Today

- Diagnostic interpretation
- Continuity
- Some clicker questions about differentiability.
- Reminders:
    - Wed - PL3.2,
    - Thurs - Assignment 2,
    - Fri - Quiz 1

Diagnostic results and interpretation

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- If you didn’t pass, talk to me.
- Everyone else, fill in your gaps asap because you will fall into your pre-calc holes in this course!

Continuous functions

When $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$

we say that f(x) is continuous at x=a.

- Examples of categories of continuous functions:
  - Polynomials
  - Exponentials
  - sin, cos
- These are all continuous for all real x.

Ensuring continuity

For what value of a is the following function continuous?

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$

(A) a = 3

(B) a = -3

(C) a = 0

(D) a = 1

(E) Don’t know.

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https://www.desmos.com/calculator/obtqmika1u

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Types of limits we'll talk about

- Points of continuity: $\lim_{x \to a} f(x) = f(a)$
- Hole-in-the-graph (like derivative limit)
- Limits at $\pm\infty$ (horizontal asymptotes)

What is $\lim_{x \to 1} \frac{x^2 - 4}{x - 2}$ **?**

(A) 3

(B) -3

(C) Not defined.

(D) ∞

(E) Don’t know.

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Limit at a point of continuity.

What is $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ **?**

(A) 0

(B) ∞

(C) 1

(D) 4

(E) Don’t know.

What is $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ **?**

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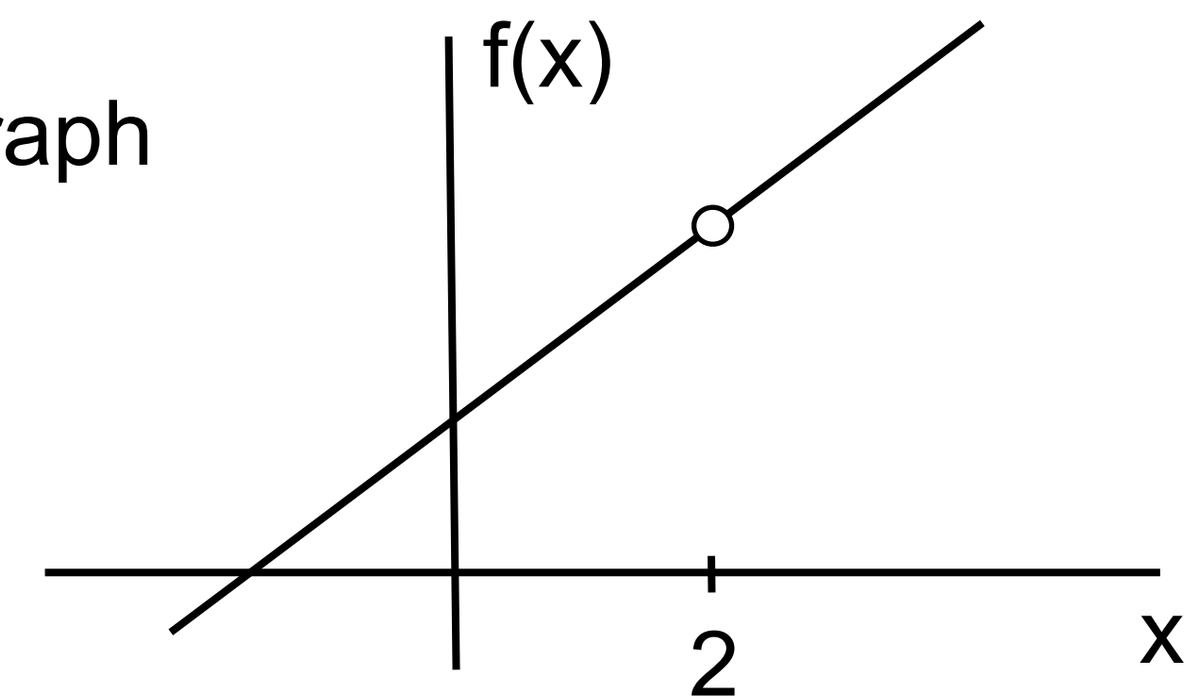
$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x - 2}$$

$$= \lim_{x \to 2} x + 2$$

$$= 4$$

Hole-in-the-graph type limit



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(for you to think about)

What is $\lim_{x \to \infty} \frac{5x^3 + 3x^2 - x + 1}{2x^4 - x^2 + 2}$ **?**

(A) 0

(B) ∞

(C) 5/2

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$$\frac{5x^3 + 3x^2 - x + 1}{2x^4 - x^2 + 2} \approx \frac{5x^3}{2x^4}$$

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$$\frac{5x^3 + 3x^2 - x + 1}{2x^4 - x^2 + 2} \approx \frac{5x^3}{2x^4} = \frac{5}{2x}$$

Use asymptotics to simplify top and bottom.

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Continuity and differentiability

- A function that is continuous at a point must also be differentiable at that point?
    - (A) True. (B) False. (E) Not sure.

is continuous

has a derivative

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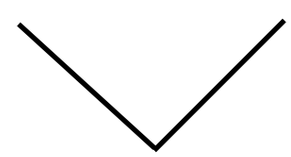

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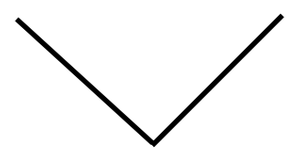

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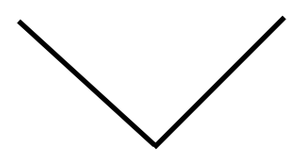

- A function that is differentiable at a point must also be continuous at that point?
  - (A) True. (B) False. (E) Not sure.
  - If a function is differentiable, then it looks like a straight line near the point and straight lines are continuous.