Today

- Midterms - come to my office
    - Today 2-2:45 pm, 4:00-5:00 pm
    - Tues. 10:30 am - 12 pm
    - Wed. 11:30 am-12:30 pm, 2:30-3:30pm
- Optimization examples (goat, Kepler).

Optimization

- Given a scenario involving a choice of some number, use calculus to find the best value.
  - Translate scenario into a mathematical problem.
  - Solve the problem.
  - Translate back (make sure it makes sense).

I have 10 meters of fence. I want the biggest enclosure possible for my goat. I only know how to make rectangular enclosures.

Find the max of

(A) $A(w) = lw$. (l=length, w=width)

(B) $A(w) = w(10-w)$

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I have 10 meters of fence. I want the enclosure to be as small as possible but it can't be narrower than my goat (1/2 meter).

How long and how wide should I make the enclosure?

(A) l = 5/2 m, w = 5/2 m.

(B) l = 0 m, w = 5 m

(C) l = 1/2 m, w = 9/2 m

(D) l = 1/2 m, w = 19/2 m

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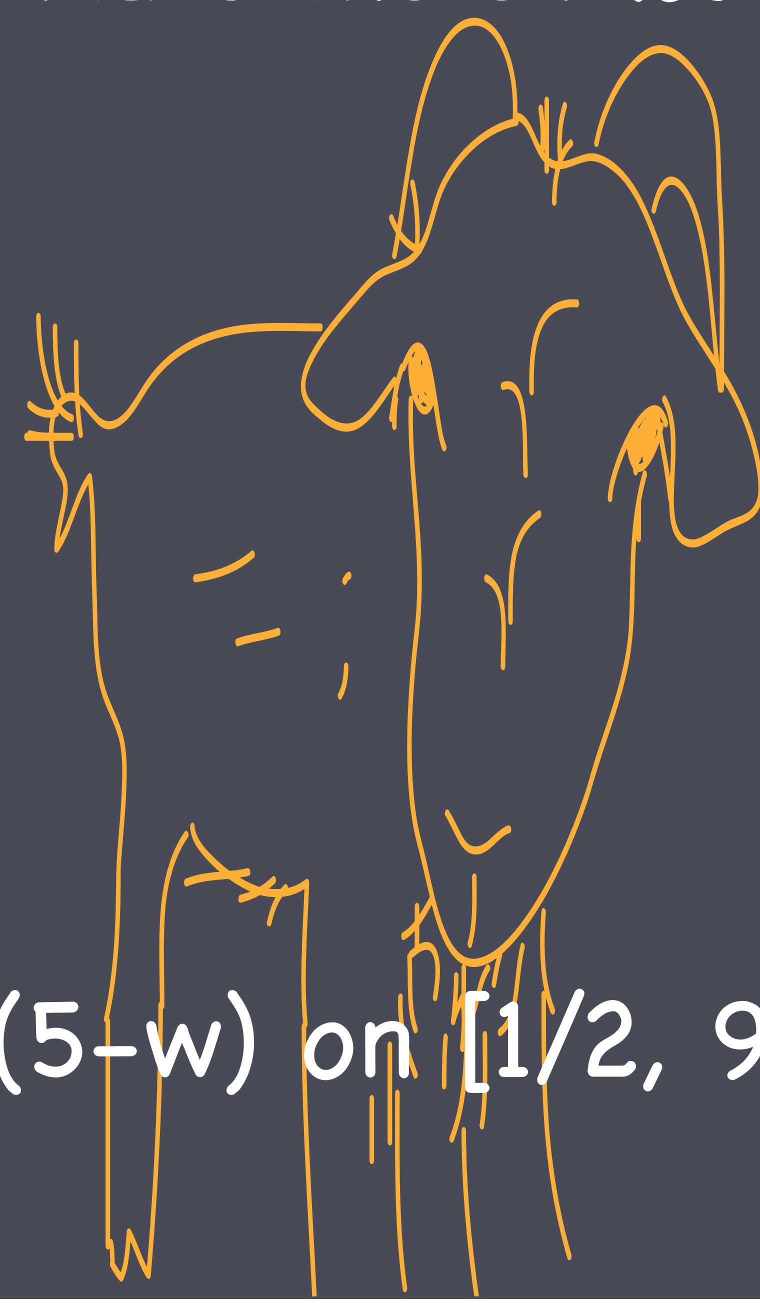

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Find absolute min of $A(w)=w(5-w)$ on $[1/2, 9/2]$.

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- The domain is restricted by "physical" considerations.

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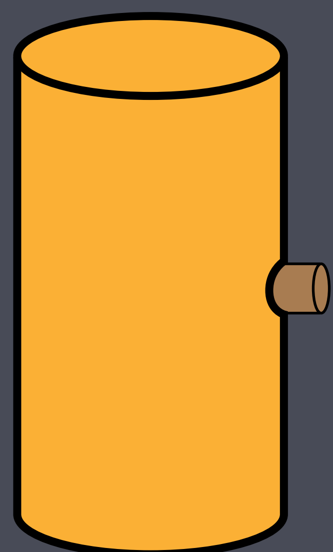
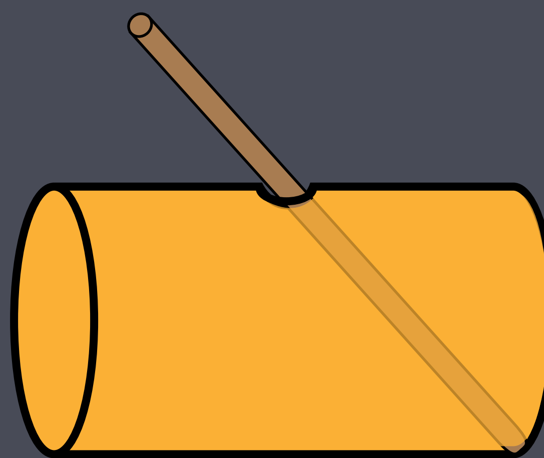
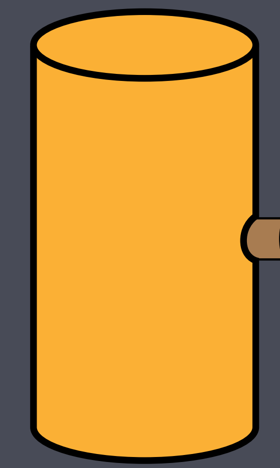
Wine for Kepler's wedding

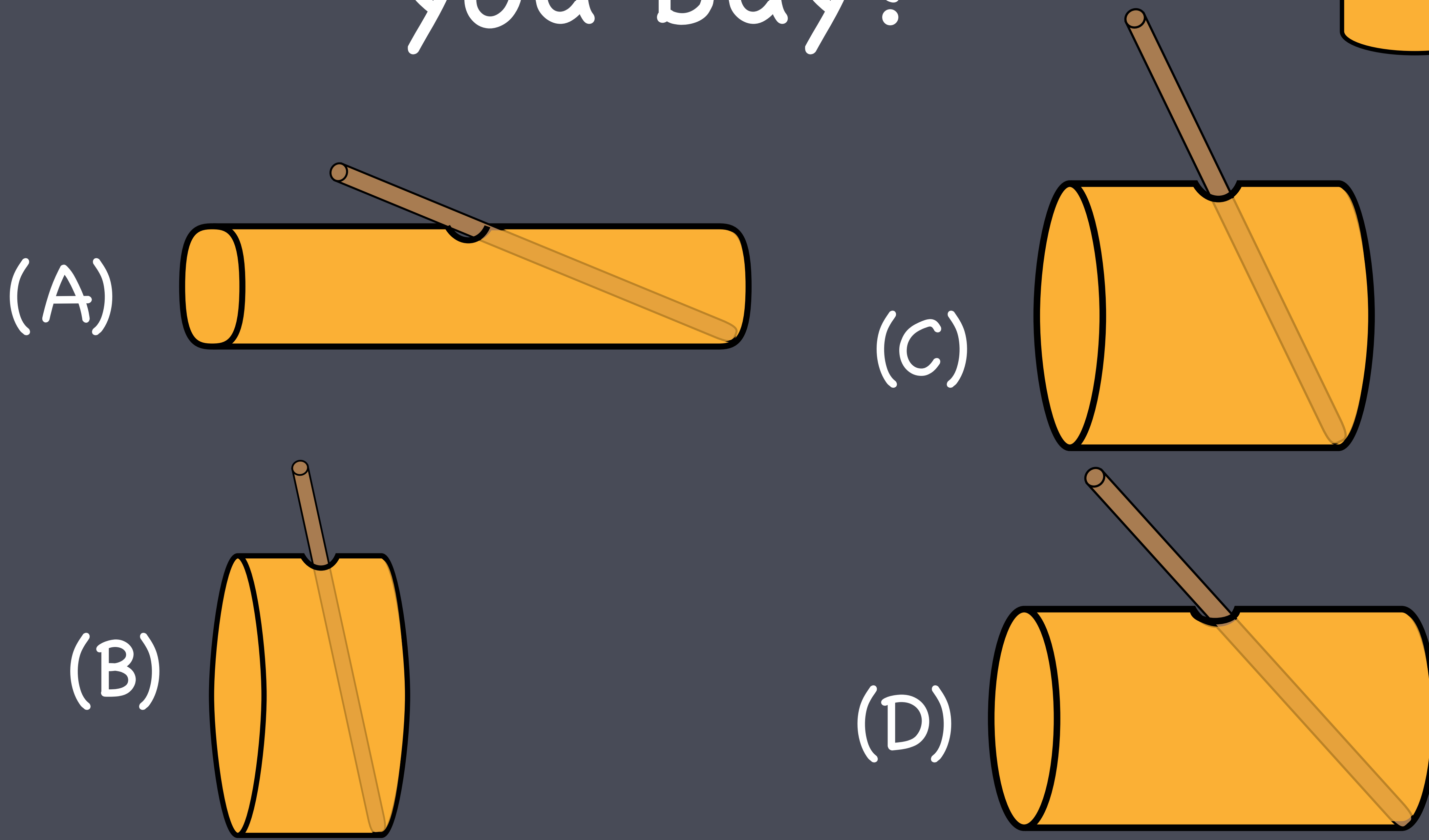




- Wine was sold by "the length of the submerged part of the rod"
- Same length of wet rod = same volume of wine?

Which barrel would you buy?





Kepler should try to

(A) Minimize the length of the rod.

(B) Maximize the volume of the barrel

(C) Maximize the volume while minimizing the length of the rod.

(D) Maximize the volume of the barrel for a fixed rod length.

(E) Minimize the rod length for a fixed volume of the barrel.

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Kepler had enough $ for a rod of length L_0. How much wine can he get?

What do you expect to be the best option?

(A) Shortest possible barrel (h=0).

(B) Tallest possible barrel (h = max h).

(C) Somewhere in between.

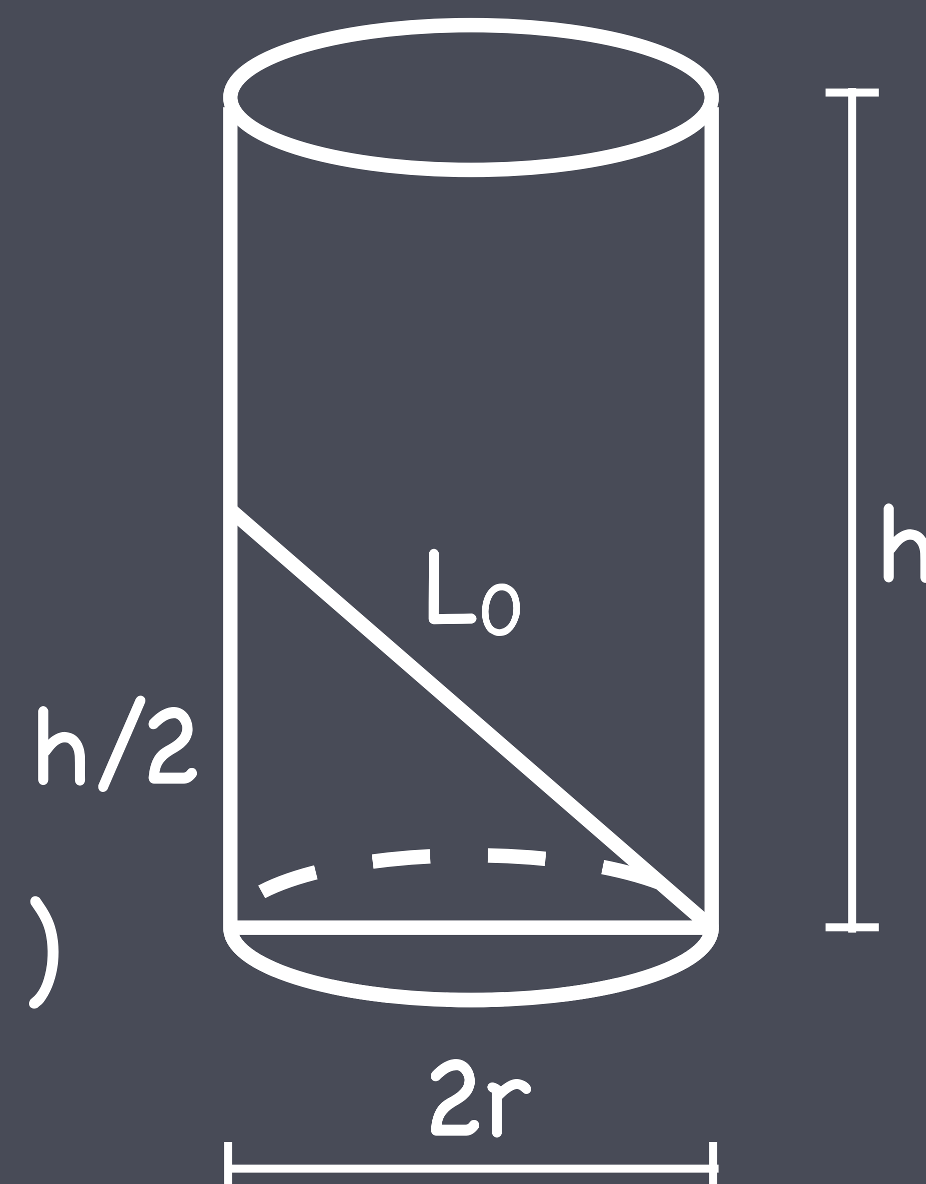
Objective function? (to be maximized)

(A) $V = 2\pi rh$

(B) $r^2 = L_0^2/4 - h^2/16$

(C) $V = \pi r^2 h$

(D) $L_0 = \mathrm{sqrt}(\ (2r)^2 + (h/2)^2\ )$



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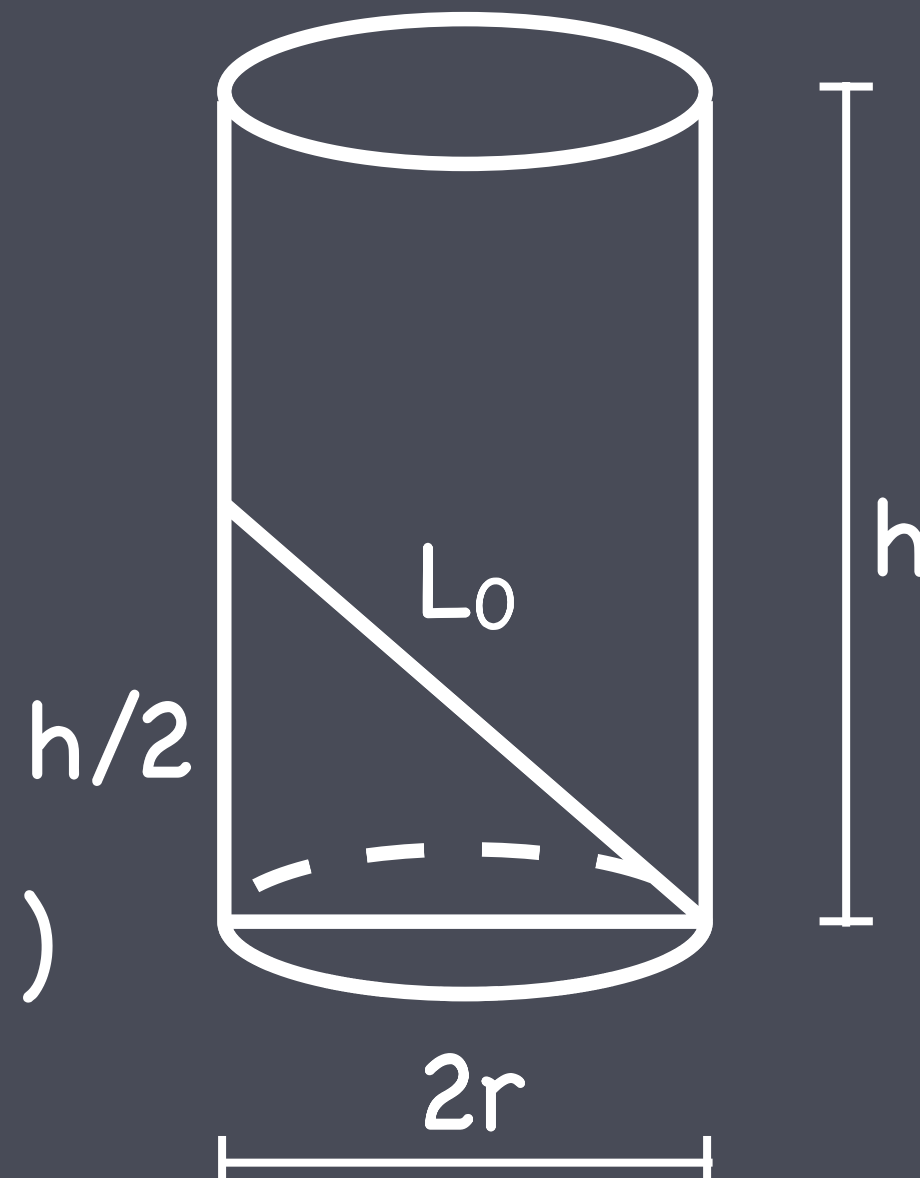

Constraint?
(used to simplify OF)

(A) $L_0^2 = (2r)^2 + (h/2)^2$

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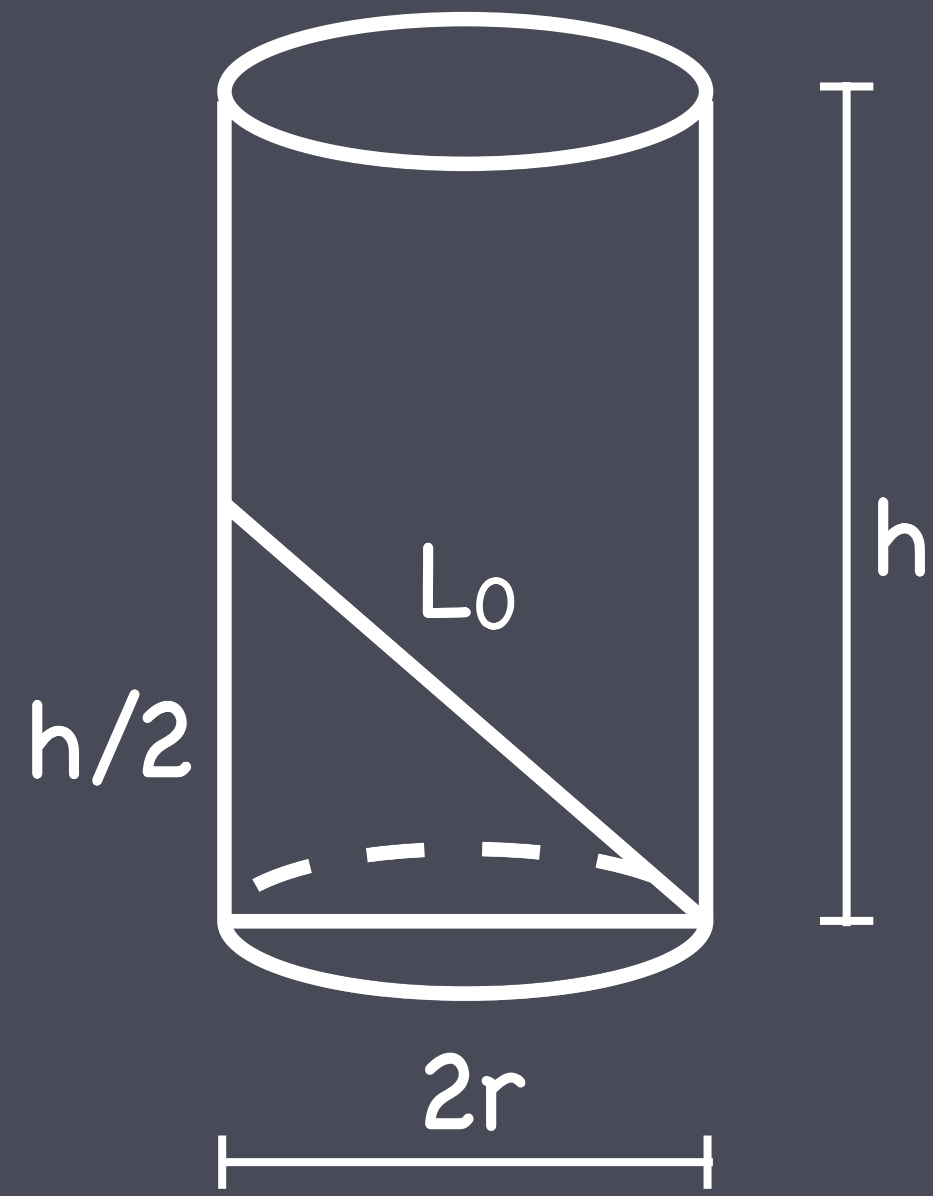
(D) $L_0 = \tan(h/4r)$



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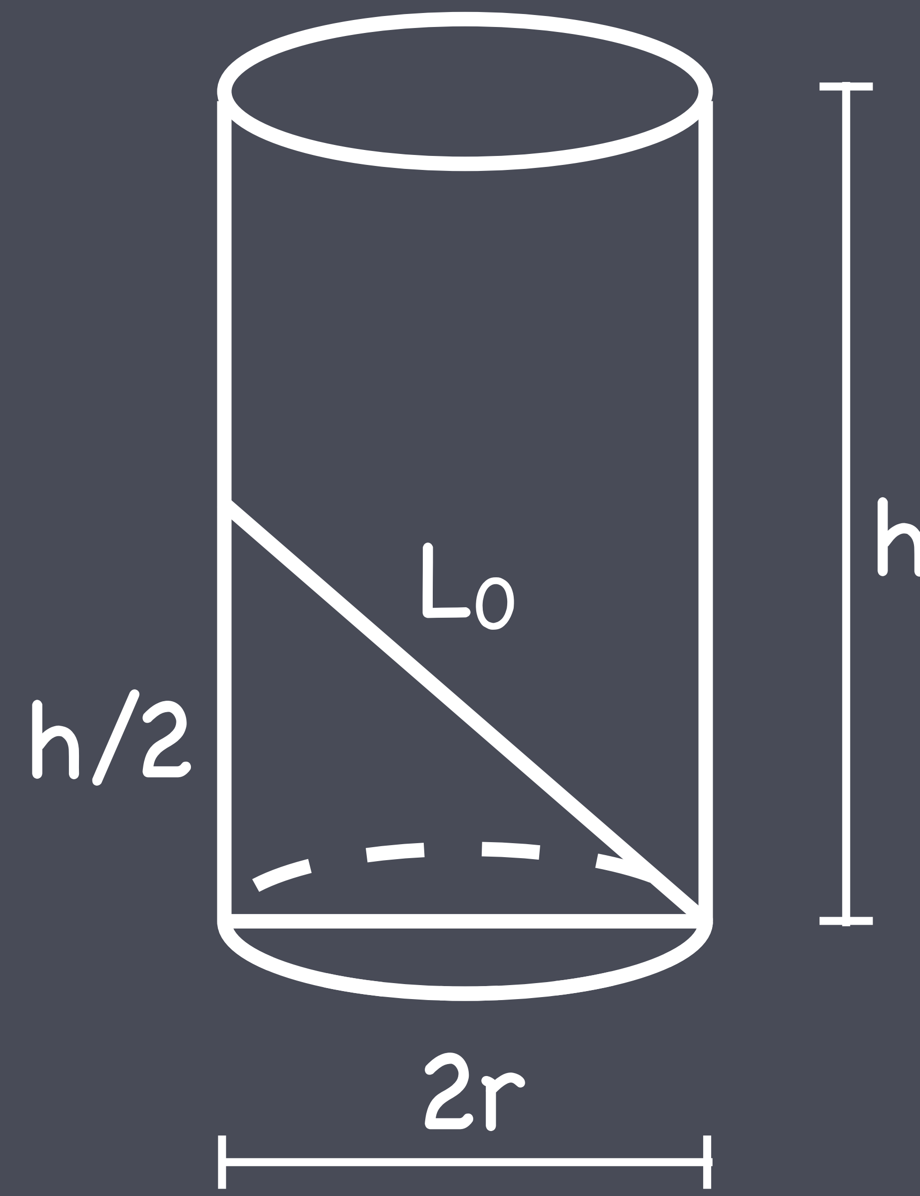

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Solve for:

(A) r

(B) r^2

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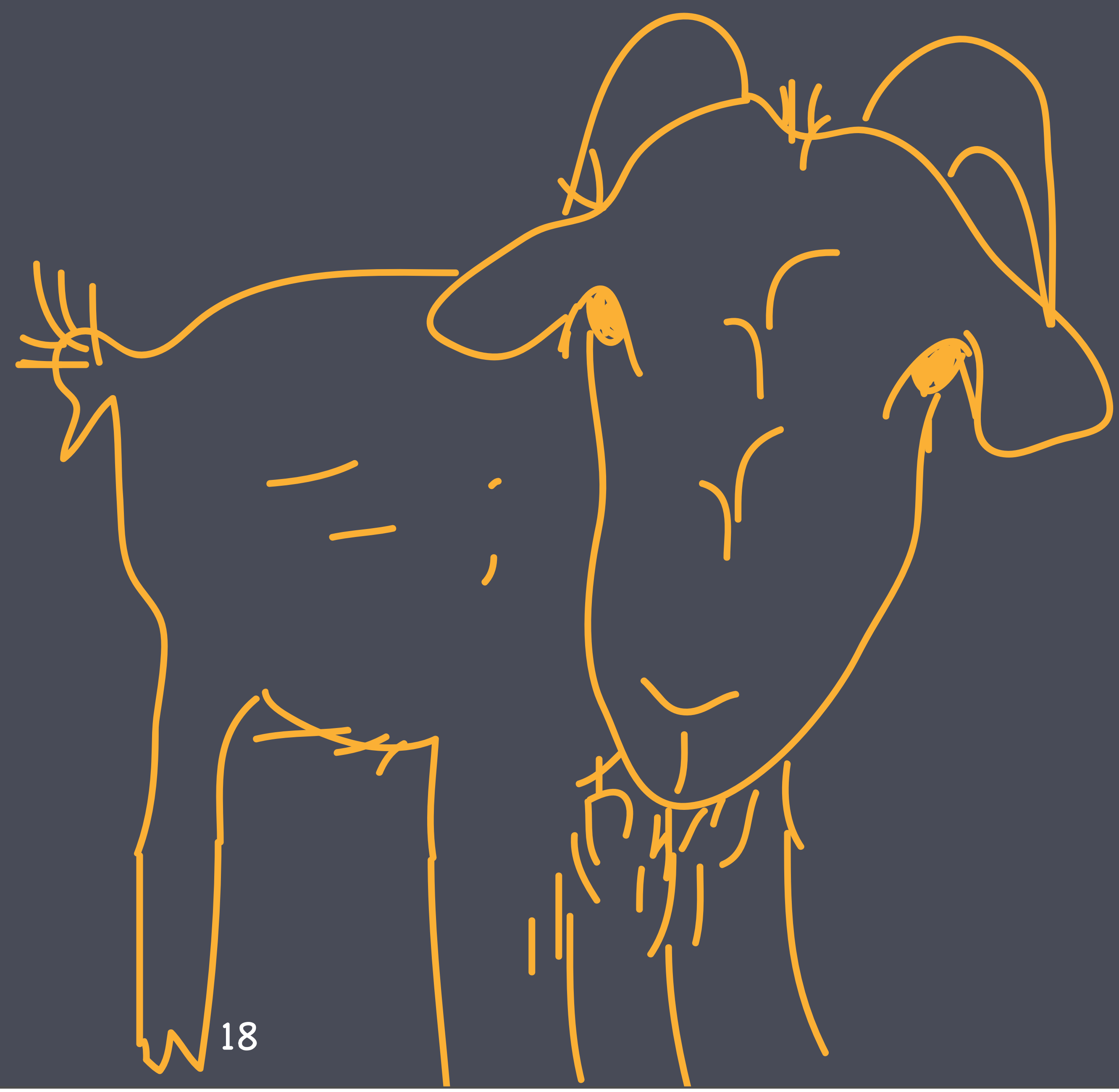

$$V = \pi h(4L_0^2 - h^2)/16$$

What is the best h?

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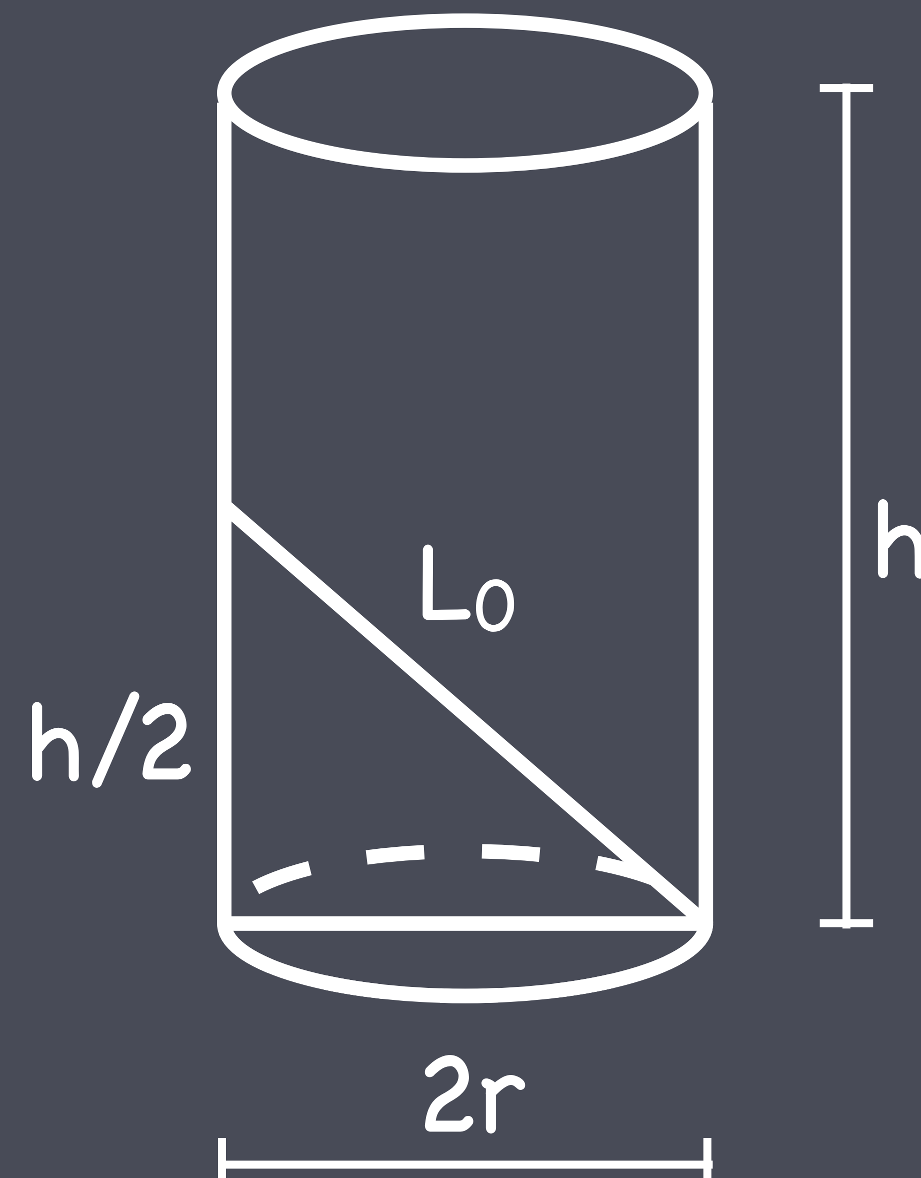

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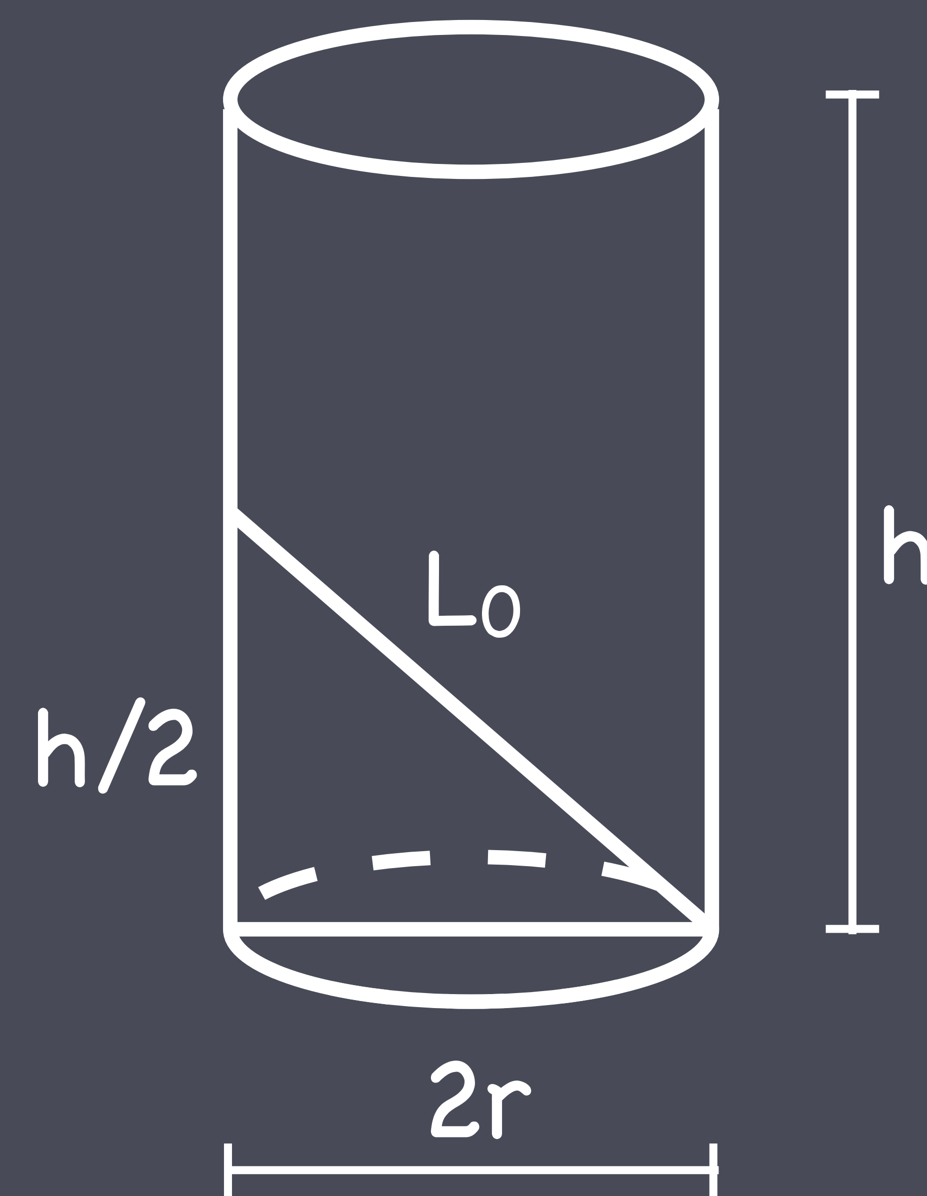

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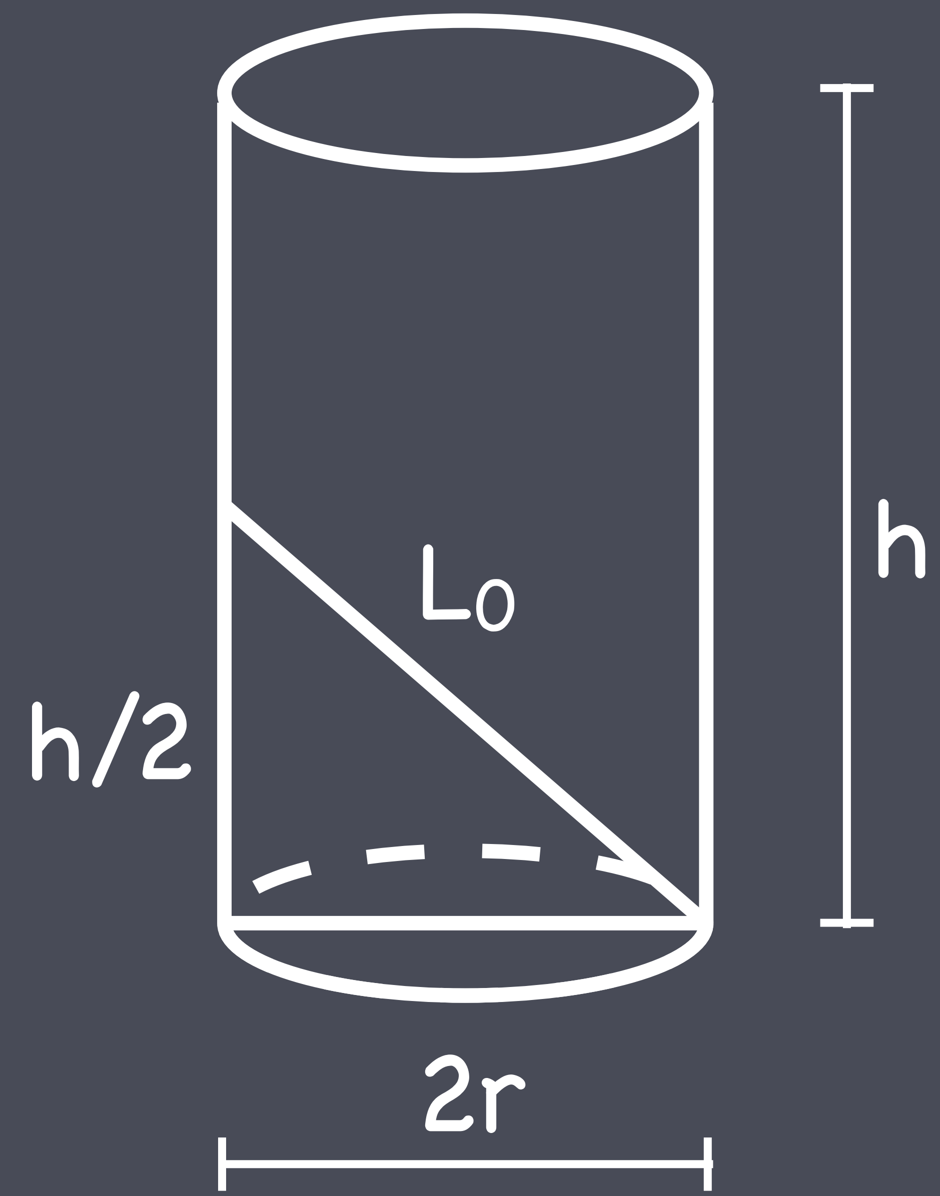

Did you do a FDT or SDT?

http://www.matematicasvisuales.com/english/html/history/kepler/doliometry.html

Overall procedure

1. Draw a sketch.
2. Determine the objective function.
3. Determine the constraint.
4. Establish an expectation (end-points or local extremum).
5. Solve constraint for one variable (make your life easy if possible).
6. Substitute it into the objective function.
7. Find the absolute extremum (check concavity!).