Today

- Chain rule
- Related rates examples
- Announcement: CLASS, the Conference for Learning and Student Success, is taking place this Saturday (Oct. 18). See my.science.ubc.ca where CLASS is one of the three highlighted programs on the home page.

Composition of functions

If $f(x) = 2x+3$ and $g(x) = -4x+2$,

(A) $h(x) = f(g(x)) = -8x+7$

(B) $h(x) = f(g(x)) = -8x-10$

(C) $h(x) = f(g(x)) = -8x^2-8x+6$

(D) $h(x) = f(g(x)) = -8x+5$

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Notation for composition:

$f \circ g(x)=f(g(x))$

Composition of functions

If $h(x)= f(g(x))$, then

(A) $h'(x) = f'(x)g'(x)$

(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$

(C) $h'(x) = f'(g'(x))$

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Composition of functions

For $f(x) = x^2$ and $g(x) = x+1$,
$h(x) = x^2+2x+1$ and
$h'(x) = 2(x+1)$. But...

If $h(x)= f(g(x))$, then

(A) $h'(x) = f'(x)g'(x)$ -----------> $h'(x) =?\ 2x(1)$

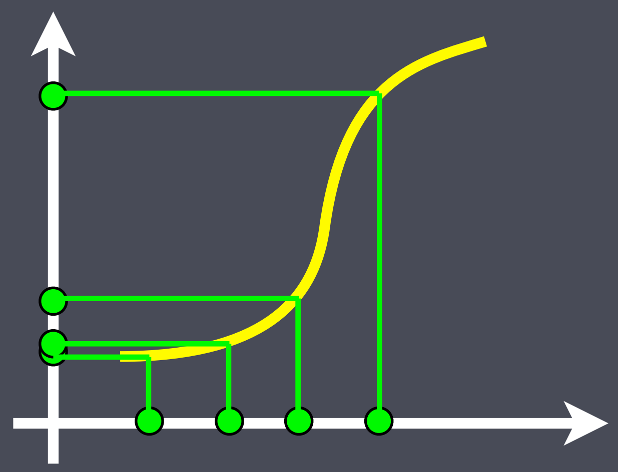
(B) $h'(x) = f'(x)g(x) + f(x)g'(x)$ ---> $h'(x) =?\ 2x(x+1) + x^2(1)$

(C) $h'(x) = f'(g'(x))$ --------------> $h'(x) =?\ 2$

(D) $h'(x) = f'(g(x))g'(x)$ ----------> $h'(x) = 2(x+1)(1)$

Geometry of function composition

- A function f(x) takes x values to y values.
- When the slope is steep(shallow) at a point, the distance between nearby points on the x axis get stretched out (squished) on the y axis.
- So f′(x) is the stretch factor near x.
- Where you are matters!



Geometry of function composition

- When you compose functions, h(x)=f(g(x)), you first stretch/squish near x according to g and then stretch/squish near g(x) according to f.
- When you do one and then the other, you multiply their effects.
- Where you are on the function matters - multiply the stretch factor of g near x: g'(x), by the stretch factor of f near g(x): f'(x).

Gas costs $1.25/litre. Your car consumes 7 litres/100 km. You've driven 130 km. How much does it cost to drive one more km?

(A) 1.25·(7/100)·130

(B) 1.25· (7/100)

(C) 125/7

(D) 7/125

The complicated way to do this:

- Cost of gas for L litres: $C(L) = 1.25\ L$
- Litres used for x km: $L(x) = 0.07x$
- Cost to go x km: $c(x) = C(L(x)) = 1.25(0.07x)$
- $c'(x) = C'(L(x))L'(x)$
- These are straight lines so the slopes are independent of L, x.

Related rates

- When two quantities (e.g. Q_1 and Q_2) are related to each other, if one changes in time so will the other.
- Knowing the relationship between Q_1 and Q_2 gives you the relationship between Q_1' and Q_2'.

The radius of a spherical tumor grows at a constant rate, k. Determine the rate of growth of the volume of the tumor when the radius is one centimeter.

Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2 k$

(C) $V' = 4\ \pi\ k^2$

(D) $V = 4/3\ \pi$ <---------

But we lose the ability to take derivatives of the related quantities!

We know r=1.

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$$V(r) = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}V(r(t)) = \frac{dV}{dr}\frac{dr}{dt}$$

$$\frac{d}{dt}V(r(t)) = 4\pi r^2\frac{dr}{dt} \quad = 4\pi r^2 k$$

Now we can plug in r=1.

Water is leaking out of a conical cup of height H and radius R. Find the rate of change of the height of water in the cup when the cup is full, if the volume is decreasing at a constant rate, k.

Which is the relevant equation relating the quantities when the water is at height h (not rates of change yet)?

(A) $V = 1/3\ \pi\ R^2 H$

(B) $V = 1/3\ \pi\ (R^2/H^2)\ h$

(C) $V = 1/3\ \pi\ (R^2/H^2)\ h^3$

(D) $V = 1/3\ \pi\ r^2 h$