MATH 102 - MIDTERM TEST 2
University of British Columbia

Name (print):

ID number:

Section number:

Date: *November 6, 2012*
Time: *6:00 p.m. to 7:00 p.m.*
Number of pages: *8 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

• *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/record-ers/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*

• *Speaking or communicating with other candidates;*

• *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1	2	
2	1	
3	3	
4	3	
5	8	
6	8	
Total	25	

Note that point values are approximate and intended to help you with time management. The final mark distribution may vary slightly from the given one.

Short Answer Problems

A correct answer in the box will receive full marks. Partial marks may be given for work outside the box.

1. Simplify the expression "$\sin(\arctan(a))$" so that it is not expressed in terms of trigonometric functions or their inverses.

Answer: $\boxed{}$ [2 pt]

2. Find which of the following functions satisfy the equation $y'' = -y$:

   (a) $y(t) = \ln(t)$
   (b) $y(t) = \sin(t) + \cos(t)$
   (c) $y(t) = \sqrt{3}\cos(2t)$
   (d) $y(t) = 2e^{2t}$

Answer: $\boxed{}$ [1 pt]

3. Find all the solutions of $\ln(x+1) + \ln(x-2) = 2\ln(2)$. If there are no solutions write NONE.

Answer:

[3 pt]

4. Suppose a function $y = f(x)$ satisfies $e^{xy} + y = x^3$. Find the value $\frac{dy}{dx}$ when $y = 0$. Your answer should be a number!

Answer:

[3 pt]

Long Answer Problems

5. During pregnancy, a fraction of twins are at risk of death due to entanglement of their umbilical cords. To prevent such deaths, the twins are removed early by surgery (Caesarean section). However, removing them earlier puts them at risk of death due to premature birth. In this problem, you will determine when to schedule delivery so as to maximize their chance of survival in the face of these opposing risk factors.

   (a) [**5 pt**] Out of 25 such pregnancies that were identified at 24 weeks into the pregnancy, 16 were still alive at 32 weeks. Write down an exponential function that describes the number of surviving twins, T(t), as a function of the time (measured in weeks) into the pregnancy.

(b) [**3 pt**] The overall probability of survival for a set of such twins, accounting for both survival to delivery and survival once delivered, is

$$S(t) = A(t - 7) \exp(-t/20)$$

where A is a constant. At which time t should the twins be removed to maximize their overall probability of survival?

6. A photographer is trying to photograph an airplane flying overhead with her camera. At first, the airplane is 100m above the ground and directly above a point on the ground 100m away from her. It's flying horizontally in a straight line at a speed of 25 m/s and will pass directly over her. The photographer rotates her camera carefully so as to keep the airplane in the middle of the image throughout its flight.

   (a) [**1 pt**] What is the initial distance between the airplane and photographer?
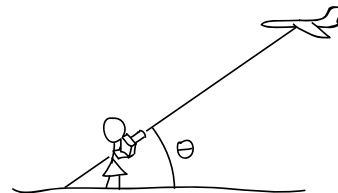
   (b) [**1 pt**] Give an expression for the horizontal component of the distance between the airplane and photographer as a function of time.

(c) **[6 pt]** How quickly is the angle between the axis of the camera (a line through the middle of the camera lens) and the ground changing as the airplane passes directly above the photographer?

This page may be used for rough work. It will not be marked.