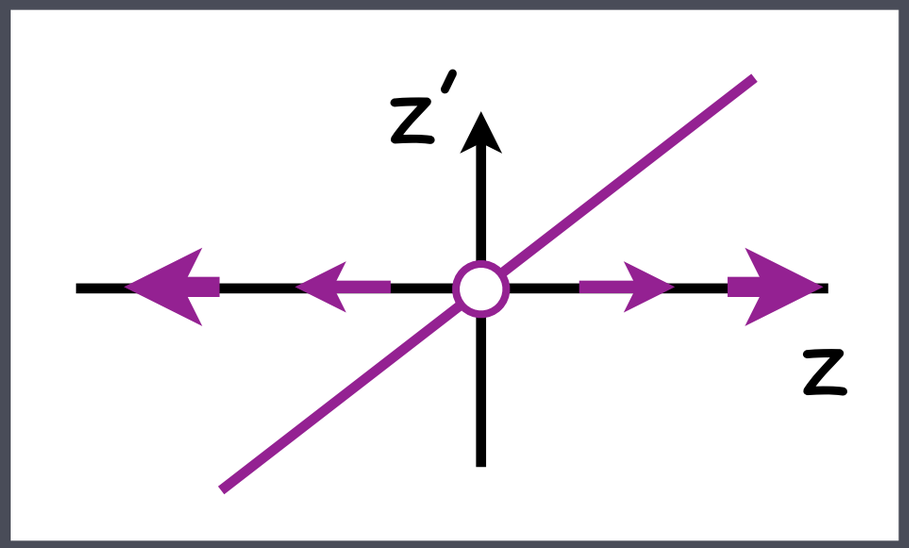

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- Velocities are exactly the same, just shifted.
- Shift y(t) down by -b/a: z(t) = y(t) - (-b/a)

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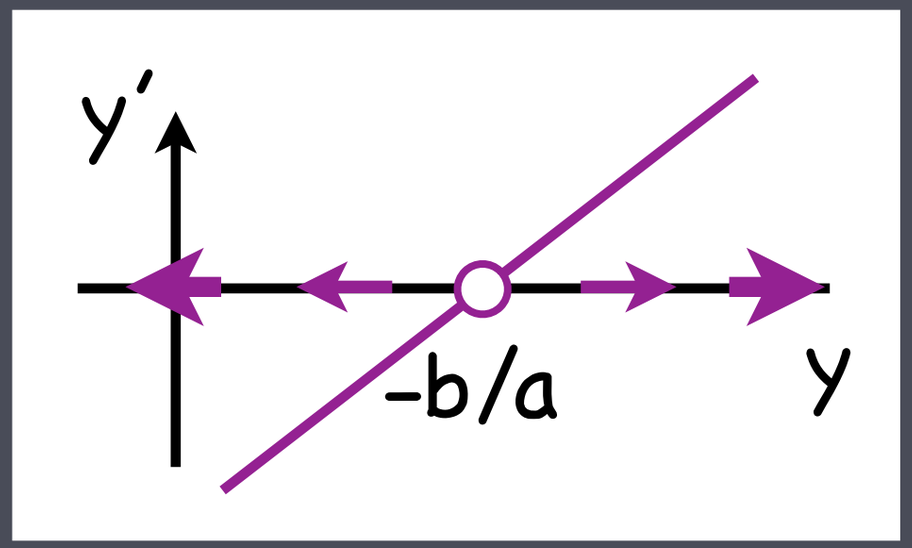

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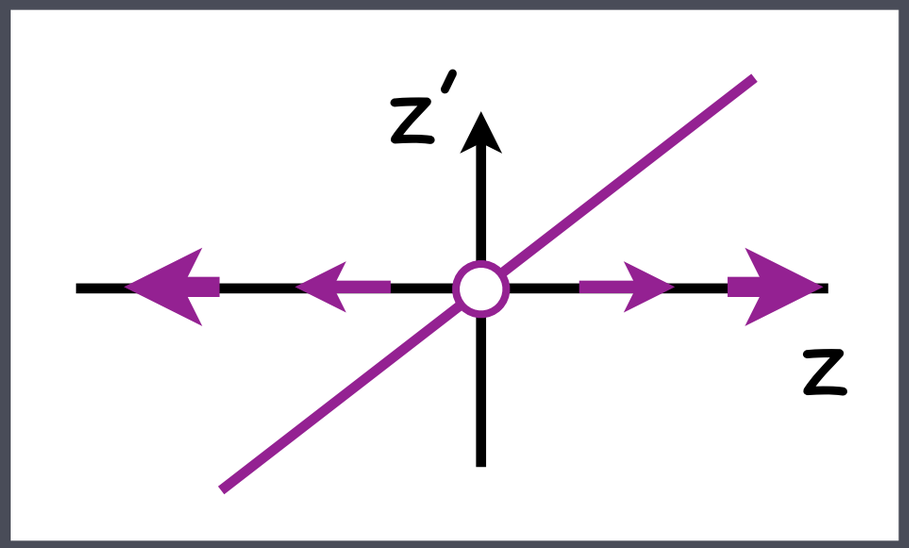

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- Shift $y(t)$ down by $-b/a$: $z(t) = y(t) - (-b/a)$
- The equation for $z(t)$ is $z'=az$. Solve it.

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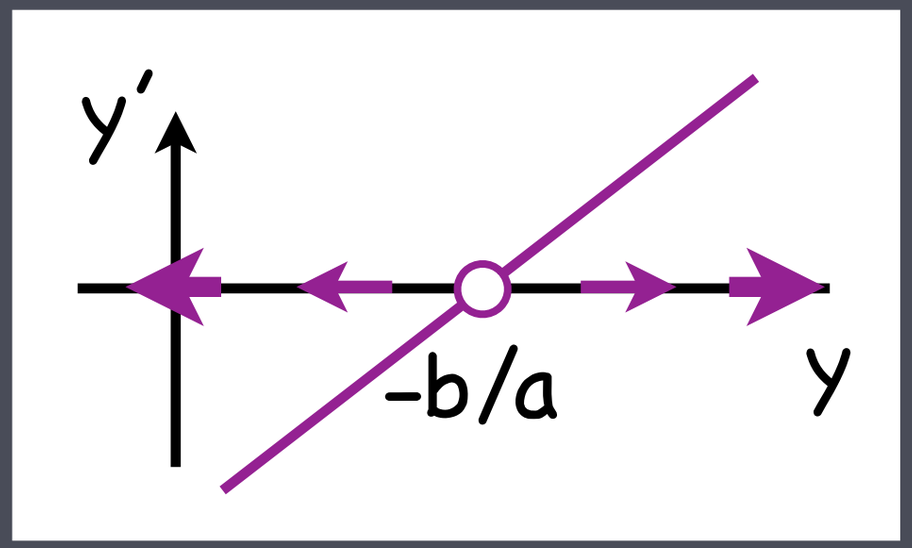

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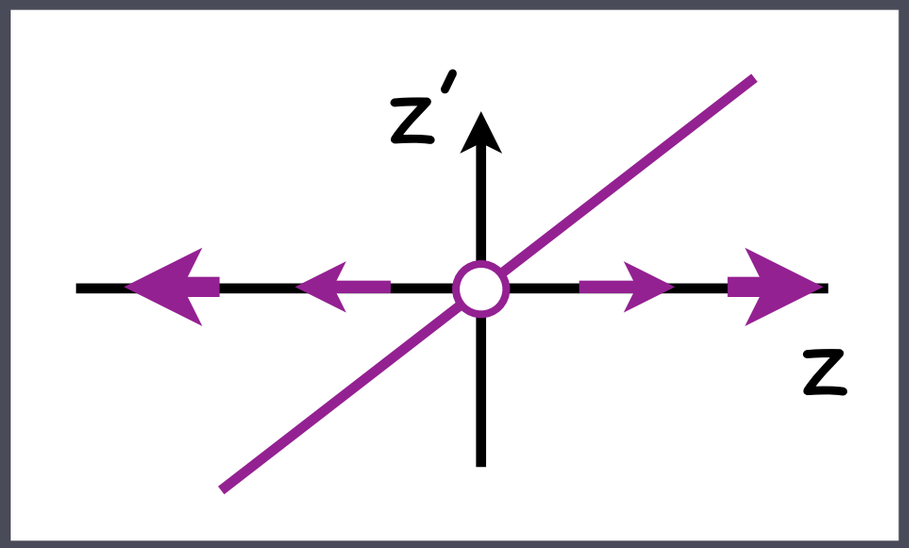

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- Velocities are exactly the same, just shifted.
- Shift $y(t)$ down by $-b/a$: $z(t) = y(t) - (-b/a)$
- The equation for $z(t)$ is $z'=az$. Solve it.
- Substitute back to get $y(t)$ which solves $y'=ay+b$.

General case: solving $y'=ay+b$

- To solve $y' = ay+b$ with $y(0)=y_0$, define a new function

  $$z(t) = y(t) - (-b/a) \quad \text{(subtract steady st.)}$$

- What equation does $z(t)$ solve?
- Note that

  $$z'(t) = y'(t)$$

- So we can replace y' by z' and $ay+b$ by az
- New equation: $z' = az$. Solved by $z(t) = z_0 e^{at}$.
- $y(t) = z_0 e^{at} - b/a$. What is z_0? Set $t=0$ . . . $z_0=y_0+b/a$
- Solution: $y(t) = (y_0 + b/a) e^{at} - b/a$.

Where do these equations come from?
An application...

A drug delivered by IV accumulates at a constant rate k_{IV}. The body metabolizes the drug proportional to the amount of the drug.

(A) $d'(t) = k_{IV} - k_m\, d(t)$

(B) $d'(t) = (k_{IV} - k_m)\, d(t)$

(C) $d'(t) = k_{IV}\, d(t) - k_m$

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$$d'(t) = k_{IV} - k_m\, d(t) \quad \text{with IC } d(0)=0$$

You measure the mass of drug in the patient's body as a function of time, d(t), and plot it. Use the graph to determine the constant k_{IV}.

(A) $k_{IV} = 1$

(B) $k_{IV} = 2$

(C) $k_{IV} = 3$

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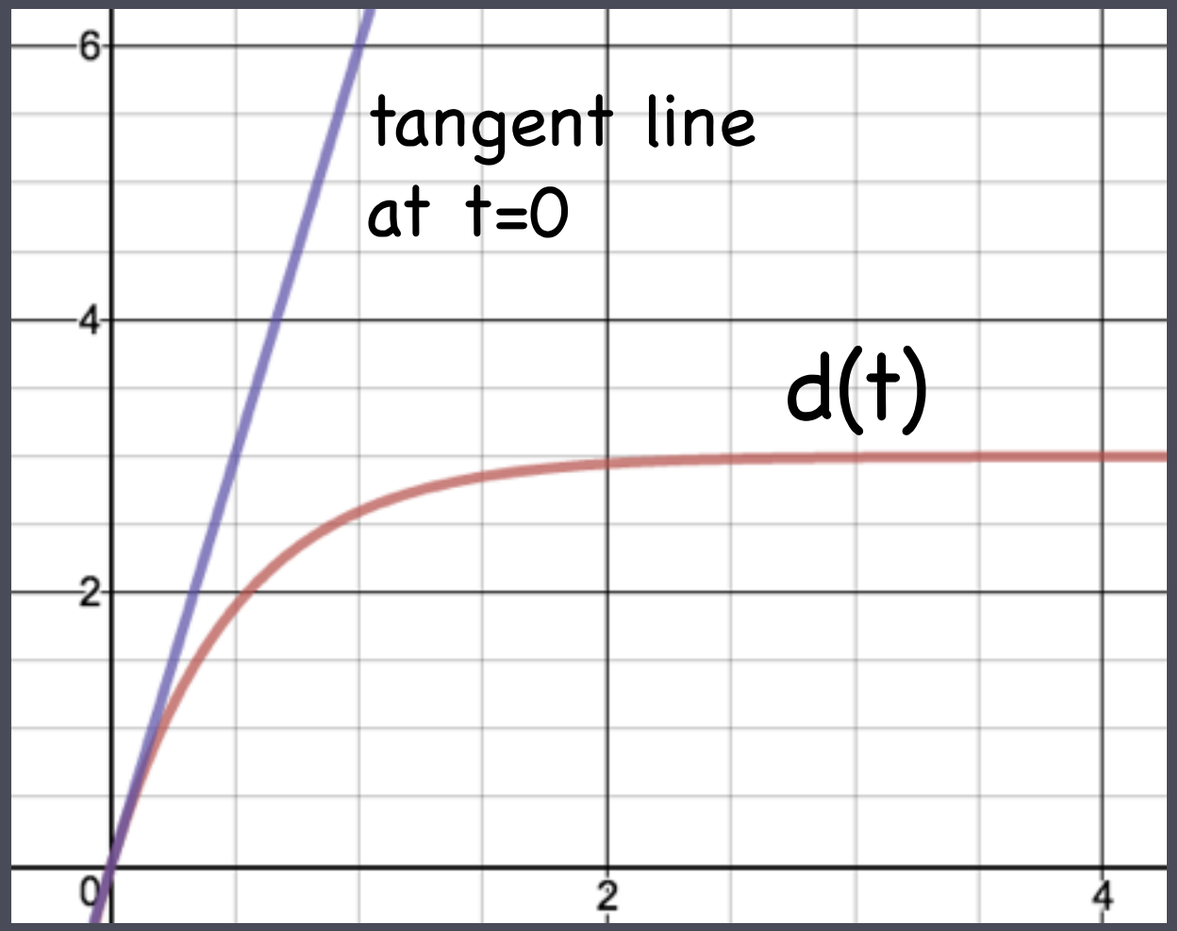

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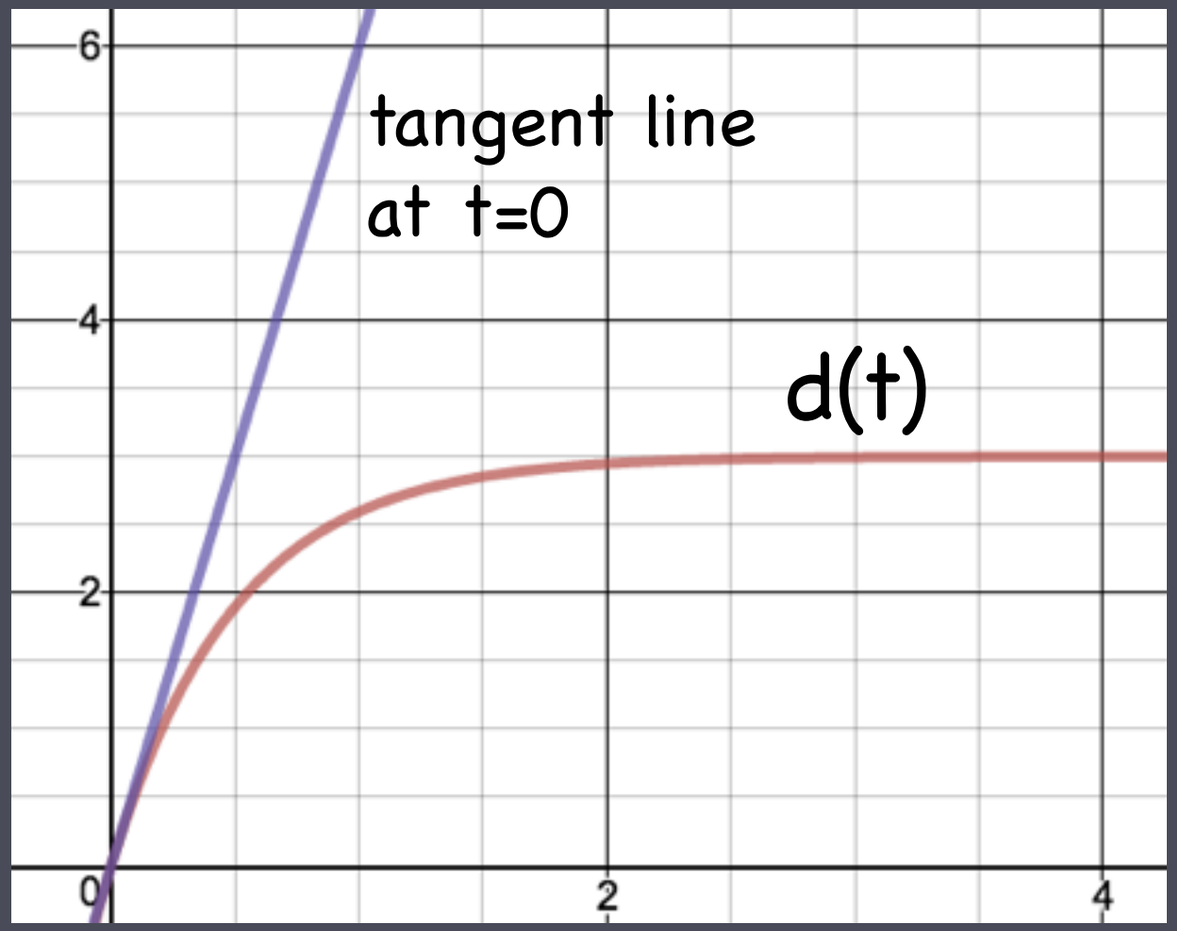

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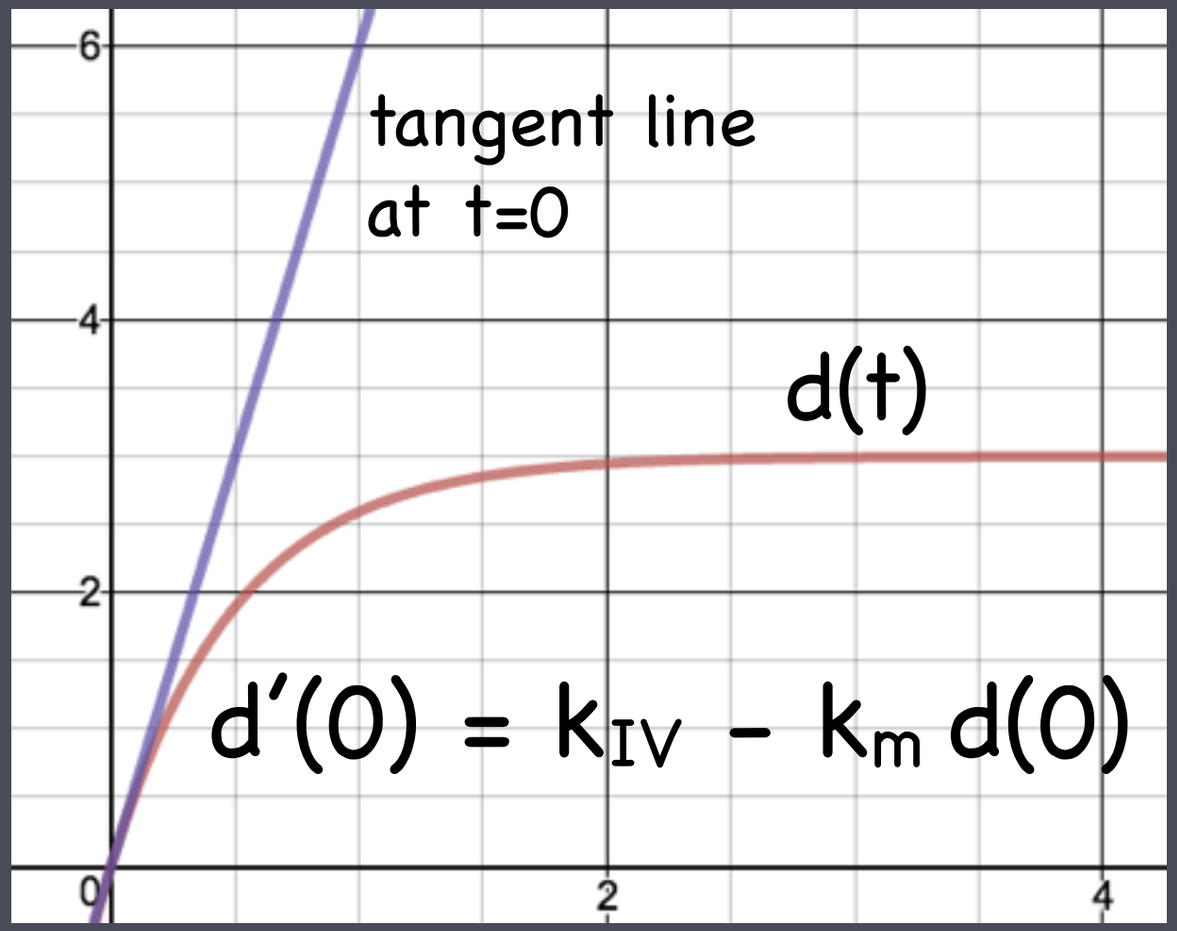

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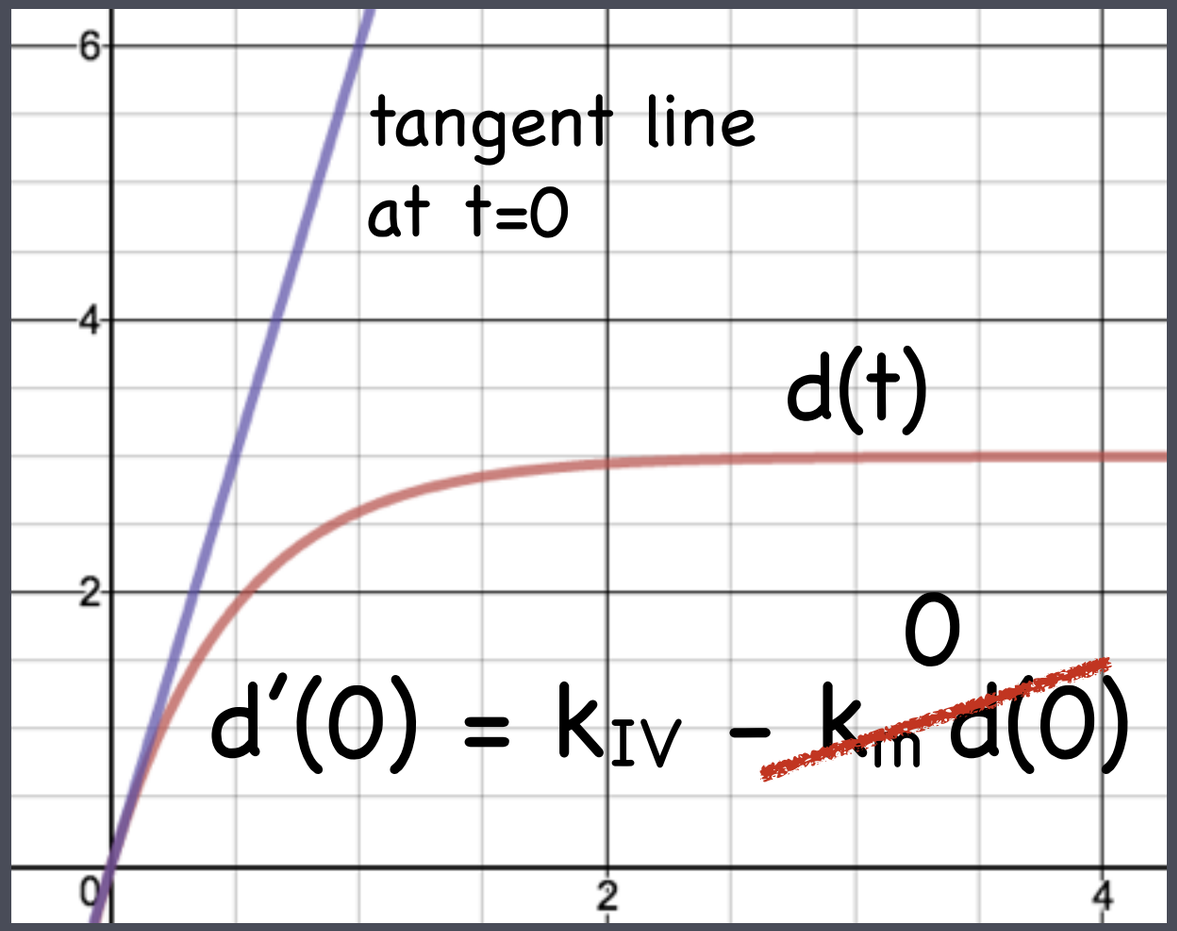

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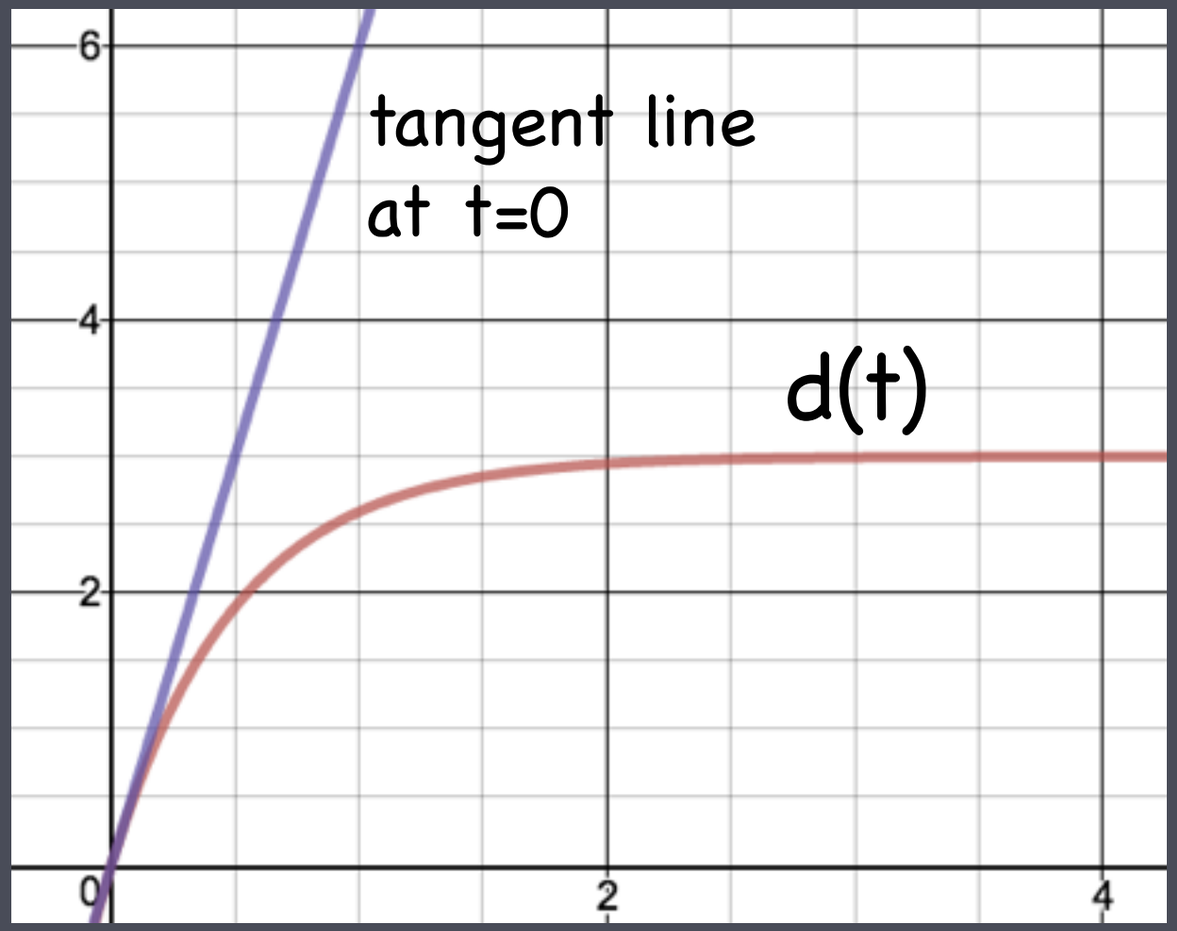

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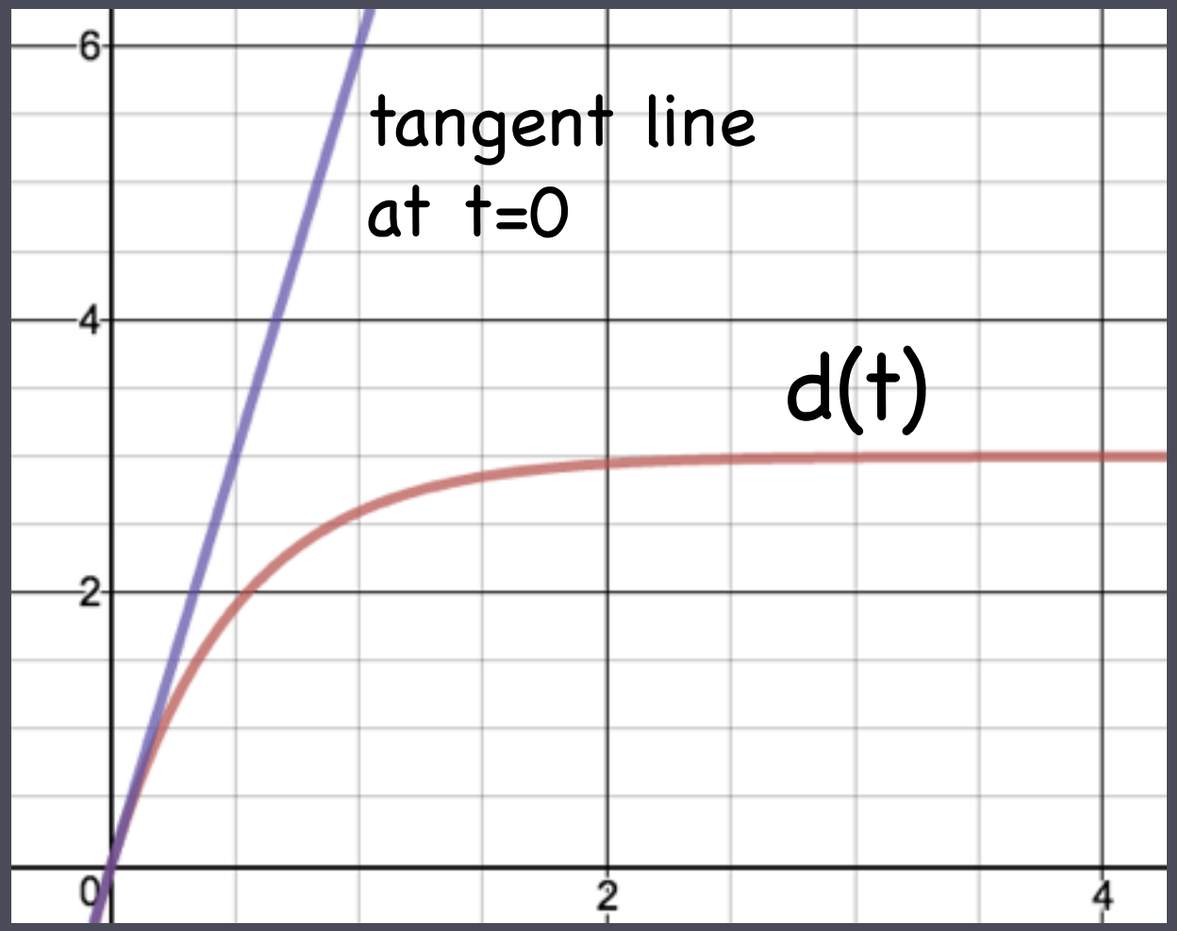

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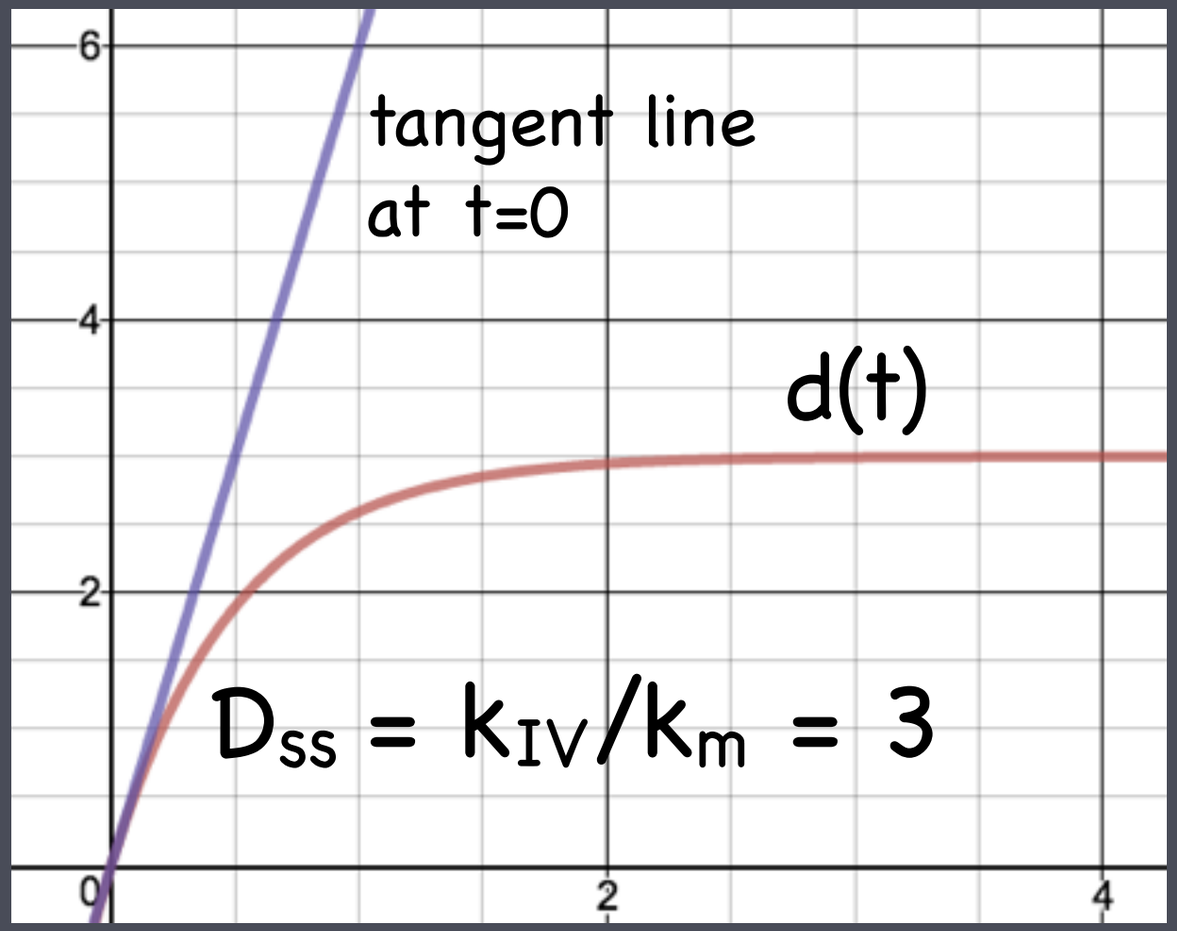

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coeff on exp makes IC work out

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