Today...

- Reminders- OSH 2 Monday! WW2 Thurs 7AM!
- Hole-in-graph examples - derivative at a point.
- Limit properties.
- Examples - continuous, infinite, indeterminate.
- Ensuring continuity (like OSH 2 #1).
- Limits at infinity (asymptotes).

Examples in which f′(2) does not exist

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

On the board...

Limit properties

1.

$$\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

2.

$$\lim_{x \to a} (cf(x)) = c \lim_{x \to a} f(x)$$

3.

$$\lim_{x \to a} (f(x) \cdot g(x)) = \left(\lim_{x \to a} f(x)\right) \cdot \left(\lim_{x \to a} g(x)\right)$$

4. Provided that $\lim_{x \to a} g(x) \neq 0$, we also have that

$$\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \left(\frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}\right).$$

Continuous functions

- Some example:
  - Polynomials
  - Exponentials
  - sin, cos, tan (cont. at all points in its domain)

$$\lim_{x \to 2} (x^3 - 2x + 1)$$

(A) 1

(B) 2

(C) 5

(D) ∞

$$\lim_{x \to 2} (x^3 - 2x + 1)$$

(A) 1

(B) 2

(C) 5

(D) ∞

Continuous at 2.

$$\lim_{x \to 2} \frac{1}{x^2 - 2}$$

(A) 1

(B) 2

(C) 5

(D) ∞

$$\lim_{x \to 2} \frac{1}{x^2 - 2}$$

(A) 1

(B) 2

(C) 5

(D) ∞

Continuous at 2.

$$\lim_{x \to 2} \frac{1}{(x-2)^2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{1}{(x-2)^2}$$

(A) 0

(B) 4

(C) ∞

(D) -∞

(E) Does not exist

Infinite limit or singular at 2.

$$\lim_{x\to 2} \frac{1}{(x-2)^2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$f'(2) = \lim_{x\to 2} x + 4 = 6$$

$$f'(2) = \lim_{x\to 2} \frac{1}{(x-2)^2} = \infty$$

Infinite limit or singular at 2.

$$\lim_{x \to 2} \frac{1}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{1}{x-2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Infinite limit or singular at 2.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form "0/0".

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) -∞

(E) Does not exist

Indeterminate form “0/0”.

$$\lim_{x \to 2} \frac{x-2}{x^2 - 4x + 4}$$

(A) 0

(B) 4

(C) ∞

(D) -∞

(E) Does not exist

$$\lim_{x \to 2} \frac{x-2}{x^2-4x+4}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form “0/0”.

Summary

1. Continuous.
2. Rational, nonzero denominator.
3. Rational, zero denominator, nonzero numerator:
    - asymptote, same direction (--> +/-infty).
    - asymptote, opposite directions (DNE) - emphasize diff lim = 2 and lim = infty.
4. Rational, zero denominator, zero numerator:
    - numer. saves the day (hole-in-graph).
    - numer. goes overkill saving the day (0).
    - numer. unable to save the day (see 3. for cases.)

Summary

1. lim (x-->2) of x^3-2x+1.
2. lim (x-->2) of x/(x^2-1).
3. Rational, zero denominator, nonzero numerator:
    - lim (x-->2) 1/(x-2)^2 = infinity.
    - lim (x-->2) 1/(x-2) does not exist.

4. Rational, zero denominator, zero numerator:
    - lim (x-->2) (x^2-4)/(x-2) = 4.
    - lim (x-->2) (x^2-4x+4)/(x-2) = 0.
    - lim (x-->2) (x-2)/(x^2-4x+4) does not exist.

Ensuring continuity

- For what value of a is the following function continuous at all points x?

(A) a=2

(B) a=-2

(C) a=0

(D) a=1

$$f(x) = \begin{cases} 4 - a^2 + 3x & x < 1 \\ x^2 + ax & x \geq 1 \end{cases}$$