Today

- Power rule for $y=x^{m/n}$
- Exponential review (brief - 2 CQs)
- The derivative of exponential functions
- Reminders:
  - Midterm 2 is in less than 2 weeks.
  - OSH 5 due Monday

For $y=x^n$, $y'=nx^{n-1}$ when n is an integer. What if n is rational?

For $y = x^{1/m}$ with m an integer, what is y'?

Rewrite the equation as $y^m = x$ and take derivatives:

(A) $m\, y^{m-1} = 1$

(B) $m\, y^{m-1}\, y' = 1$

(C) $m\, y^{m-1} = x$

(D) $y'^m = 1$

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For $y = x^{1/m}$ with m an integer, what is y′?

Solve $m\, y^{m-1}\, y' = 1$ for y′:

(A) $y' = 1/(m\, y^{m-1})$

(B) $y' = 1/(m\, x^{m-1})$

(C) $y' = 1/(m\, x^{(m-1)/m})$

(D) $y' = (1/m)\, x^{1-1/m}$

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  - We just used implicit diff. to show this.
- When p=m/n and $y = x^p$, $y'=px^{p-1}$.
  - You do this (exercise in course notes).

Exponential functions

Which of the following is an exponential function?

(A) x^n

(B) 2^x

(C) e^2

(D) $\ln(x)$

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Exponential functions a^x where $a>1$...

(A) All go through the point (1,1).

(B) All go through the point (0,0).

(C) All go through the point (1,0).

(D) If $a<b$ then $a^x<b^x$ for all $x>0$ and $a^x>b^x$ for all $x<0$.

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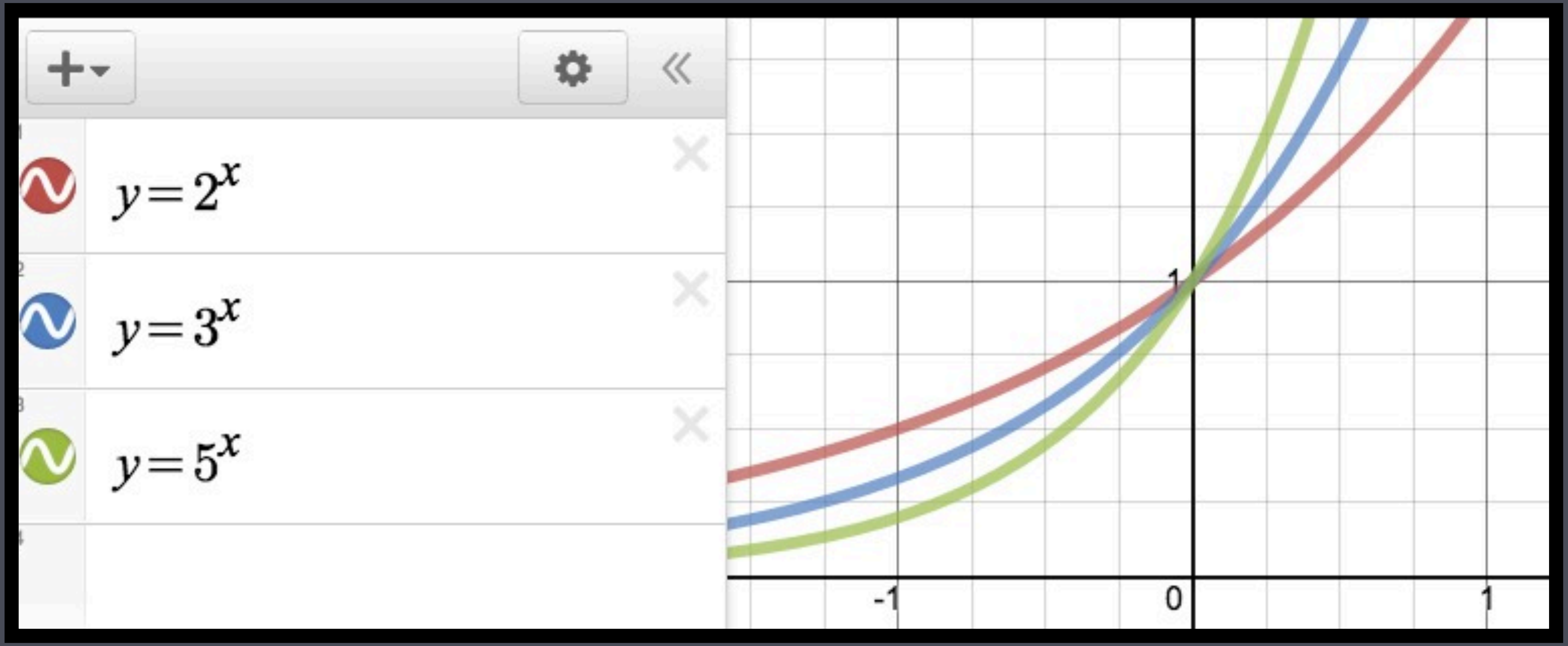

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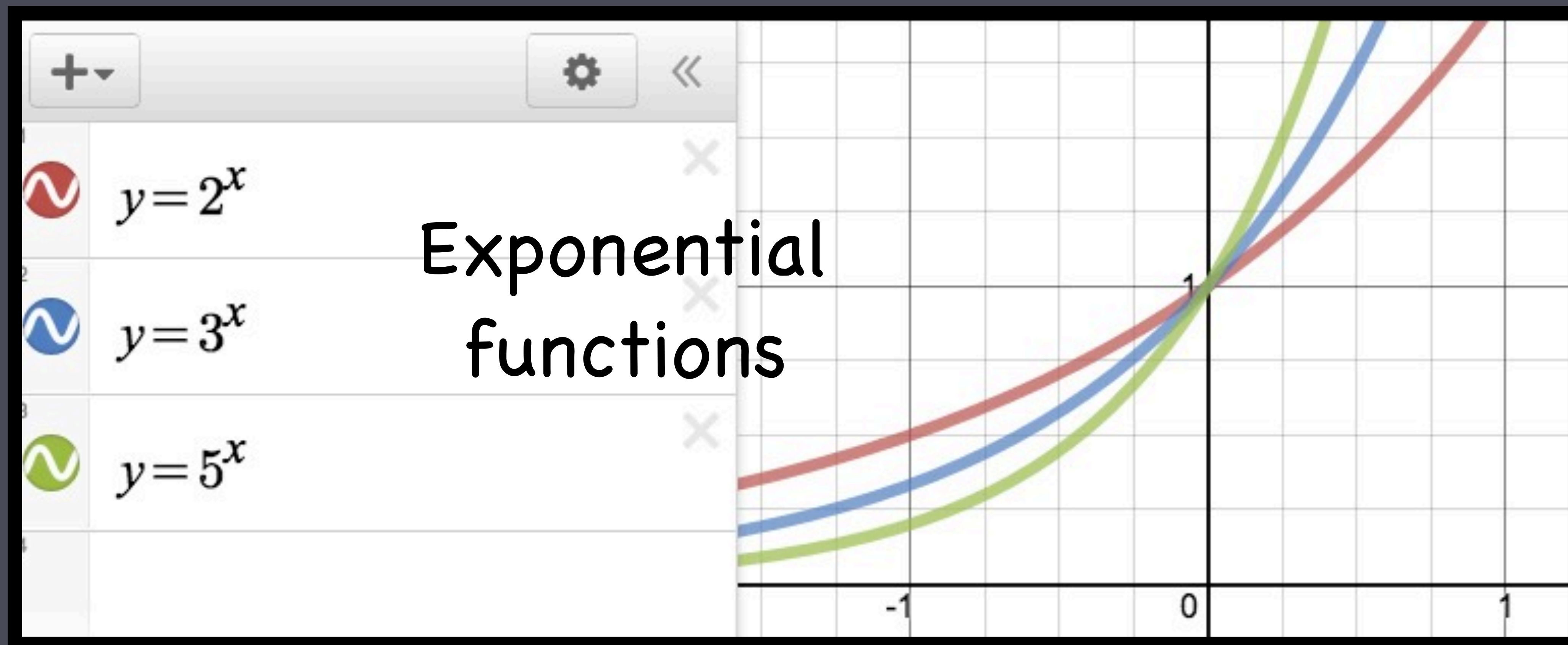

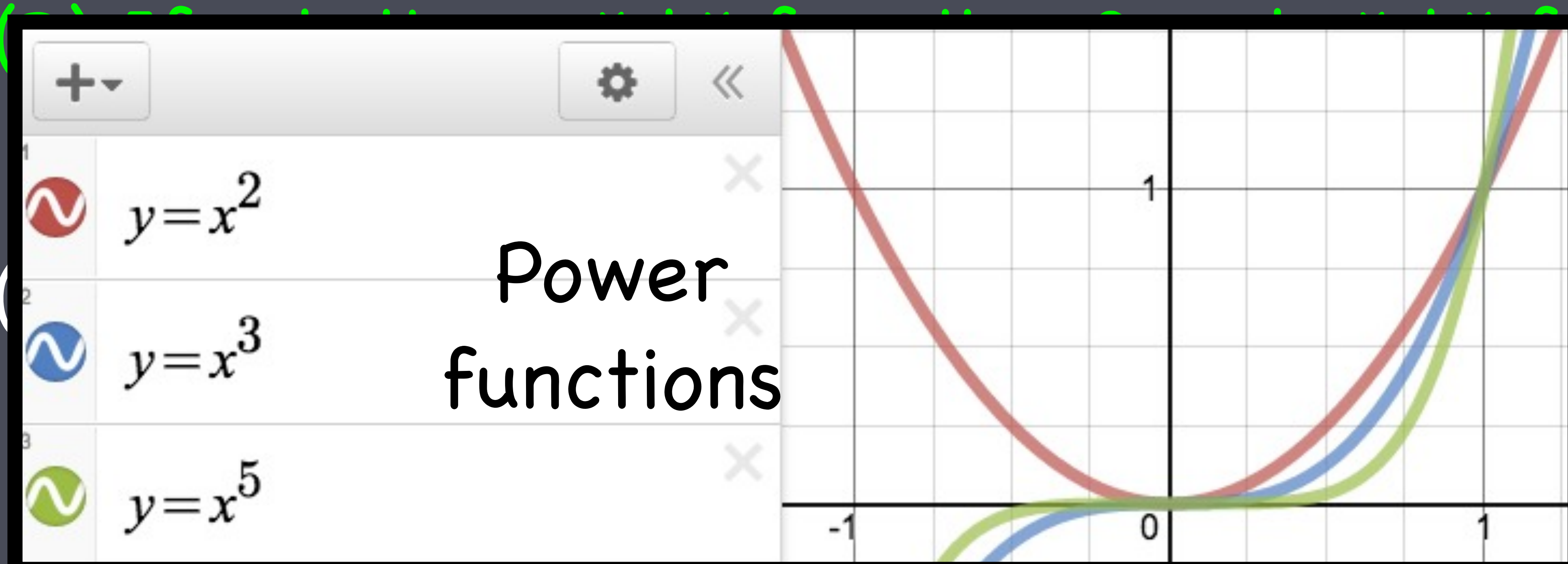

The derivative of $f(x)=a^x$ is…

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(C) $f'(x) = (a^{x+h}-a^x)/h$

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Checking your high school memory here.

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What is this special a value? a=e!

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This is precisely how e is defined - the number whose exponential function is its own derivative.

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- What function is equal to its own derivative?
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  - Call this a "differential equation". (DE)

DE example: Which of the following satisfies $f'(x)=f(x)$?

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