Today

• Quiz 2

• Minima, maxima and inflection points

• Graphing
If you want to find a min/max of \( f'(x) \), look for points at which . . .

(A) \( f'(x) = 0 \).

(B) \( f'(x) = 0 \) and \( f''(x) \neq 0 \).

(C) \( f''(x) = 0 \).

(D) \( f''(x) = 0 \) and \( f'''(x) \neq 0 \).

(E) Don’t know.
If you want to find a min/max of \( f'(x) \), look for points at which . . .

(A) \( f'(x) = 0 \). \( \rightarrow \) potential extremum of \( f(x) \)

(B) \( f'(x) = 0 \) and \( f''(x) \neq 0 \).

(C) \( f''(x) = 0 \).

(D) \( f''(x) = 0 \) and \( f'''(x) \neq 0 \).

(E) Don’t know.
If you want to find a min/max of $f'(x)$, look for points at which... 

(A) $f'(x) = 0$. --> potential extremum of $f(x)$

(B) $f'(x) = 0$ and $f''(x) \neq 0$. --> extremum of $f(x)$

(C) $f''(x) = 0$.

(D) $f''(x) = 0$ and $f'''(x) \neq 0$.

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(C) \( f''(x) = 0 \). \( \rightarrow \) potential extremum of \( f'(x) \)

(D) \( f''(x) = 0 \) and \( f'''(x) \neq 0 \).

(E) Don’t know.
If you want to find a min/max of \( f'(x) \), look for points at which...

(A) \( f'(x) = 0 \). \quad \rightarrow \text{potential extremum of } f(x)

(B) \( f'(x) = 0 \) and \( f''(x) \neq 0 \). \quad \rightarrow \text{extremum of } f(x)

(C) \( f''(x) = 0 \). \quad \rightarrow \text{potential extremum of } f'(x)

(D) \( f''(x) = 0 \) and \( f'''(x) \neq 0 \). \quad \rightarrow \text{extremum of } f'(x)

(E) Don't know.
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If you want to find a min/max of $f'(x)$, look for points at which . . .

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(B) $f'(x) = 0$ and $f''(x) \neq 0$. --> extremum of $f(x)$

(C) $f''(x) = 0$. --> potential extremum of $f'(x)$

(D) $f''(x) = 0$ and $f'''(x) \neq 0$. --> extremum of $f'(x)$

(E) Don’t know.

This is “SDT” where the function considered is $f'$ instead of $f$! Would usually use “FDT”.
Potential IPs
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A potential IP is a point $a$ at which $f''(a)=0$ because that MIGHT be a min/max of $f'(x)$. 
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A potential IP is a point \( a \) at which \( f''(a) = 0 \) because that MIGHT be a min/max of \( f'(x) \).

If \( f''(x) \) changes sign at a potential IP of \( f(x) \), then it is an IP of \( f(x) \) because it's an extrema of \( f'(x) \).
Potential IPs

- A potential IP is a point \( a \) at which \( f''(a) = 0 \) because that MIGHT be a min/max of \( f'(x) \).

- If \( f''(x) \) changes sign at a potential IP of \( f(x) \), then it is an IP of \( f(x) \) because it’s an extrema of \( f'(x) \).

- If \( f''(x) \) does not change sign at a potential IP of \( f(x) \), then the potential IP is not an IP of \( f(x) \)!
Summary
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Use $f'(x)$ to determine intervals of **increase/decrease** of $f(x)$. 
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- Use $f'(x)$ to determine intervals of increase/decrease of $f(x)$.

- Solve $f'(x)=0$ to find potential extrema $(x=a)$. Check that $f'(x)$ changes sign at $a$ (FDT) or that $f''(a) \neq 0$ (SDT) to make sure.
Summary

- Use $f'(x)$ to determine intervals of increase/decrease of $f(x)$.

- Solve $f'(x)=0$ to find potential extrema $(x=a)$. Check that $f'(x)$ changes sign at $a$ (FDT) or that $f''(a) \neq 0$ (SDT) to make sure.

- Use $f''(x)$ to determine intervals of concave up/down.
Summary

Use $f'(x)$ to determine intervals of increase/decrease of $f(x)$.

Solve $f'(x)=0$ to find potential extrema ($x=a$). Check that $f'(x)$ changes sign at $a$ (FDT) or that $f''(a) 
eq 0$ (SDT) to make sure.

Use $f''(x)$ to determine intervals of concave up/down.

Solve $f''(x)=0$ to find potential inflection points ($x=a$). Check that $f''(x)$ changes sign at $a$ ("FDT") or that $f'''(a) 
eq 0$ ("SDT") to make sure.
Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.
(B) $f''(0) = 0$ so yes.
(C) $f'''(0) = 0$ so no.
(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.
(E) Don’t know.
Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

(E) Don't know.
Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

(E) Don’t know.

“Second DT” applied to $f'(x)$ - fails so no conclusion.
Does $f(x) = x^4$ have an inflection point?

(A) $f'(0) = 0$ so yes.

(B) $f''(0) = 0$ so yes.

(C) $f'''(0) = 0$ so no.

(D) $f''(0) = 0$ and $f''(x) > 0$ for all $x \neq 0$ so no.

(E) Don't know.

Not sure about (C)? Try this for $f(x) = x^5$. 

“Second DT” applied to $f'(x)$ - fails so no conclusion.
Does \( f(x) = x^4 \) have an inflection point?

(A) \( f'(0) = 0 \) so yes.

(B) \( f''(0) = 0 \) so yes.

(C) \( f'''(0) = 0 \) so no.

(D) \( f''(0) = 0 \) and \( f''(x) > 0 \) for all \( x \neq 0 \) so no. \( f''(x) = 12x^2 \)

(E) Don’t know.

Not sure about (C)? Try this for \( f(x)=x^5 \).