Today

- Euler's method for approximating DE solutions
- Logistic equation in many contexts
  - Classic example of the power of mathematics – one unifying description for many apparently unrelated phenomena.

Euler's method

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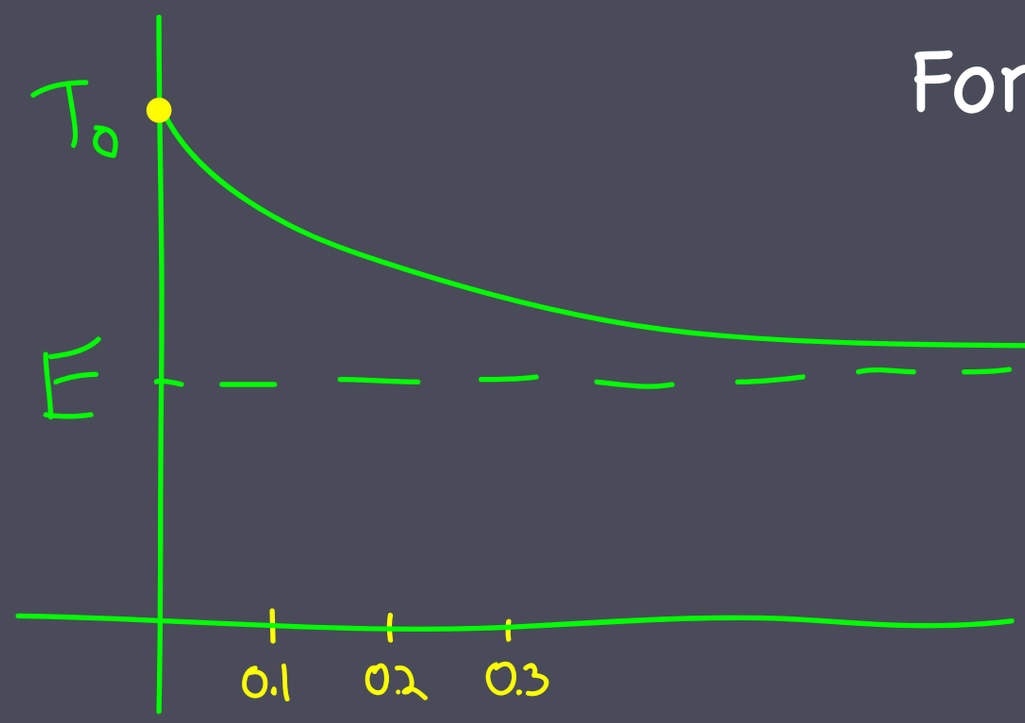
- Instead of the actual solution to NLC

$$T(t) = E + (T_0 - E)e^{-kt},$$

we find T_1, T_2, T_3,... which approximate T(0.1), T(0.2), T(0.3) ...

Euler's method for $T'(t) = 0.02\,(14 - T(t))$ with $T(0)=37$.

Forensic calculus !



What is the slope of the solution at t=0?

(A) -0.02

(B) -0.46

(C) 0.02

(D) -0.28

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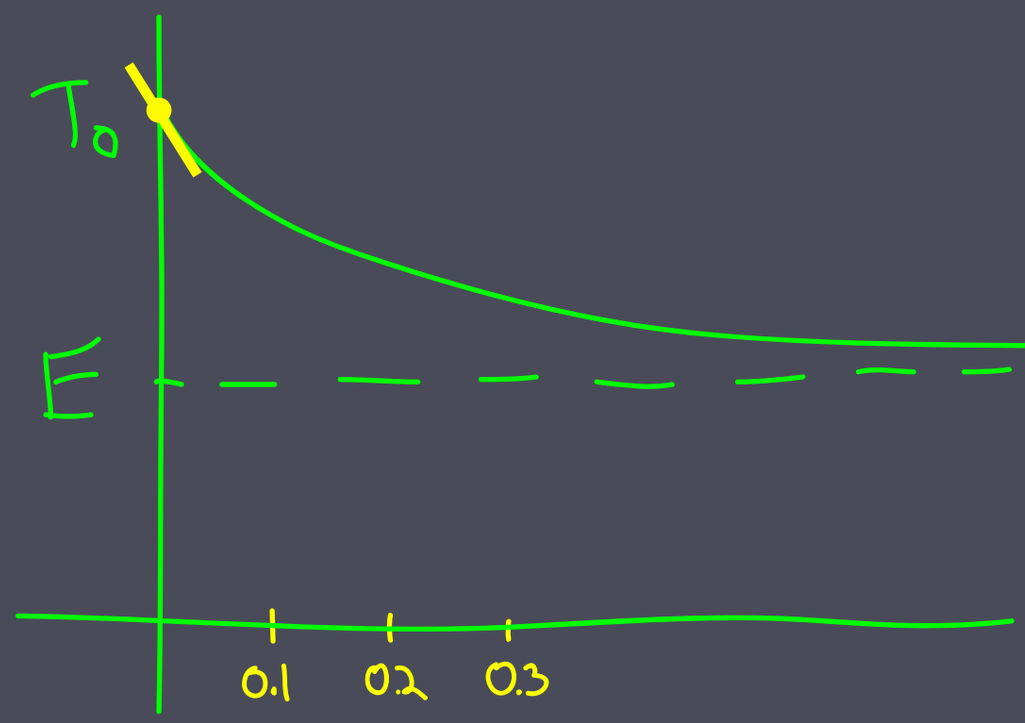

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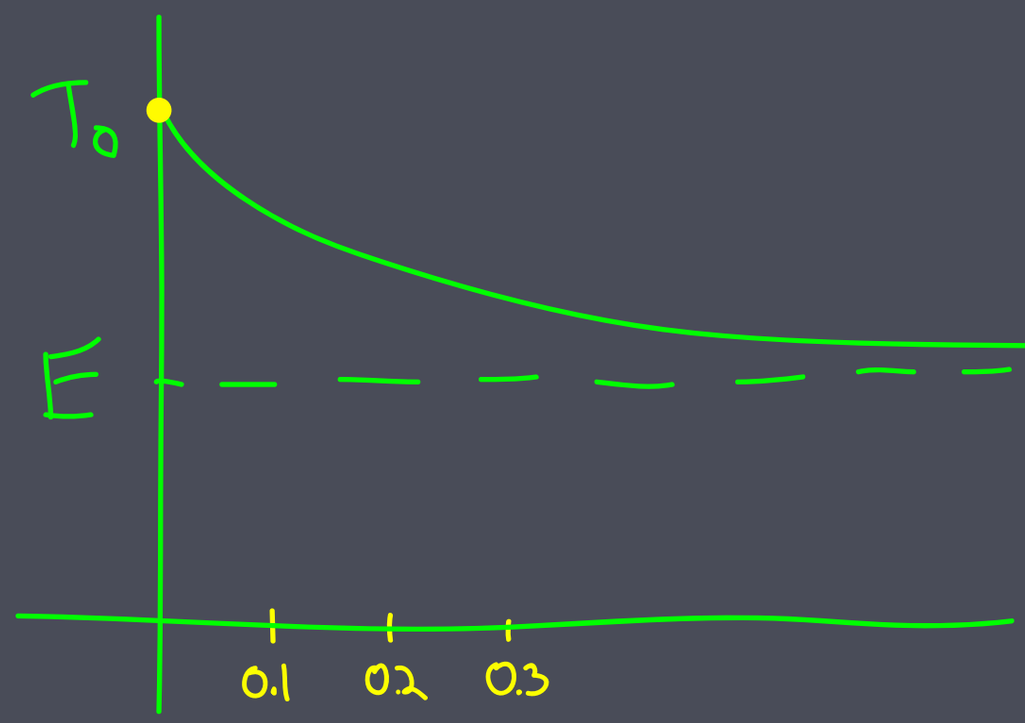

What is the slope of the solution at $t=0$?

(A) $k(E-T)$

(B) $k(E-T_0)$

(C) $T'(0)$

(D) kE

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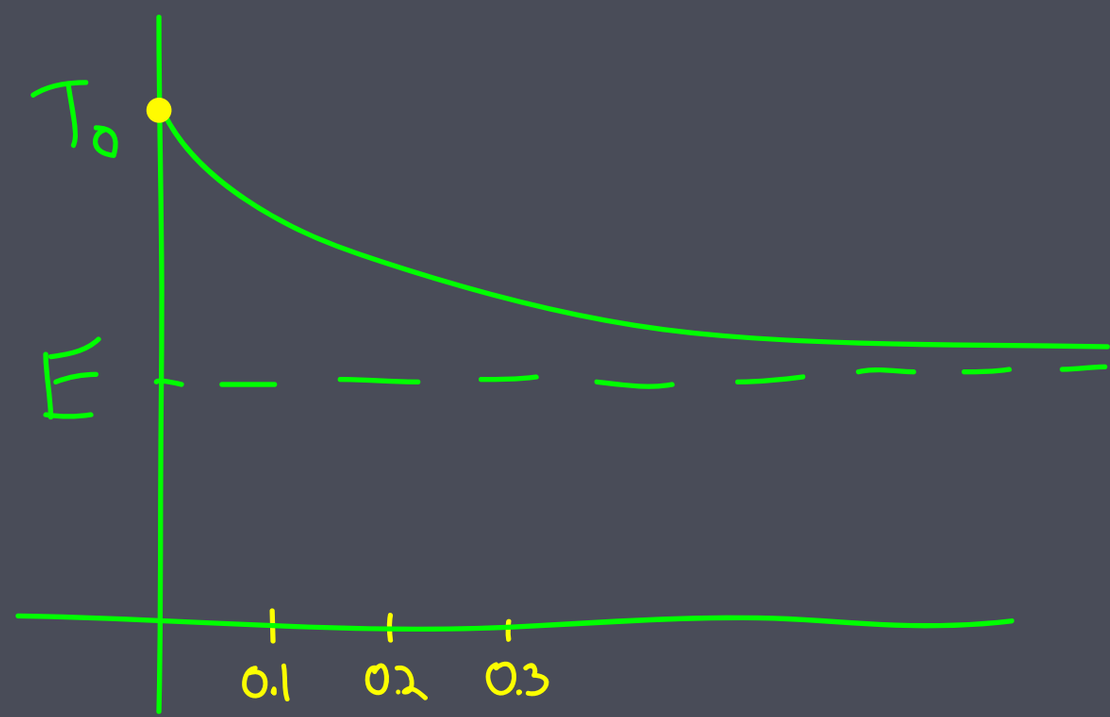

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<--- yes, but can do better

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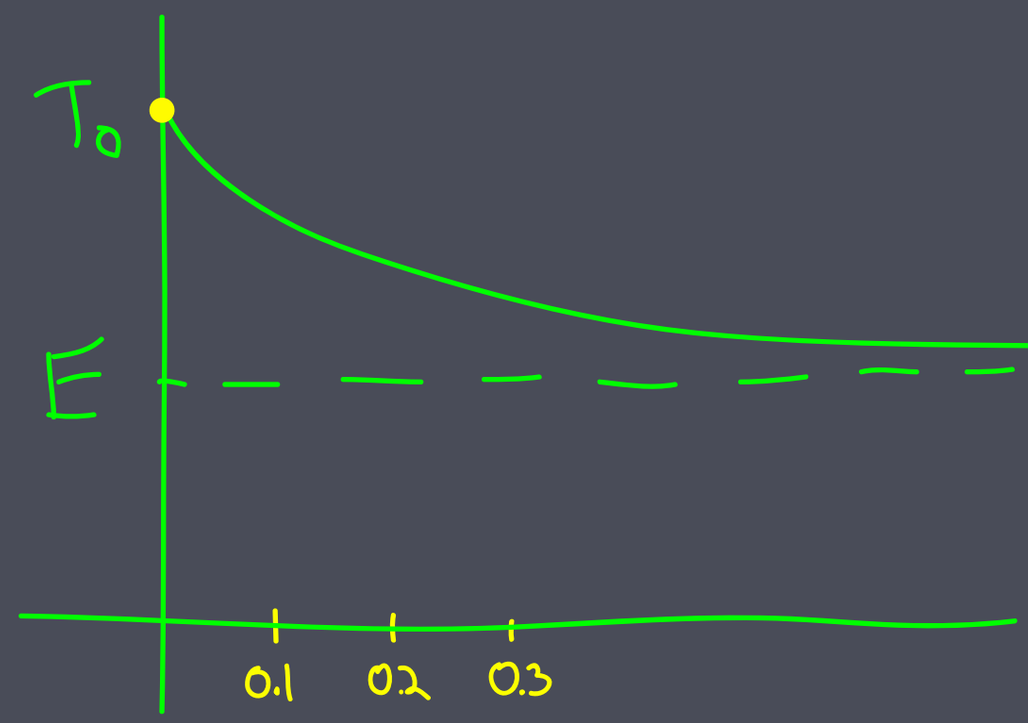

What is the equation of the tangent line to $T(t)$ at $t=0$?

(A) $y = 37 - 0.46t$

(B) $y = 14 + 23e^{-0.02t}$

(C) $y = 37 - 0.28\,t$

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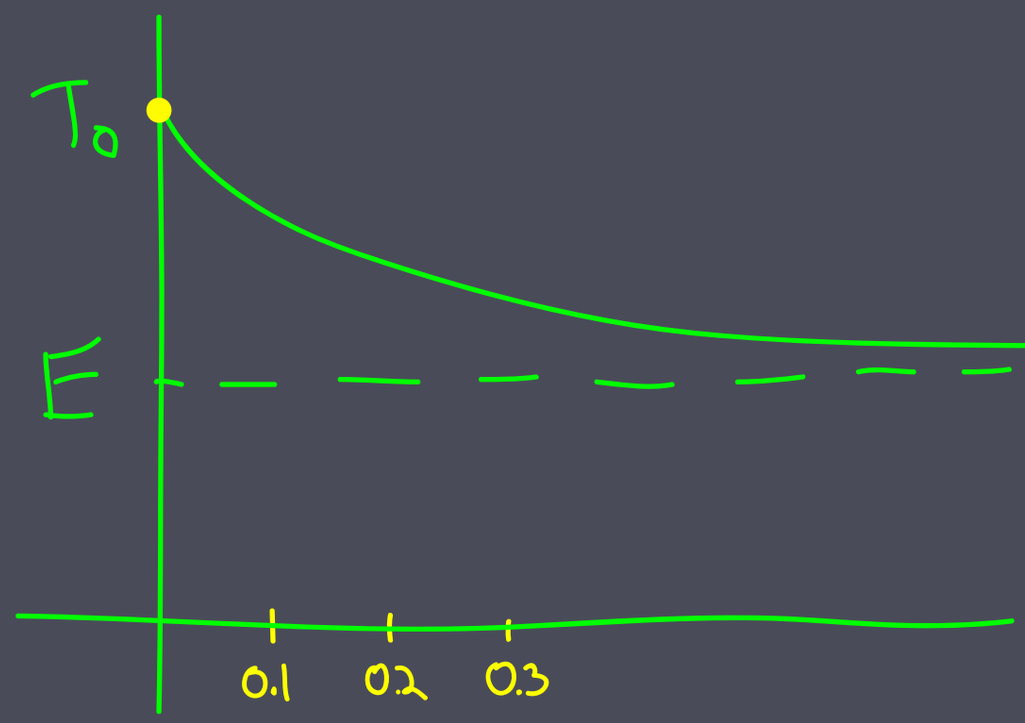

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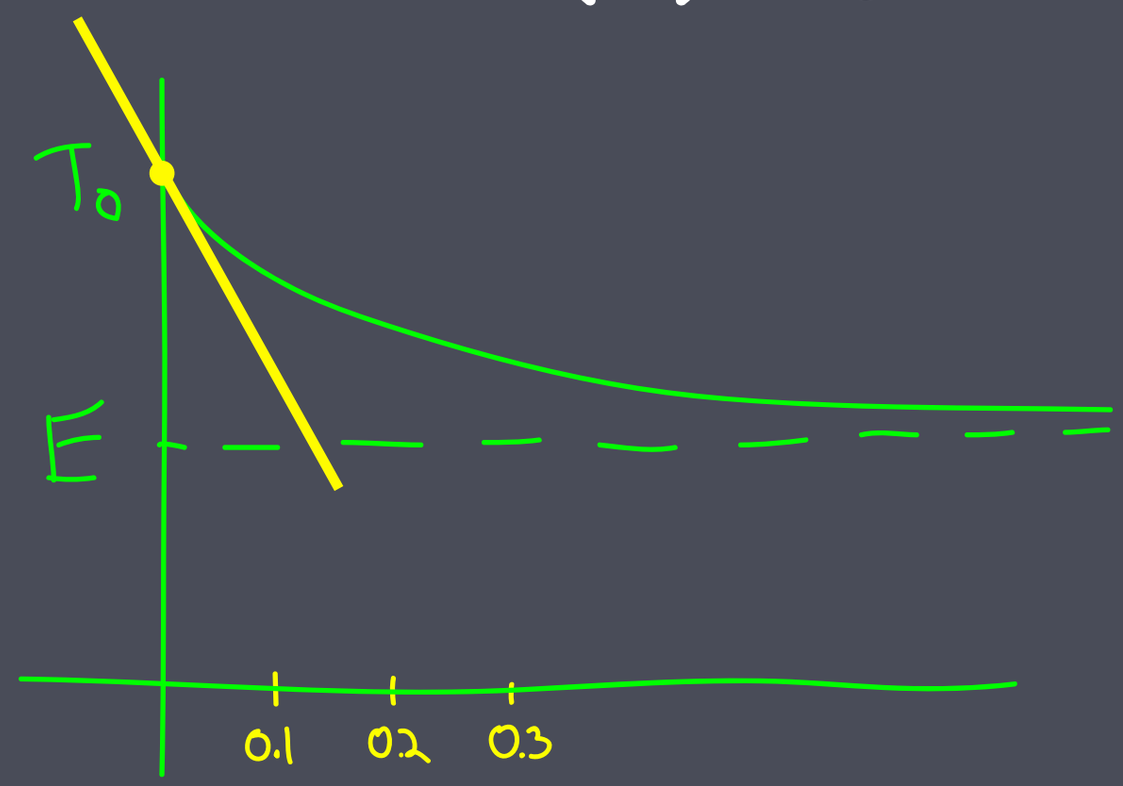

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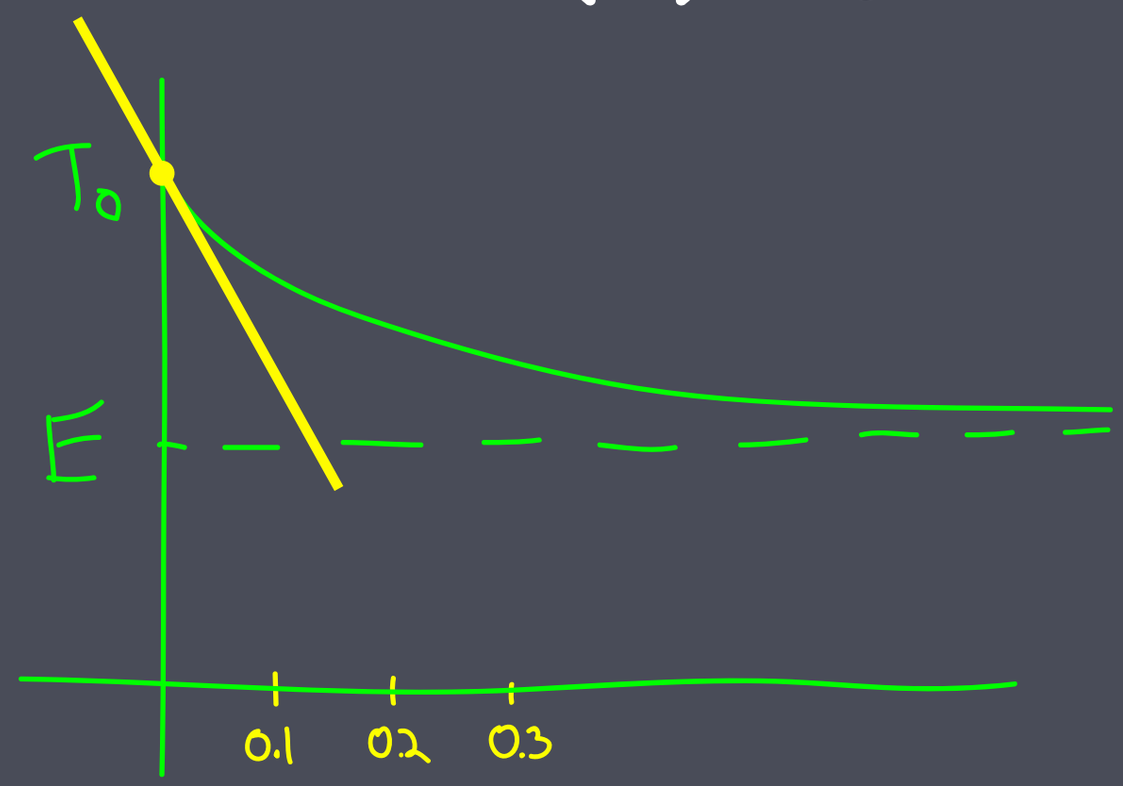

Use the tangent line at $t=0$ to estimate $T(0.1)$:

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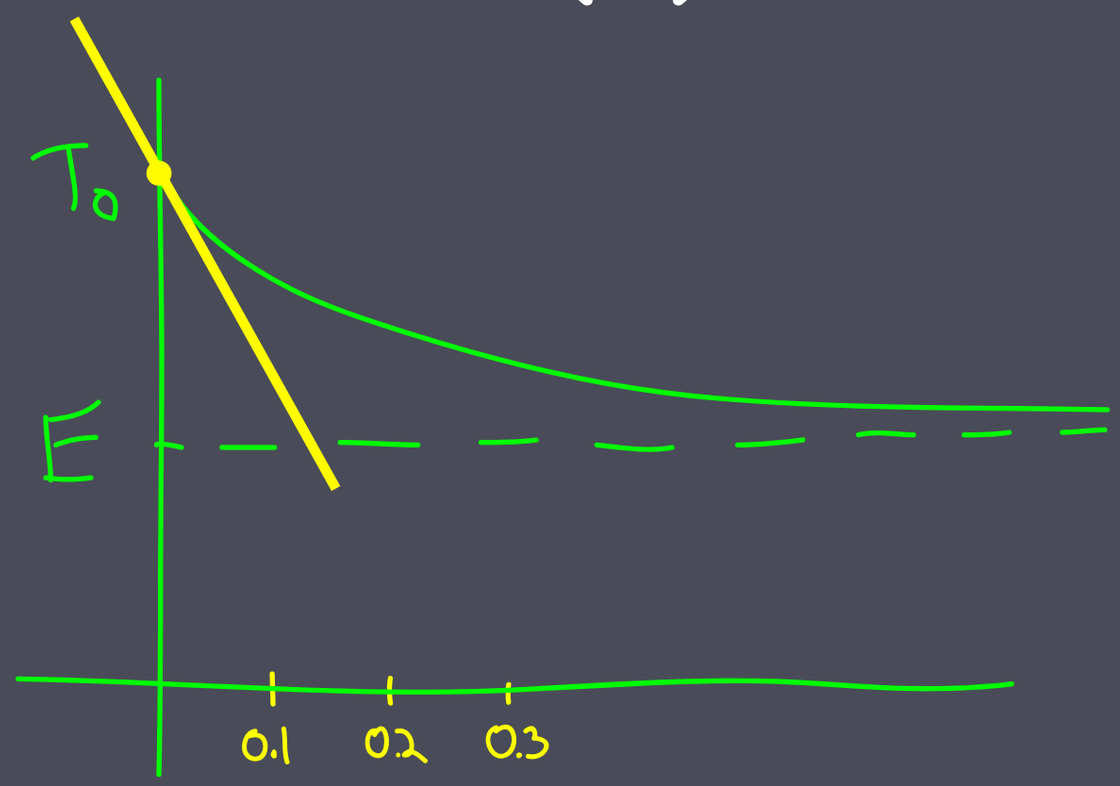

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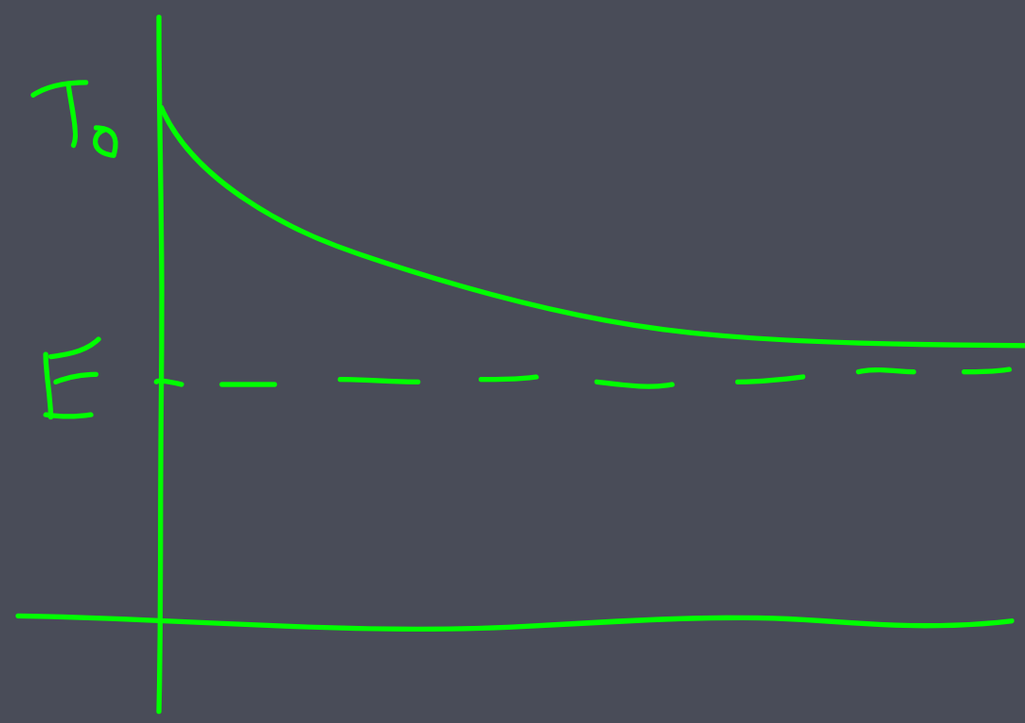

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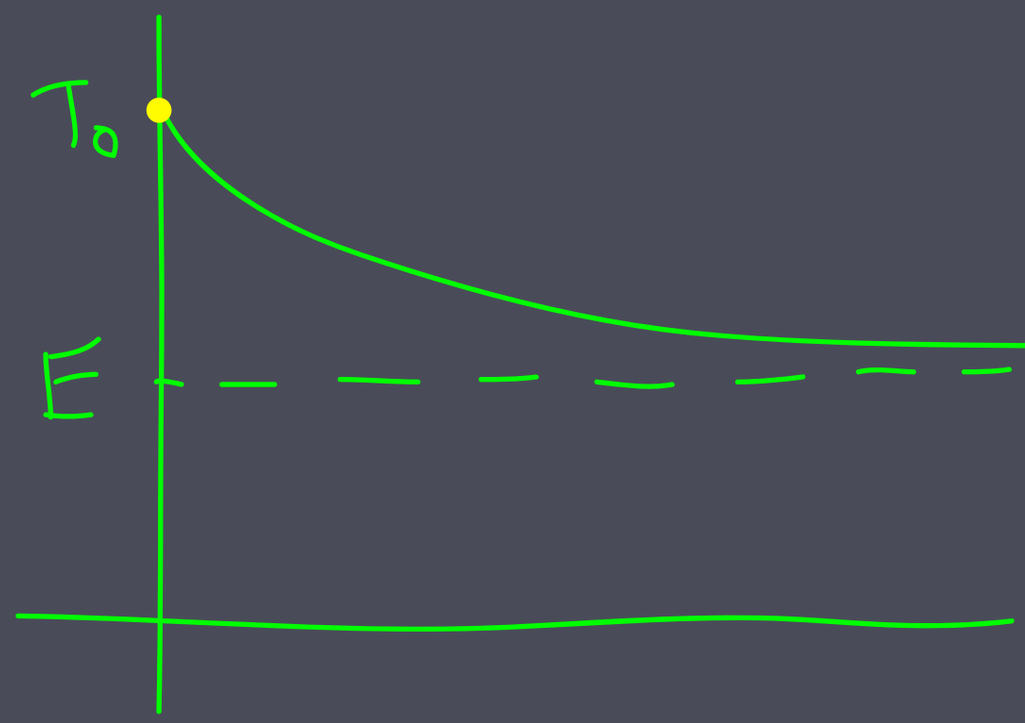

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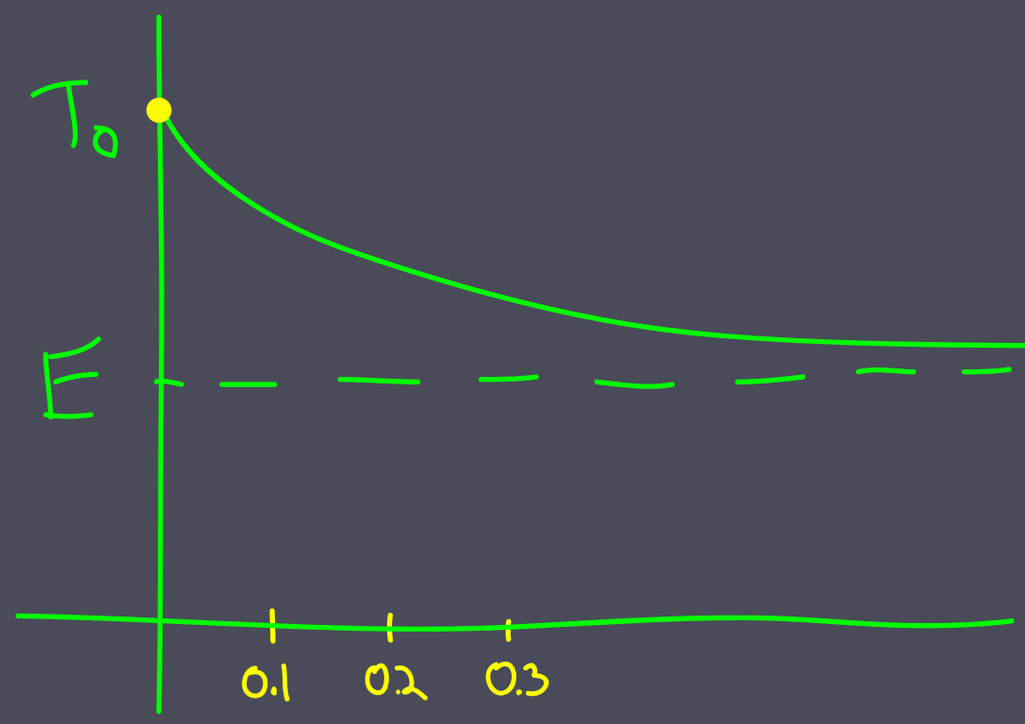

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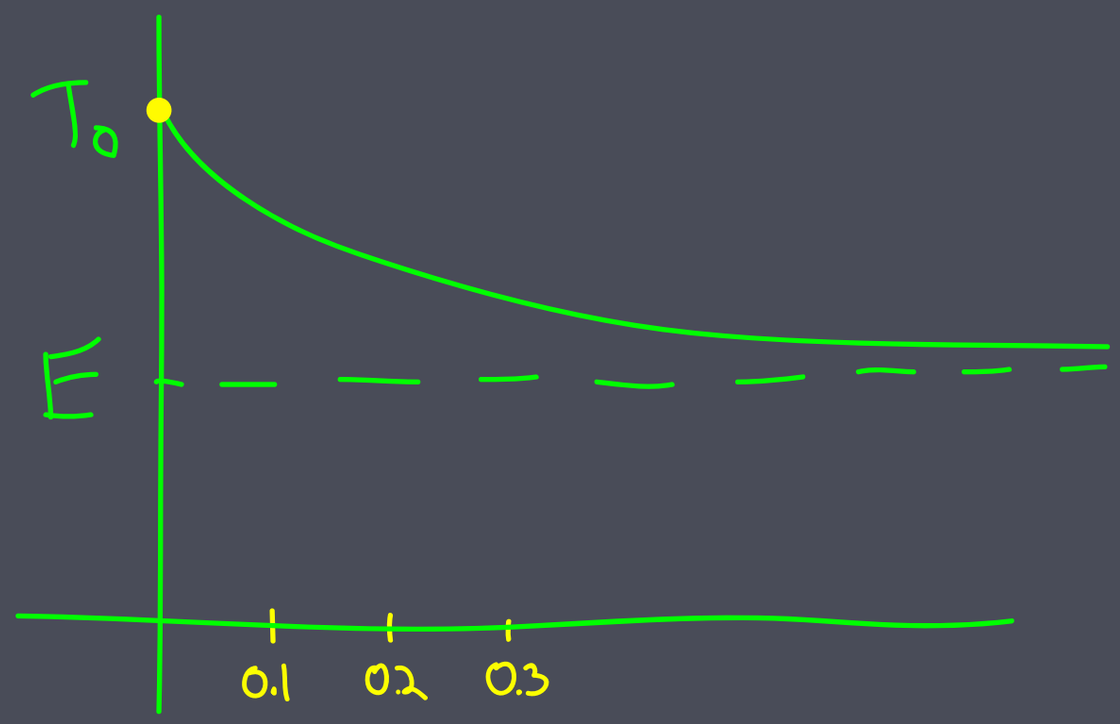

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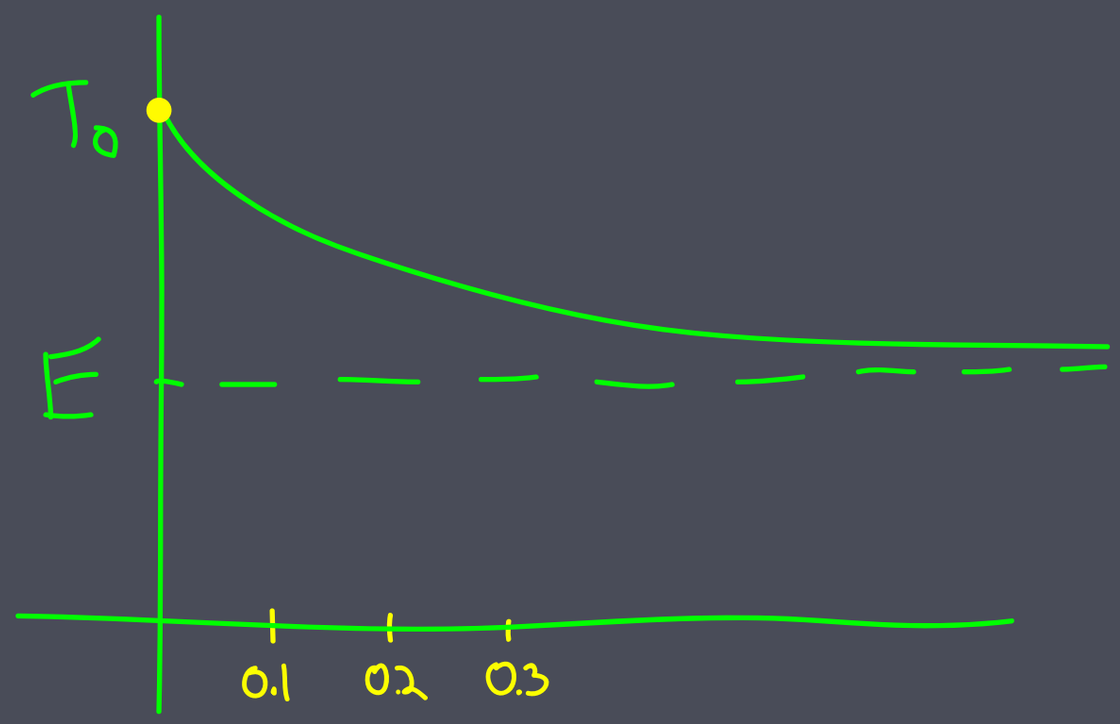

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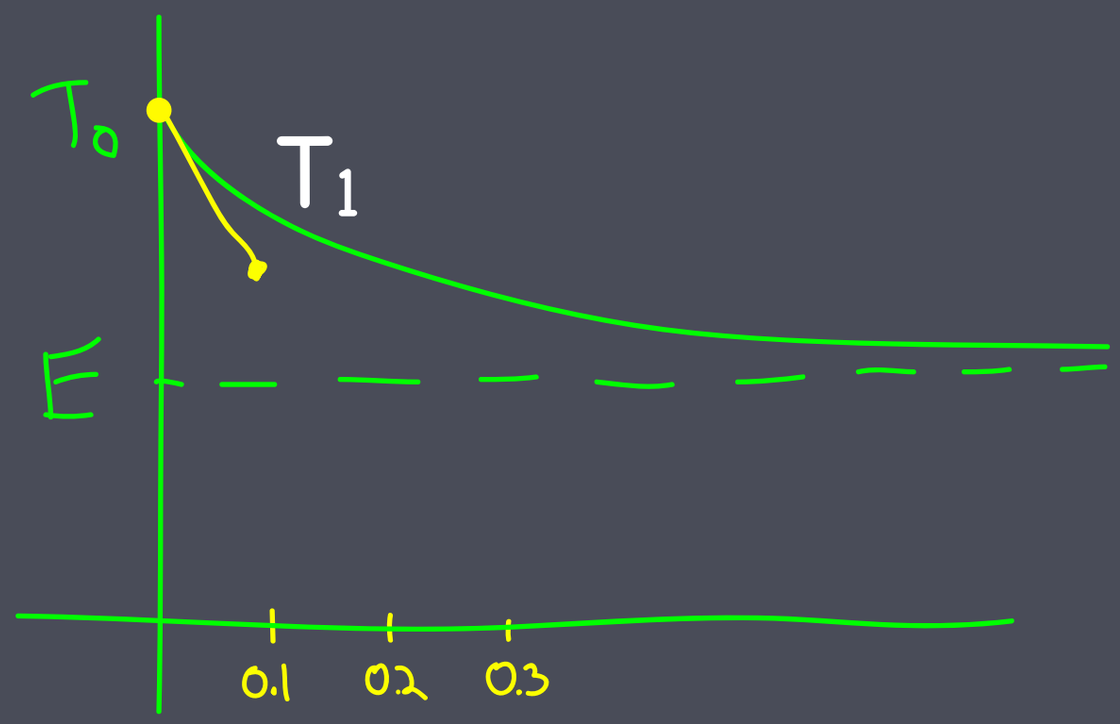

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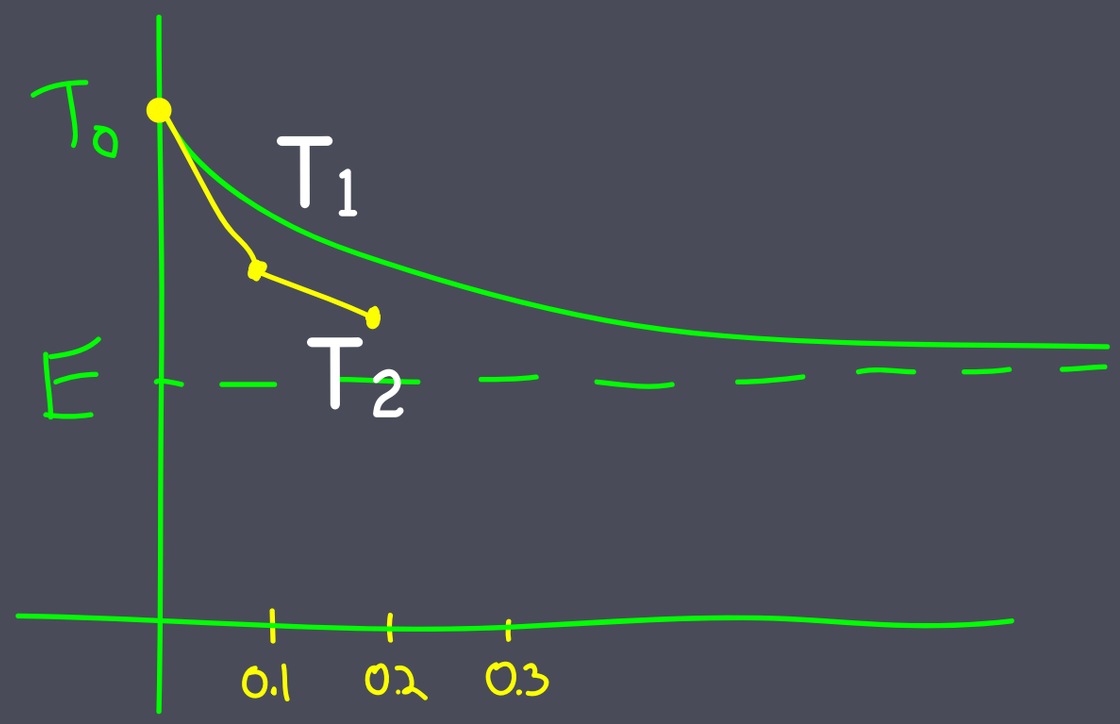

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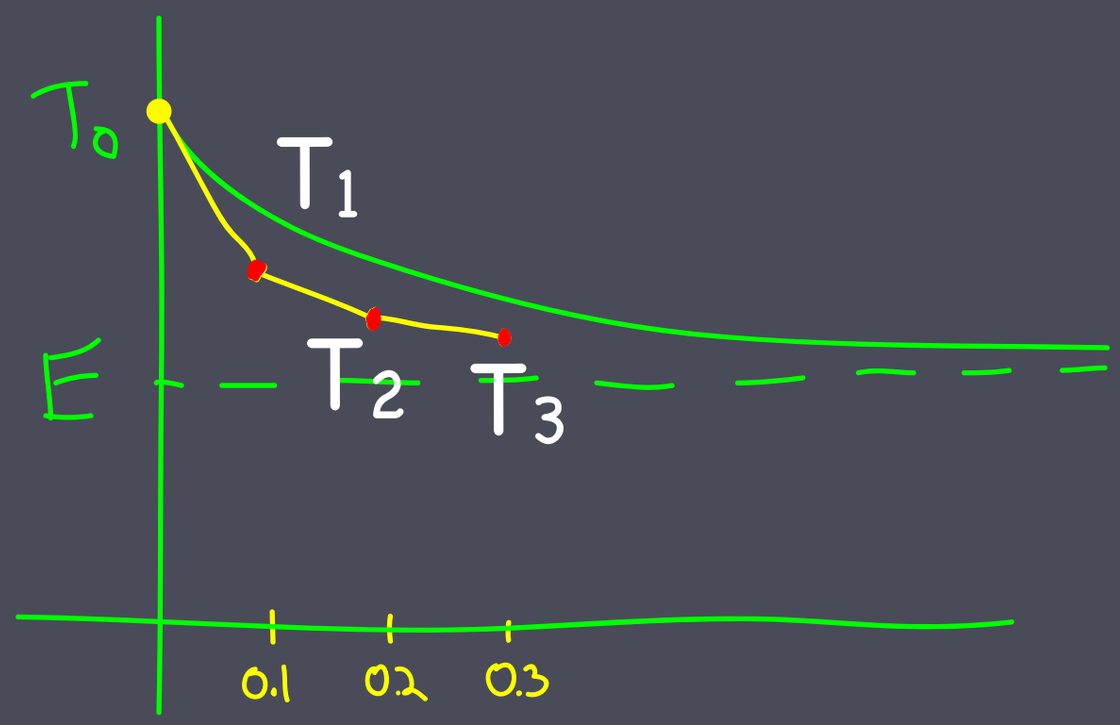

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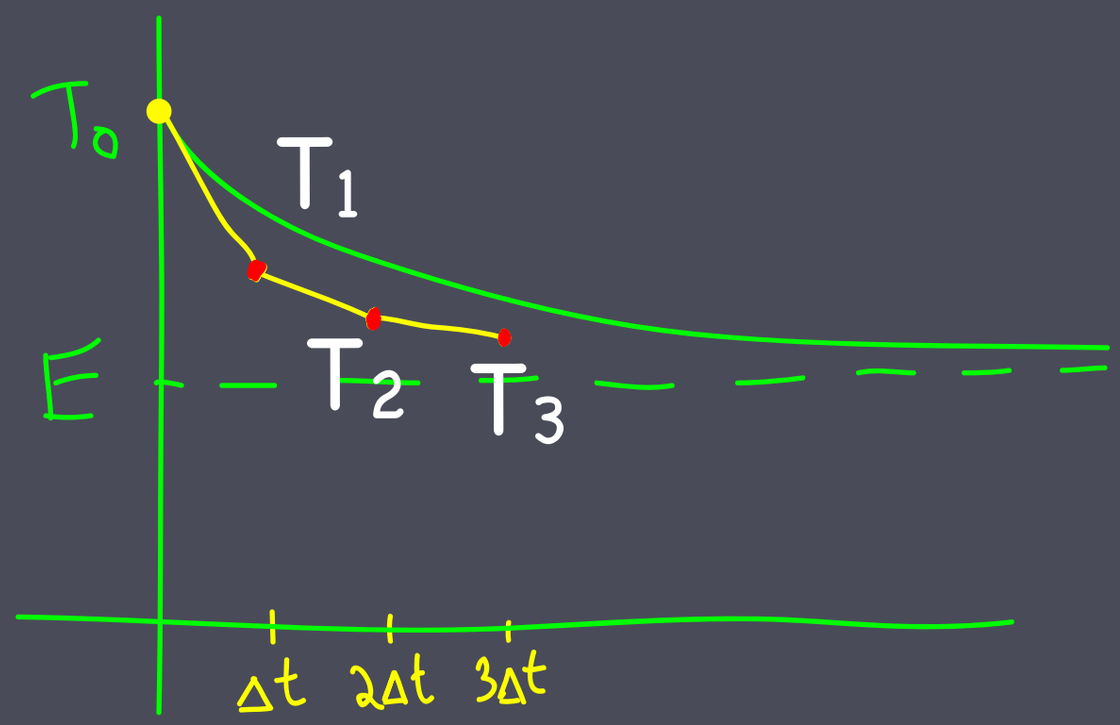

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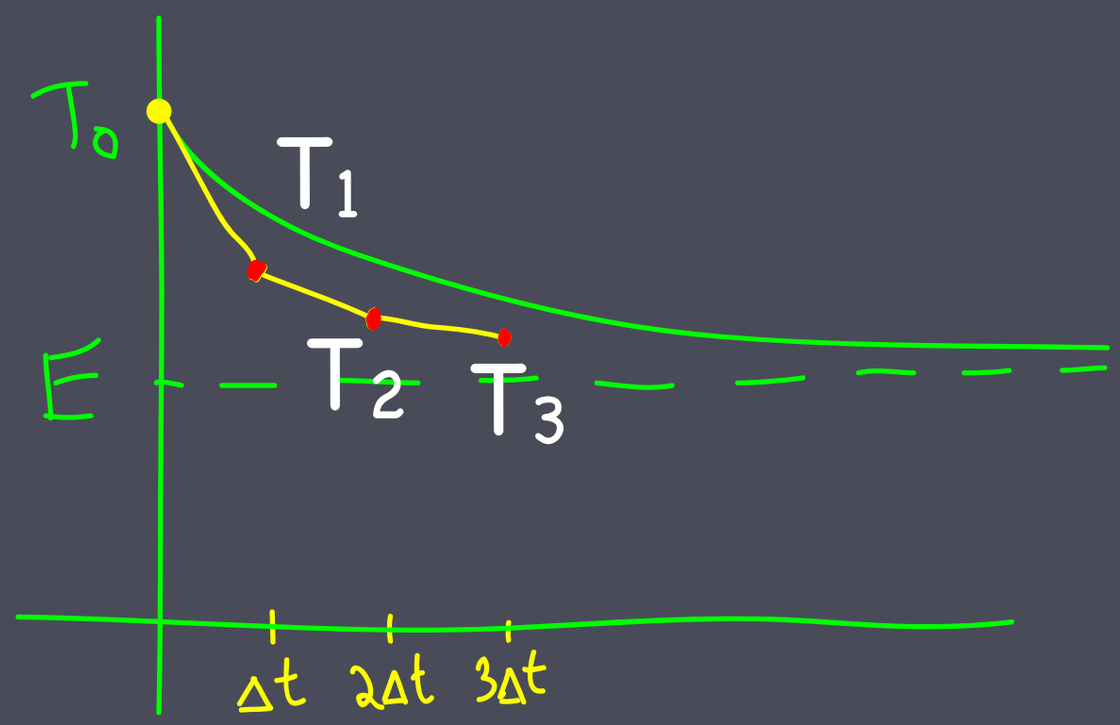

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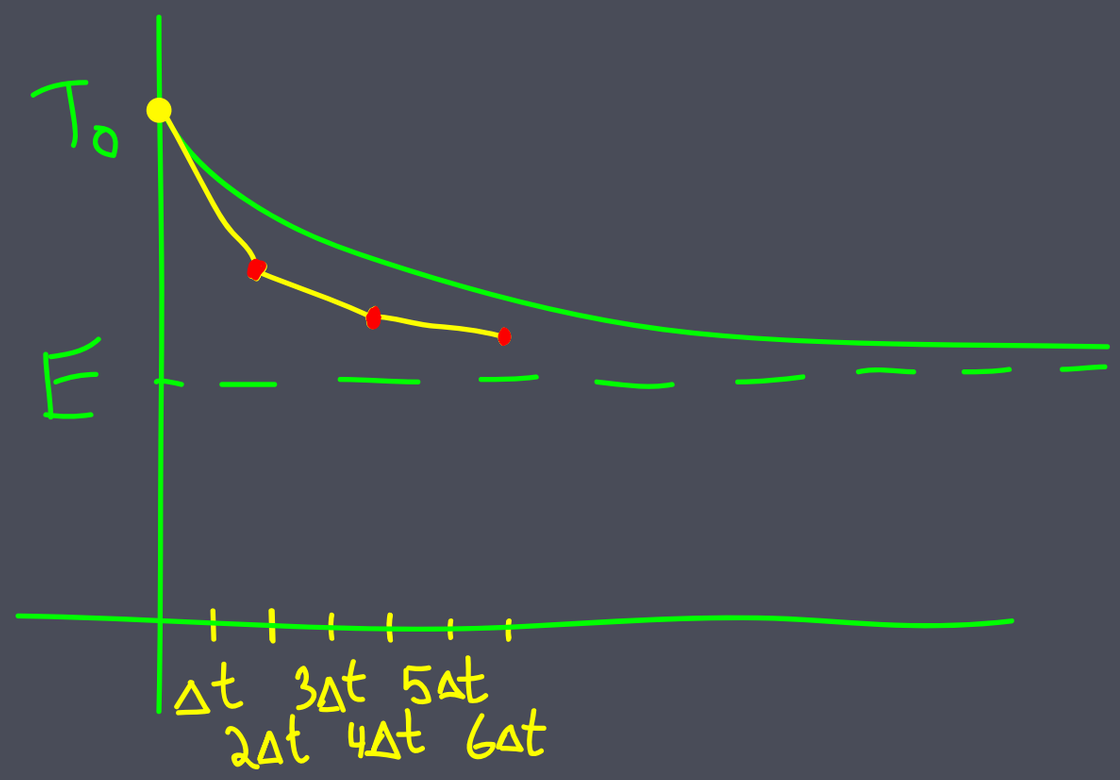

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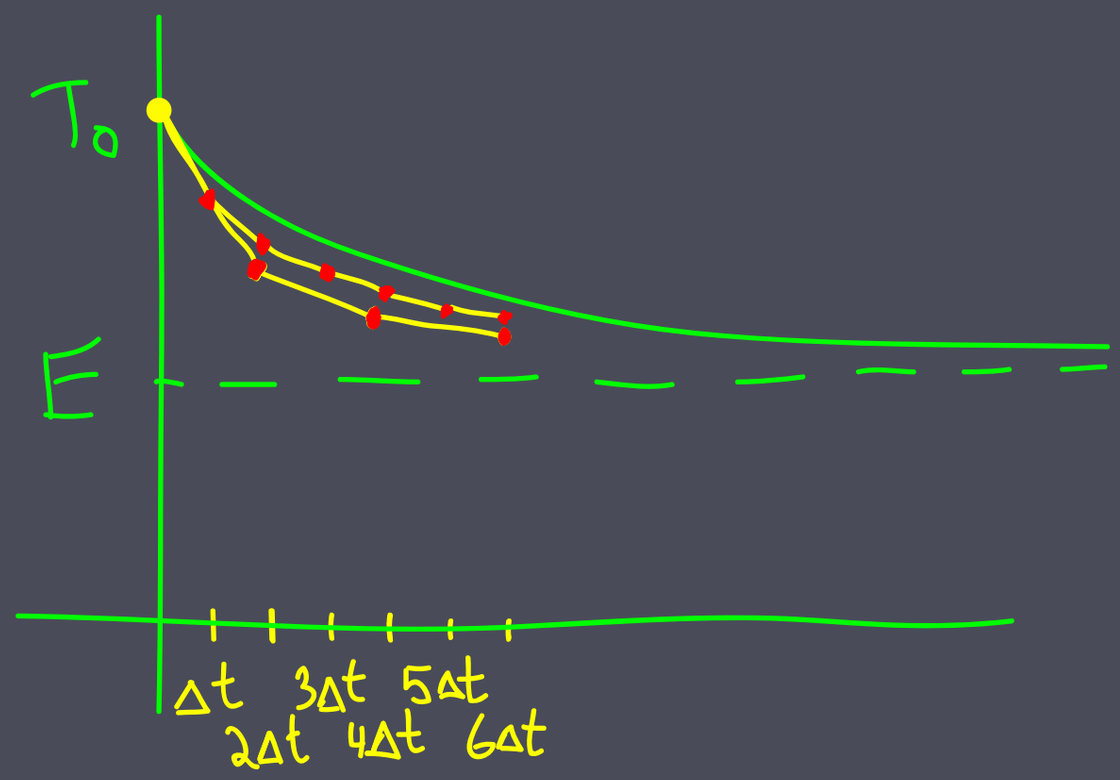

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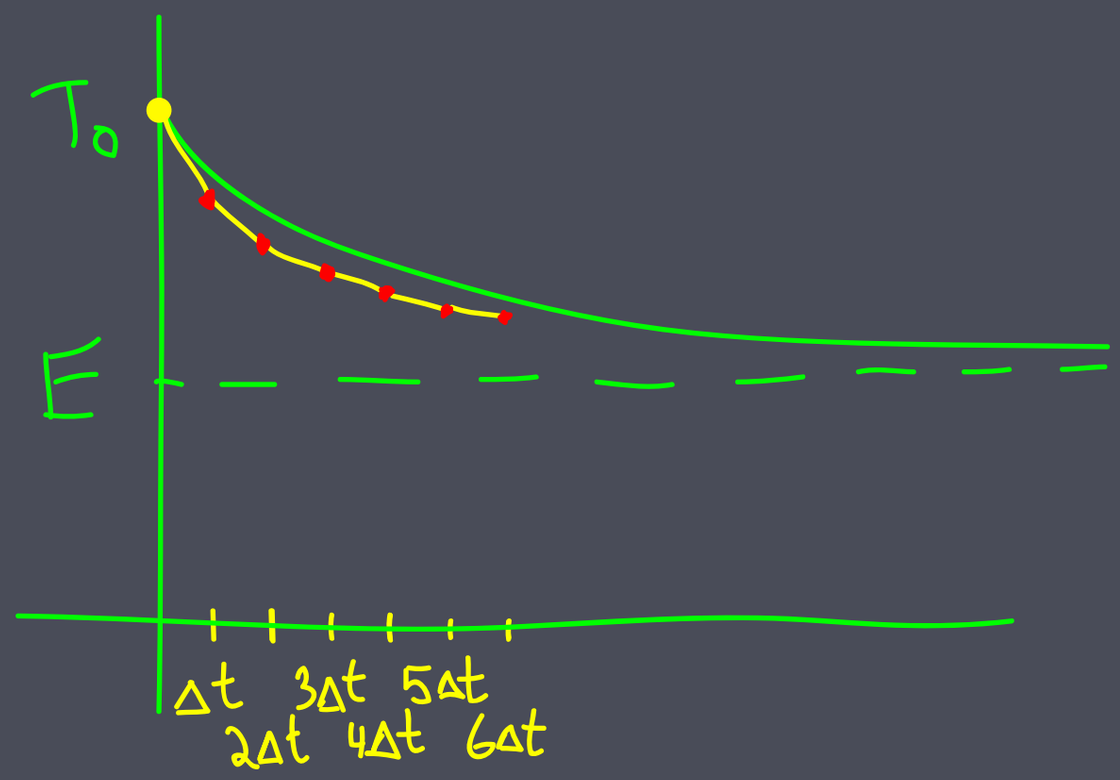

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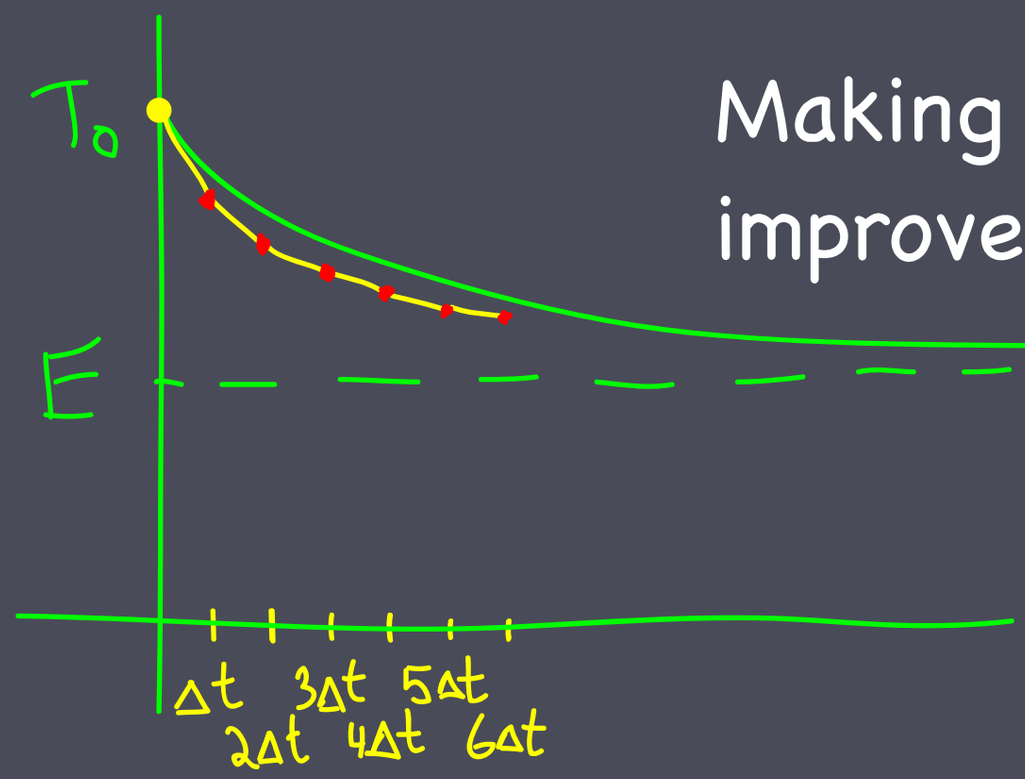

Making Δt smaller improves the estimate.

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When will Euler's method underestimate the true solution?

(A) When the derivative of the true solution is positive.

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Euler's method

- Spreadsheet demo - NLC with a varying environment (fore):

$$T'(t) = k\ (\ E(t) - T(t)\ )$$

with $T(0)=37^{o}C$,

$k=0.15$ and

$E(t) = 14 + 3\ \cos(2\ \pi\ t\ /24)$.

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- Think about how you would determine time of death if given body temp at two later times.

Rates of change that are proportional to two things

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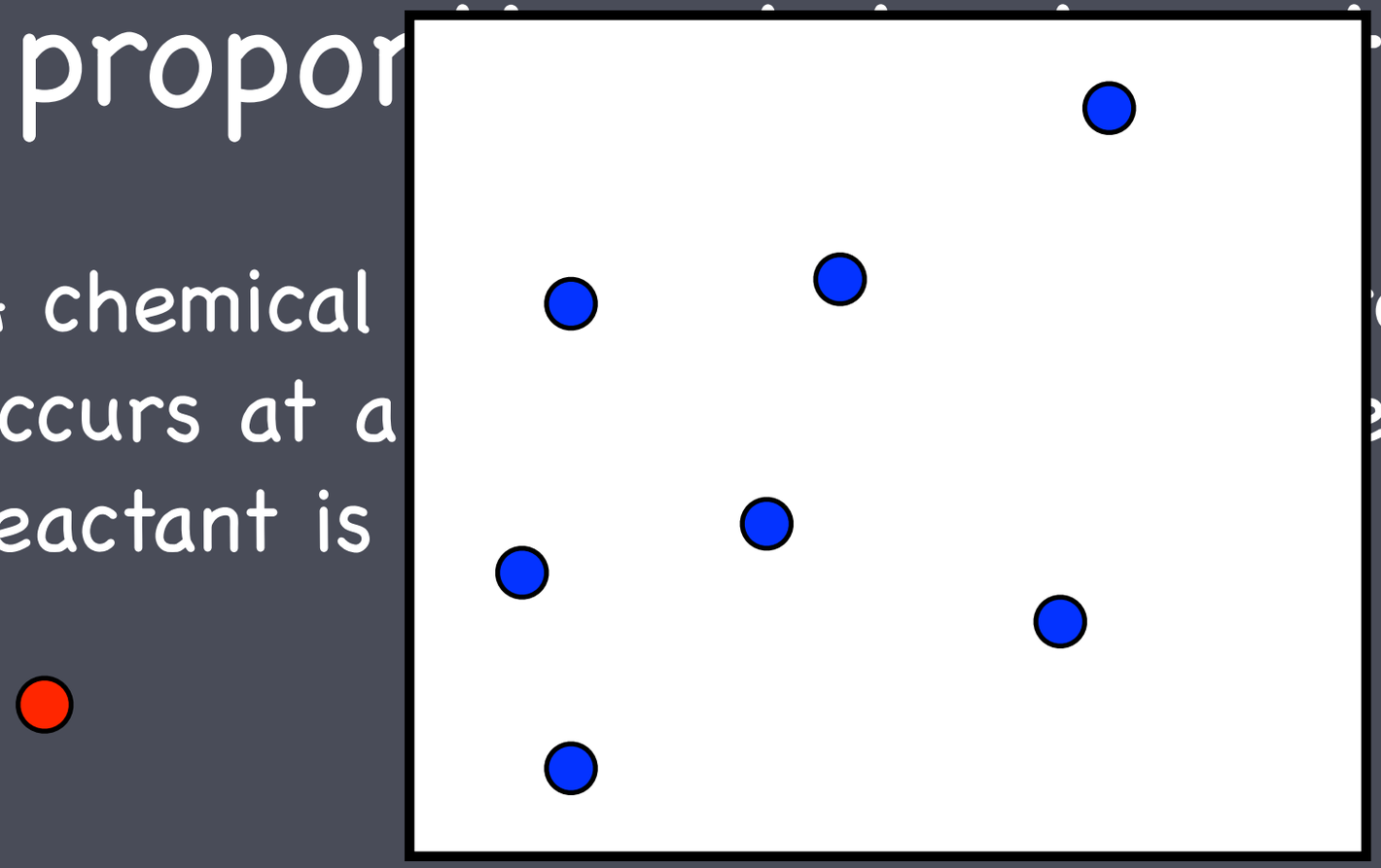

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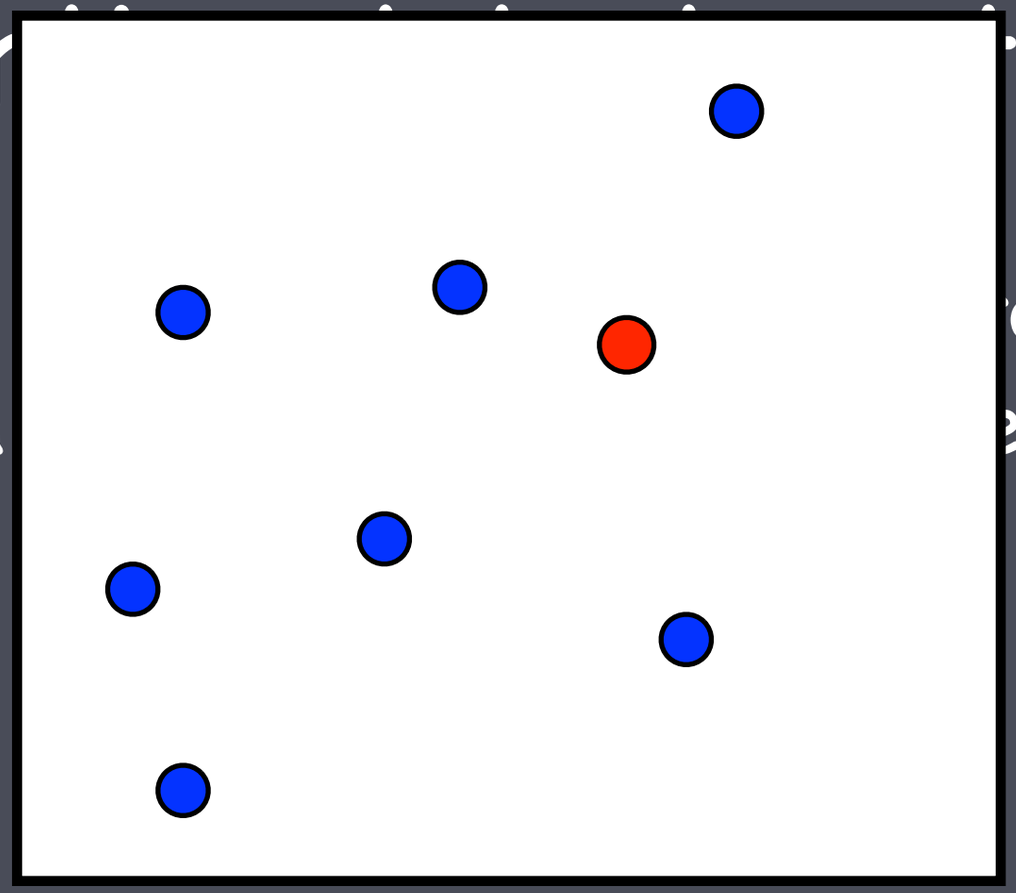

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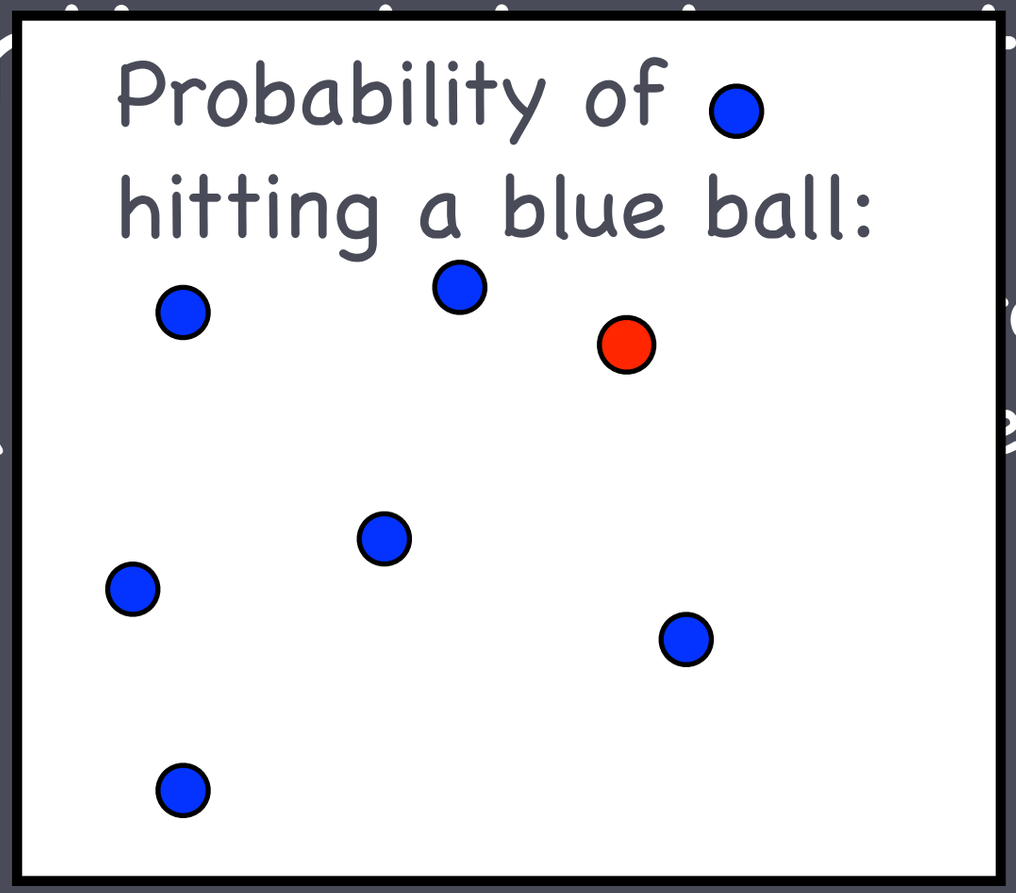

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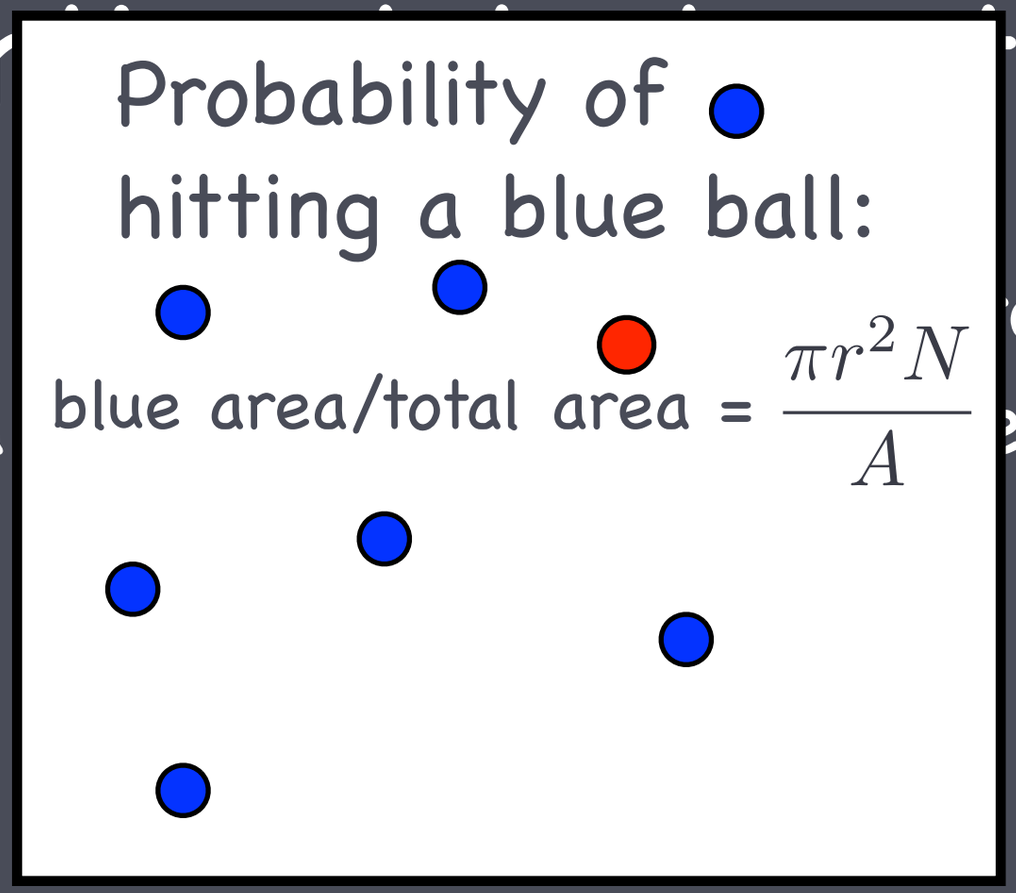

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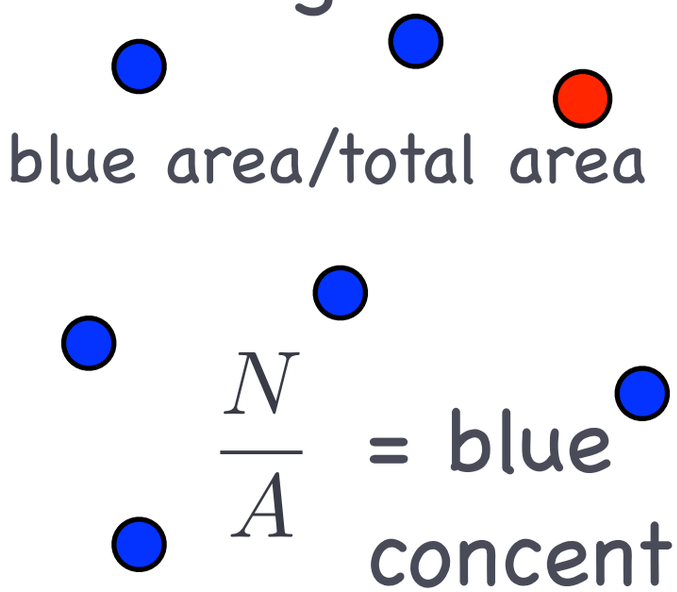
Probability of hitting a blue ball:



blue area/total area = $\frac{\pi r^2 N}{A}$

$\frac{N}{A}$ = blue concentration

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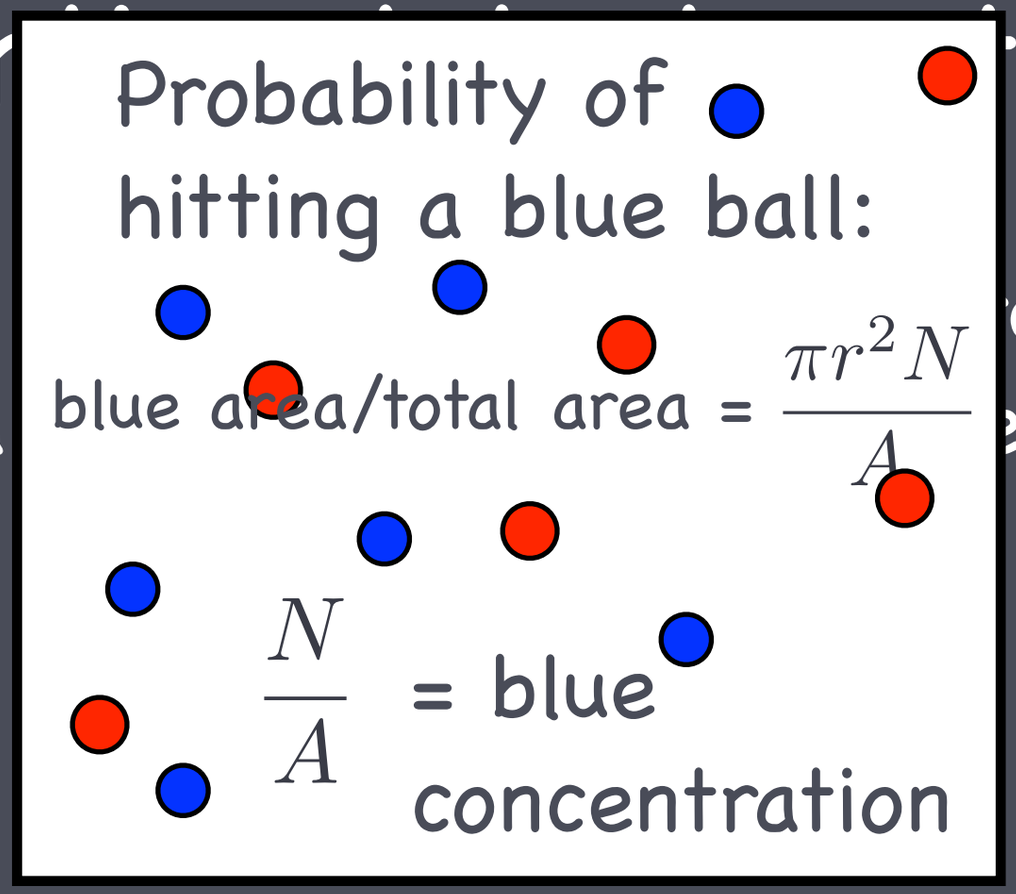

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Logistic equation in different contexts...

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- Active oil exploration sites: bUD (undiscovered and discovered).
- Waterlillies in a pond: bSW (waterlillies and space for waterwillies).

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