

Review for Midterm 1

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

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(E) Does not exist

Indeterminate form "0/0".

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$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Indeterminate form "0/0".

$$\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x - 2}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

(B) 4

(C) ∞

(D) $-\infty$

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Indeterminate form "0/0".

$$\lim_{x \rightarrow 2} \frac{x - 2}{x^2 - 4x + 4}$$

(A) 0

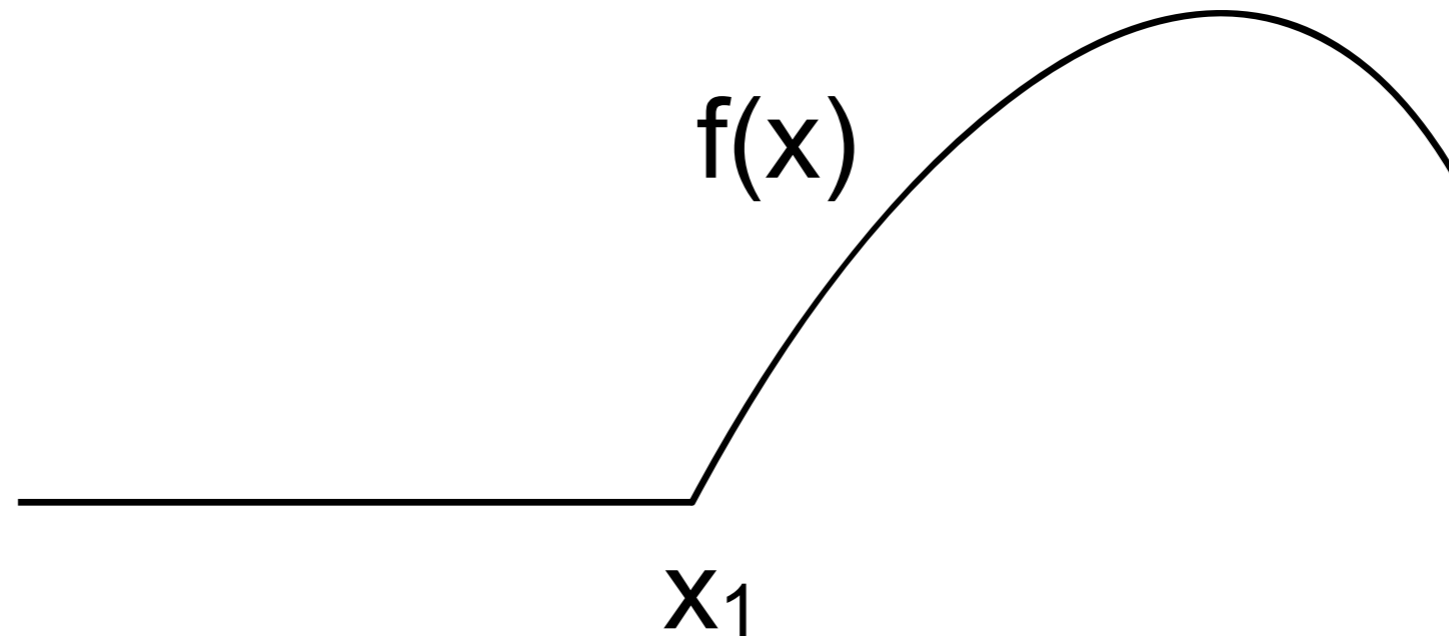
(B) 4

(C) ∞

(D) $-\infty$

(E) Does not exist

Limits



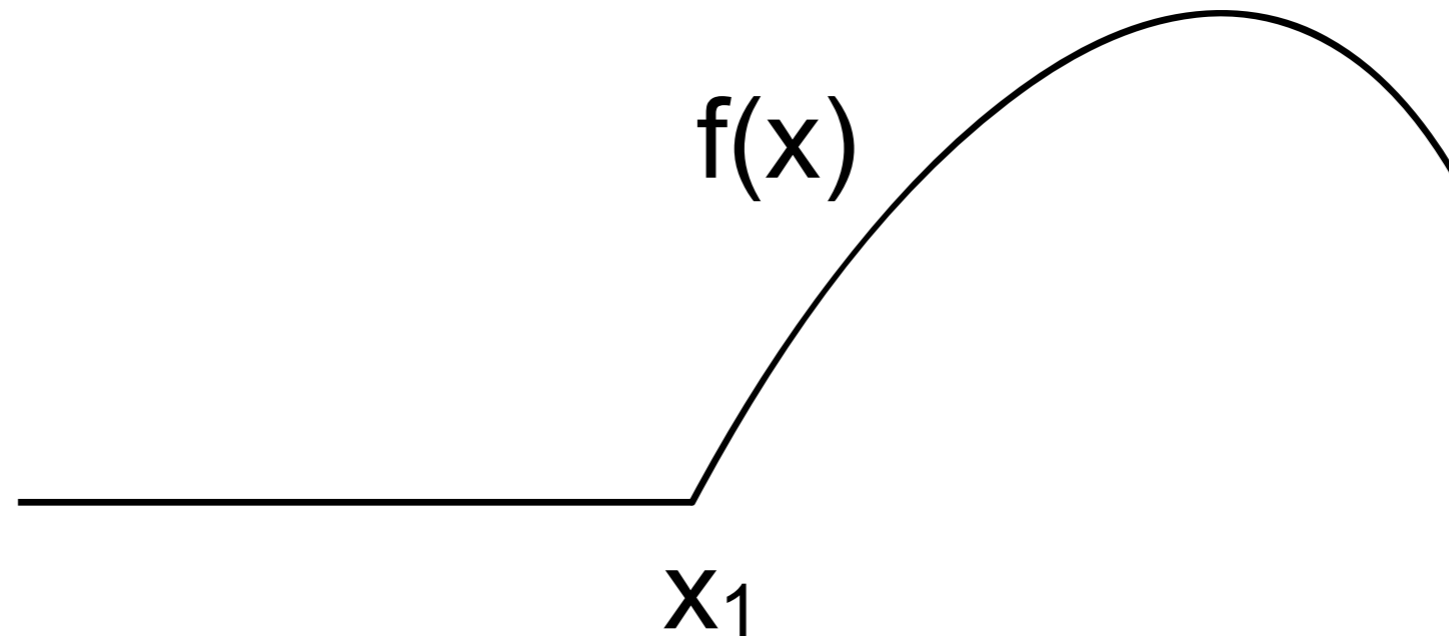
(A) $\lim_{h \rightarrow 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0$

(B) $\lim_{h \rightarrow 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0$

(C) Both (A) and (B)

(D) The limit does not exist.

Limits



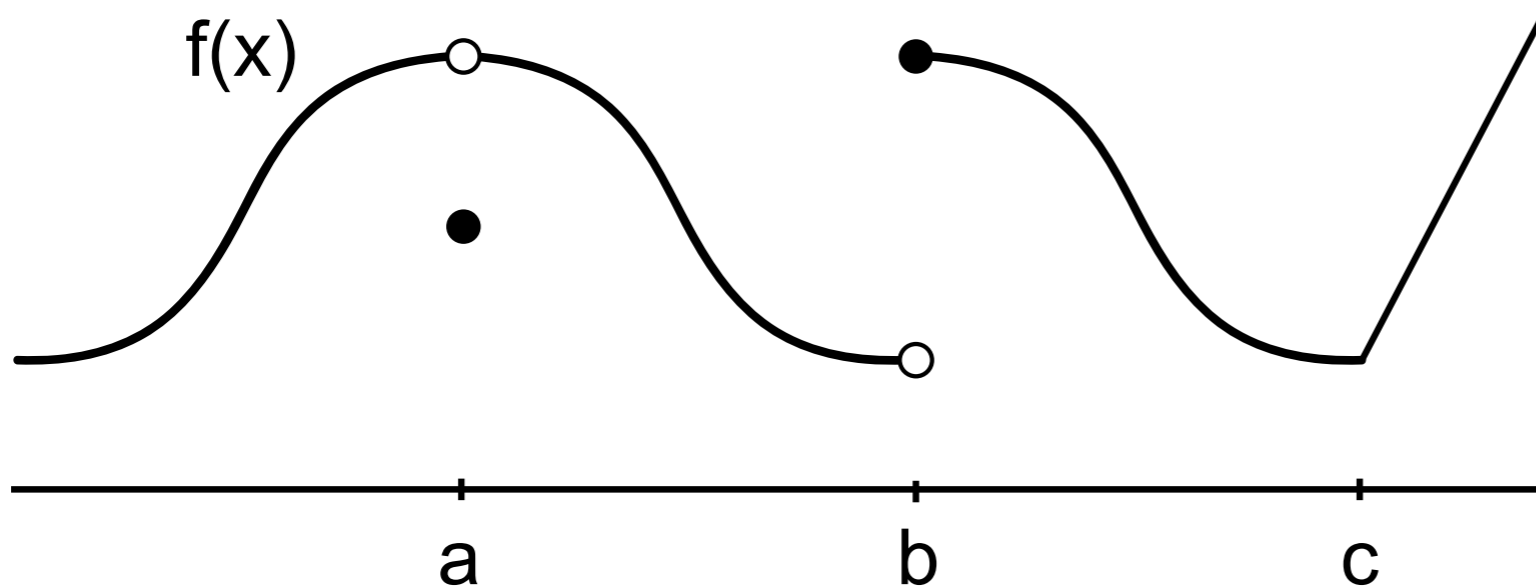
(A) $\lim_{h \rightarrow 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0$

(B) $\lim_{h \rightarrow 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0$

(C) Both (A) and (B)

(D) The limit does not exist.

Limits



(A) 1, 4

Which of the following are true?

(B) 2, 5

1. $\lim_{x \rightarrow a} f(x) = f(a)$

4. $\lim_{x \rightarrow a} f(x)$ exists.

(C) 3

2. $\lim_{x \rightarrow b} f(x) = f(b)$

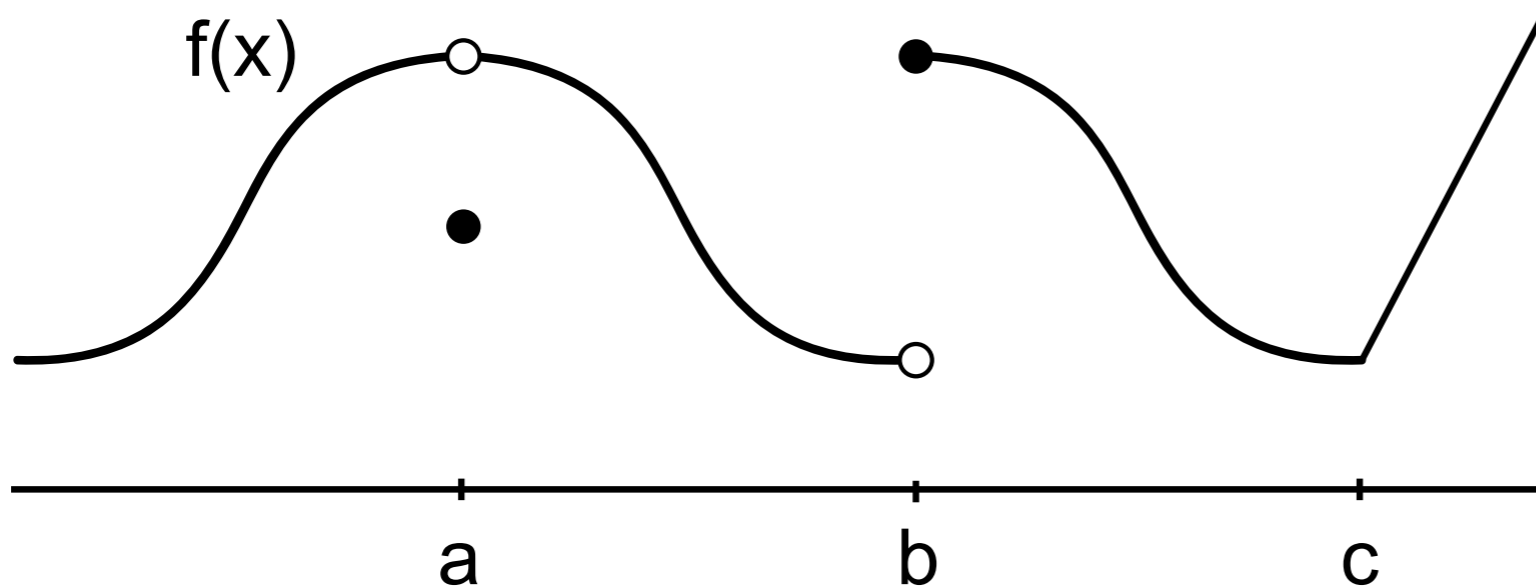
5. $\lim_{x \rightarrow b} f(x)$ exists.

(D) 4

3. $\lim_{x \rightarrow c} f(x)$ does not exist.

(E) 5

Limits



(A) 1, 4

(B) 2, 5

(C) 3

(D) 4

(E) 5

Which of the following are true?

1. $\lim_{x \rightarrow a} f(x) = f(a)$

4. $\lim_{x \rightarrow a} f(x)$ exists.

2. $\lim_{x \rightarrow b} f(x) = f(b)$

5. $\lim_{x \rightarrow b} f(x)$ exists.

3. $\lim_{x \rightarrow c} f(x)$ does not exist.

What is $\lim_{x \rightarrow -\infty} \frac{3x^n + x^2 - 1}{x^3 + 4}$?

- (A) If $n=2$, the limit is $-\infty$.
- (B) If $n=3$, the limit is ∞ .
- (C) If $n>3$ and even, the limit is $-\infty$.
- (D) If $n>3$ and odd, the limit is $-\infty$.

What is $\lim_{x \rightarrow -\infty} \frac{3x^n + x^2 - 1}{x^3 + 4}$?

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- (C) If $n>3$ and even, the limit is $-\infty$.
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Find tangent line to $f(x)=x^2$
that goes through $(1,-1)$.

Point of tangency is at

(A) $(1 + \sqrt{2}, 3 - 2\sqrt{2})$

(B) $(1 + \sqrt{2}, 3 + 2\sqrt{2})$

(C) $(1, -1)$

(D) $(1 - \sqrt{2}, 3 - 2\sqrt{2})$

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Which is true?

- (A) If $f'(1/2)=0$, then $x=1/2$ must be a max or min of $f(x)$.
- (B) If $f(0)=0$ and $f'(x)<0$ for $0<x<1$, then $f(1/2) <0$.
- (C) If $f''(1/2) = 0$, then $x=1/2$ must be an inflection point of $f(x)$.
- (D) If $f'(1/2)=0$ and $f''(1/2)<0$, then $x=1/2$ must be a min.

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Sketch $x/(1+x^2)$.

(A) $f'(x) = 1 / (2x)$

(B) $f'(x) = (1-x^2) / (1+x^2)^2$

(C) $f'(x) = (1+x^2-2x) / (1+x^2)^2$

(D) $f'(x) = 1/(1+x^2) - 2x / (1+x^2)^2$

Sketch $x/(1+x^2)$.

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Sketch $x/(1+x^2)$.

(A) $f''(x) = 2x(x^2-3) / (1+x^2)^3$

(B) $f''(x) = x(x^2-3) / (1+x^2)^2$

(C) $f''(x) = x(x^2+1) / (1+x^2)^3$

(D) $f''(x) = 2(x^2-1) / (1+x^2)^3$

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Critical points

$$f'(x) = (1-x^2) / (1+x^2)^2$$

$$f''(x) = 2x(x^2-3) / (1+x^2)^3$$

- (A) $x=1$ is a min and $x=-1$ is a max.
- (B) $x=1$ is a max and $x=-1$ is a min.
- (C) $x=1$ is a max and $x=-1$ is neither.
- (D) $x=1$ is neither and $x=-1$ is a max.

Critical points

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$$f''(x) = 2x(x^2-3) / (1+x^2)^3$$

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Inflection points

$$f''(x) = 2x(x^2 - 3) / (1 + x^2)^3$$

- (A) $x=0$, $x=\sqrt{3}$, $x=-\sqrt{3}$ are all inflection points.
- (B) $x=0$ is not an inflection because f'' doesn't change sign at $x=0$.
- (C) $x=\sqrt{3}$ is not an inflection point because f'' doesn't change sign at $x=\sqrt{3}$.
- (D) Both $\sqrt{3}$ and $-\sqrt{3}$ are not inflection points because f'' doesn't change sign at either of them.

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Inflection points - what if

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Make a table

	$(-10, -\sqrt{3})$	$-\sqrt{3}$	$(-\sqrt{3}, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, \sqrt{3})$	$\sqrt{3}$	$(\sqrt{3}, 10)$
$f(x)$	-	-	-	-	-	0	+	+	+	+	+
$f'(x)$	-	-	-	0	+	+	+	0	-	-	-
$f''(x)$	-	0	+	+	+	0	-	-	-	0	+