$g(x) = 12x^3 - 12x^2$ has...

(A) a maximum at $x=0$ and a minimum at $x=1/3$.

(B) a minimum at $x=0$ and a maximum at $x=1/3$.

(C) a maximum at $x=0$ and an inflection pt at $x=1/3$.

(D) an inflection pt at $x=0$ and a minimum at $x=1/3$.

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$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at $x=0$ and a minimum at $x=1$.

(B) a minimum at $x=0$ and a maximum at $x=1$.

(C) a maximum at $x=0$ and an inflection pt at $x=1$.

(D) an inflection pt at $x=0$ and a minimum at $x=1$.

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(B) a minimum at $x=0$ and a maximum at $x=1$.

(C) a maximum at $x=0$ and an inflection pt at $x=1$.

(D) an inflection pt at $x=0$ and a minimum at $x=1$.

How do you know? Next few slides will explain...

$f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\,(x^3 - x^2) = 0$ --> $x=0$, $x=1$.

$f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\ (x^3 - x^2) = 0$ --> $x=0$, $x=1$.
- $f''(x) = 12\ (3x^2 - 2x)$.

$f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\ (x^3 - x^2) = 0$ --> $x=0$, $x=1$.
- $f''(x) = 12\ (3x^2 - 2x)$.
- SDT: $f''(1) = 1 > 0$

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  --> $f'(x)$ is increasing near $x=1$.

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  --> $f'(x)$ goes from - to 0 to + near $x=1$.

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$f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\ (x^3 - x^2) = 0 \text{ --> } x=0,\ x=1.$
- $f''(x) = 12\ (3x^2 - 2x).$
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Could also do FDT: $f'(0^{+/-})$

--> $f'(x)$ is increasing near $x=1$.

--> $f'(x)$ goes from - to 0 to + near $x=1$.

--> $f(x)$ has a minimum at $x=1$.

- SDT: $f''(0) = 0$ --> Min/max? Inflection point?

Is x=0 an inflection point of $f(x) = 3x^4-4x^3$?

(A) Yes because $f''(0) = 0$.

(B) Yes because $f''(0) = 0$ and $f'''(0) < 0$.

(C) No because $f''(-1) = 60$ and $f''(1) = 12$.

(D) Yes because $f''(-1) = 60$ and $f''(1/2) = -3$.

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Note: $f'(x) = g(x) = 12x^3-12x^2$ from earlier
and we agreed that g(x) had a max at x=0!

Is $x=0$ an inflection point of $f(x) = 3x^4-4x^3$?

(A) Yes because $f''(0) = 0$.

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Is x=0 an inflection point of $f(x) = 3x^4-4x^3$?

(A) Yes because $f''(0) = 0$.

"SDT" applied to $f'(x)$.

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"FDT" applied to $f'(x)$.

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Jumped over a zero of $f''(x)$

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x					
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x	$(-\infty,0)$	0	$(0,2/3)$	2/3	$(2/3,\infty)$
$f''(x)$		0		0	

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$f''(x)$	+	0		0	

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$f''(x)$	+	0	–	0	+

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