

Math 102 Midterm Examination
Thursday, October 26

Email address:

(This may be used to help match your exam to your name in our records.)

These files will be scanned, and your name will be read by a computer, so please write clearly and darkly.

Time: 90 minutes

Content: 7 multiple choice, 5 full answer
55 points total

Clear communication is an important skill to practice, so simplify and justify all answers unless otherwise directed, show your work (except for multiple choice), and use proper notation.

Multiple Choice: 7 questions

1. Evaluate $\lim_{x \rightarrow 1} \frac{x + 1}{\sqrt{x^4 + 3}}$.

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- A. 0
- B. $\frac{1}{4}$
- C. $\frac{1}{2}$
- D. 1
- E. 2
- F. 4
- G. ∞
- H. the limit does not exist

2. What is the derivative of $f(x) = \sqrt{x}$ at $x = 1$?

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- A. $\frac{1}{9}$
- B. $\frac{1}{8}$
- C. $\frac{1}{4}$
- D. $\frac{1}{2}$
- E. 1
- F. 2
- G. 4
- H. 8
- I. 9

3. Consider the function

$$f(x) = \frac{x^2}{x^2 + 10}$$

and the spreadsheet with the following entries. An arrow indicates that the entry from a cell will be copied down its column.

	A	B	C	D
1	0.1	0	=B1^2/(B1^2+10)	
2		=B1+\$A\$1	↓	=(C2-C1)/\$A\$1
3		↓	↓	↓
4		↓	↓	↓

The numbers in Column D...

- approximate $f'(x)$, where x is the entry in Column B.
- approximate $f(x)$, where x is the entry in Column B.
- approximate a root of $f(x)$.
- approximate the horizontal asymptote of $f(x)$.
- approximate the value of $f(0)$.
- approximate an inflection point of $f(x)$, when we input smaller and smaller positive values in cell A1.

4. You are told that a function has $f'(1) = 0$, $f'(2) = 0$, $f''(1) = -1$, $f''(2) = 1$, and $f''(3) = 0$. Which of the following can you conclude about the function $f(x)$? Choose the **best** answer.
- A. It has a local min at $x = 1$ and a local max at $x = 2$.
 - B. It has a local max at $x = 1$ and a local min at $x = 2$.
 - C. It has a local min at $x = 1$, a local max at $x = 2$, and a root at $x = 3$.
 - D. It has a local max at $x = 1$, a local min at $x = 2$, and a root $x = 3$.
 - E. It has inflection points at $x = 1, 2, 3$.

5. Which of the following linear approximations will yield an **overestimate**?

There may be more than one—circle **all** that apply.

- A. Linear approximation at $x_0 = 1$ to estimate $\sqrt{0.5}$, using $f(x) = \sqrt{x}$.
- B. Linear approximation at $x_0 = 1$ to estimate $\sqrt{1.5}$, using $f(x) = \sqrt{x}$.
- C. Linear approximation at $x_0 = 13$ to estimate 12.8^2 , using $f(x) = x^2$.
- D. Linear approximation at $x_0 = 12$ to estimate 12.2^2 , using $f(x) = x^2$.
- E. Linear approximation at $x_0 = 6$ to estimate $f(5.9)$, using $f(x) = \frac{1}{12}x^4 - 2x^3 + \frac{35}{2}x^2 + 9x - 8$.
- F. Linear approximation at $x_0 = 0$ to estimate $f(0.1)$, using $f(x) = \frac{1}{12}x^4 - 2x^3 + \frac{35}{2}x^2 + 9x - 8$.
- G. None of the above

6. Calculate the first two approximations x_1 and x_2 for a solution of the equation

$$x^3 - 4x = 8$$

using Newton's method starting with $x_0 = 2$. What is x_2 ?

A. $x_2 = 2 - \frac{7}{23}$

D. $x_2 = 3 - \frac{7}{23}$

G. $x_2 = 4 - \frac{7}{23}$

B. $x_2 = 2$

E. $x_2 = 3$

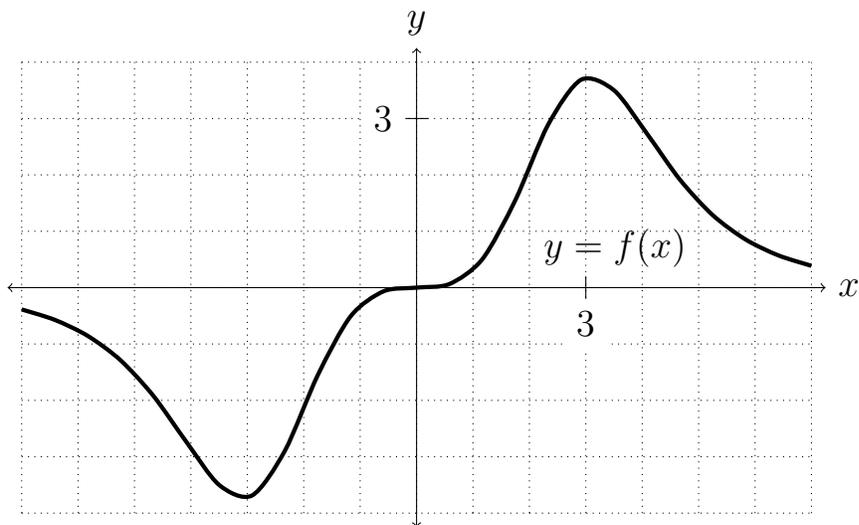
H. $x_2 = 4$

C. $x_2 = 2 - \frac{23}{7}$

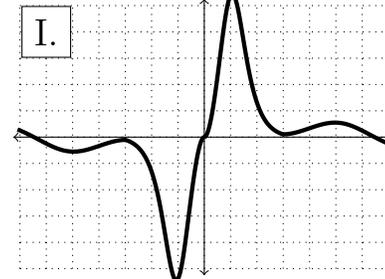
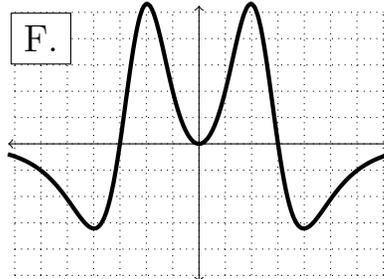
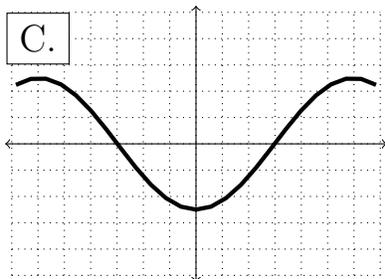
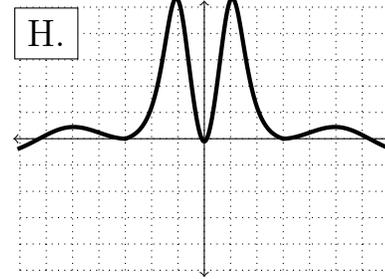
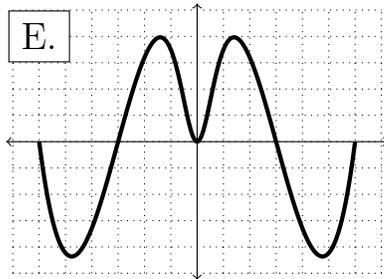
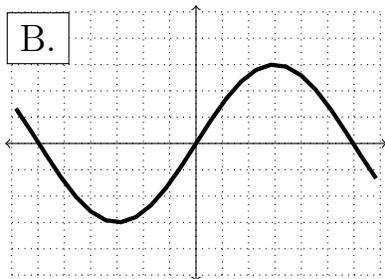
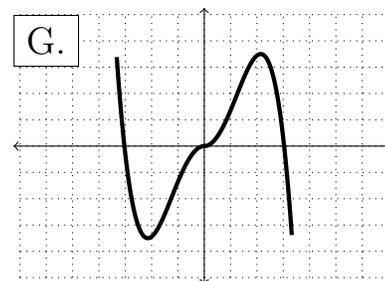
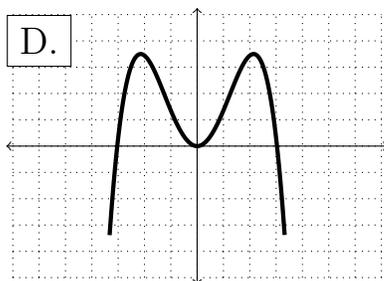
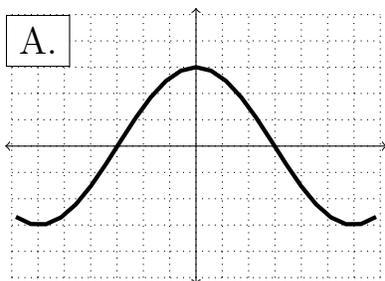
F. $x_2 = 3 - \frac{23}{7}$

I. $x_2 = 4 - \frac{23}{7}$

7. The graph $y = f(x)$ is shown below.



Choose $f'(x)$ from among the following graphs.



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Full Answer: 5 questions

Justify your answers and show all work

8. **Using the definition of the derivative**, calculate the derivative of the function $f(x) = x^2 + x$.

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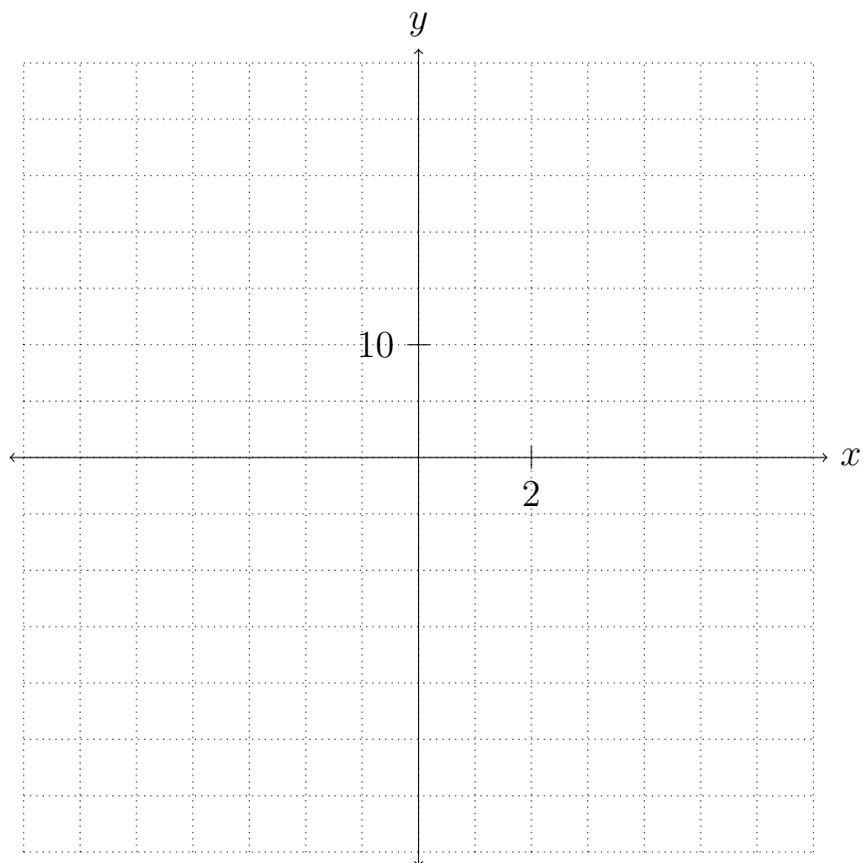
Simplify your final answer, and remember to show your work and use proper notation throughout. No marks for an answer that does not show your ability to use the definition of a derivative—for example, an answer using the power rule.

9. Carefully sketch the graph $y = x^3(x - 4)$. Be sure to accurately include all roots, local extrema, and inflection points.

You may assume without proof the following:

- $\frac{dy}{dx} = 4x^2(x - 3)$
- $\frac{d^2y}{dx^2} = 12x(x - 2)$

Note the x -axis and y -axis below have different scales.



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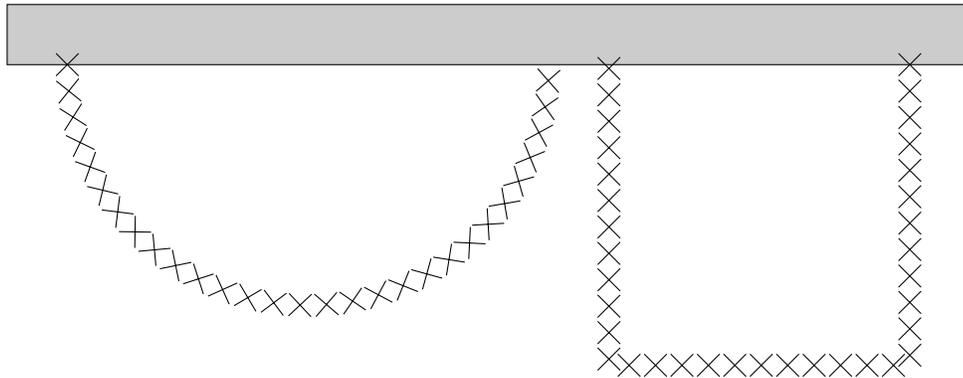
10. You are told that the velocity of a bacterial cell at time t is observed to be

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$$v(t) = 3t^2 - 6t, \quad -1 \leq t \leq 3$$

- (a) When is the acceleration of the cell equal to 0? (2 points)
- (b) At what times, if any, does the cell reverse its direction of motion?
(2 points)
- (c) What is the average velocity of the cell over the time interval $-1 \leq t \leq 1$?
(3 points)

11. A section of fence 10 m long is cut into two pieces. One piece is bent into three sides of a square and the other is bent into a semi-circle. Both shapes are placed against a wall so as to form a square and semi-circular enclosure. How much of the fence should go to the square to maximize the total area enclosed by both figures?



Remember to show all your work, and justify your answer fully. You may use the fact that $\pi \approx 3.14$.

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12. The size of this year's salmon population, y , depends on last year's salmon population, x . This relationship is often modeled as:

$$y = f(x) = \frac{k_1 x}{1 + k_2 x}, \quad k_1, k_2 > 0.$$

- (a) As x increases, what asymptote is $f(x)$ approaching? (Your answer will be in terms of constants in the problem). (2 points)
- (b) For what population size this year will the population next year be exactly half the asymptote (which you found in part (a))? (2 points)
- (c) What positive population level this year, if any, will lead to next year's population being exactly the same? (2 points)
- (d) What conditions (if any) on the constants are essential for the existence of this kind of static population, i.e. so that the population does not change from one year to the next? (1 point)

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13. Predict your score: fill in the appropriate box.

2 (bonus)

45-60 points

30-45 points

15-30 points

0-15 points

If your score (excluding the bonus points for this question!) falls within the selected range, you'll get two bonus points. The purpose of this question is to encourage you to think critically about your own understanding. It is possible to earn 100% on this exam without answering this question.

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