Today

- Increasing/decreasing
- Extrema
- Concavity
- Inflection points
- Critical points

I like ice cream

(A)Yes

(B)No

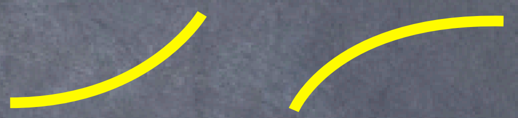
Definitions

Definitions

- Increasing/decreasing
- Local minimum/maximum
- Concave up/down
- 2-like and s-like inflection points

Increasing/decreasing

- We say a function is **increasing** on some interval if for any points a and b with $a < b$ we have that $f(a) < f(b)$.



- We say a function is **decreasing** on some interval if for any points a and b with $a < b$ we have that $f(a) > f(b)$.

- Notice - no reference to $f'(x)$!!

Local minimum/maximum

- A point **a** is a **local minimum** of a function f(x) provided that **f(x) > f(a)** for all x on an interval around a (excluding a, of course).

- A point **a** is a **local maximum** of a function f(x) provided that **f(x) < f(a)** for all x on an interval around a (excluding a, of course).

Local minimum/maximum

- A point **a** is a **local minimum** of a function f(x) provided that **f(x) > f(a)** for all x on an interval around a (excluding a, of course).

Which of the following is a local minimum?

- A point **a** is a **local maximum** of a function f(x) provided that **f(x) < f(a)** for all x on an interval around a (excluding a, of course).

(A) (B) (C)

Local minimum/maximum

- A point **a** is a **local minimum** of a function f(x) provided that **f(x) > f(a)** for all x on an interval around a (excluding a, of course).

Which of the following is a local minimum?

- A point **a** is a **local maximum** of a function f(x) provided that **f(x) < f(a)** for all x on an interval around a (excluding a, of course).

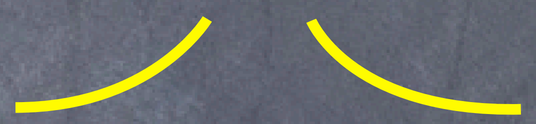
(A) (B) (C)

Local minimum/maximum

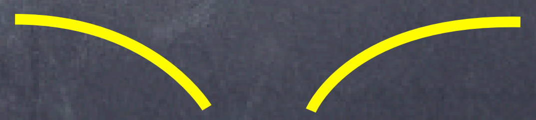
- A point **a** is a **local minimum** of a function f(x) provided that **f(x) > f(a)** for all x on an interval around a (excluding a, of course).
- A point **a** is a **local maximum** of a function f(x) provided that **f(x) < f(a)** for all x on an interval around a (excluding a, of course).

Concave up/down

- We say a function is **concave up** on some interval if for any points a and b with $a < b$ we have that $f'(a) < f'(b)$.



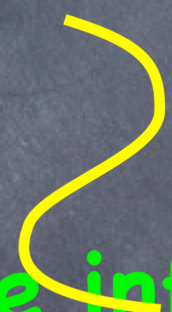
- We say a function is **concave down** on some interval if for any points a and b with $a < b$ we have that $f'(a) > f'(b)$.



- Notice - no reference to $f''(x)$!!

Inflection points

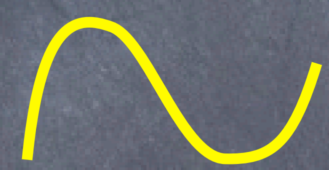
- A point **a** is a **2-like inflection point** of a function f(x) provided that $f'(x) > f'(a)$ for all x on an interval around a (excluding a, of course).



- A point **a** is an **s-like inflection point** of a function f(x) provided that $f'(x) < f'(a)$ for all x on an interval around a (excluding a, of course).

Inflection points

- A point **a** is a **2-like inflection point** of a function f(x) provided that **f'(x) > f'(a)** for all x on an interval around a (excluding a, of course).



- A point **a** is an **s-like inflection point** of a function f(x) provided that **f'(x) < f'(a)** for all x on an interval around a (excluding a, of course).

Inflection points

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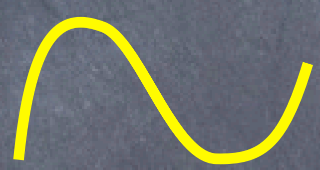

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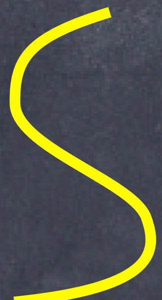
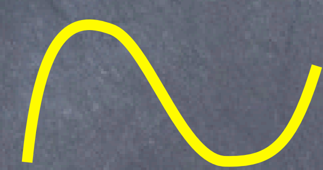


Inflection points

- A point **a** is a **2-like inflection point** of a function f(x) provided that $f'(x) > f'(a)$ for all x on an interval around a (excluding a, of course).



- A point **a** is an **s-like inflection point** of a function f(x) provided that $f'(x) < f'(a)$ for all x on an interval around a (excluding a, of course).

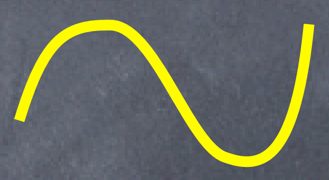
Concave up/down (equivalent)

- We say a function is **concave up** on some interval if f′(x) is increasing on that interval.

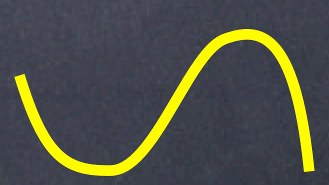
- We say a function is **concave down** on some interval if f′(x) is decreasing on that interval.

Inflection points (equivalent)

- A point **a** is a **2-like inflection point** of a function $f(x)$ provided that **a** is a **local minimum of $f'(x)$.**

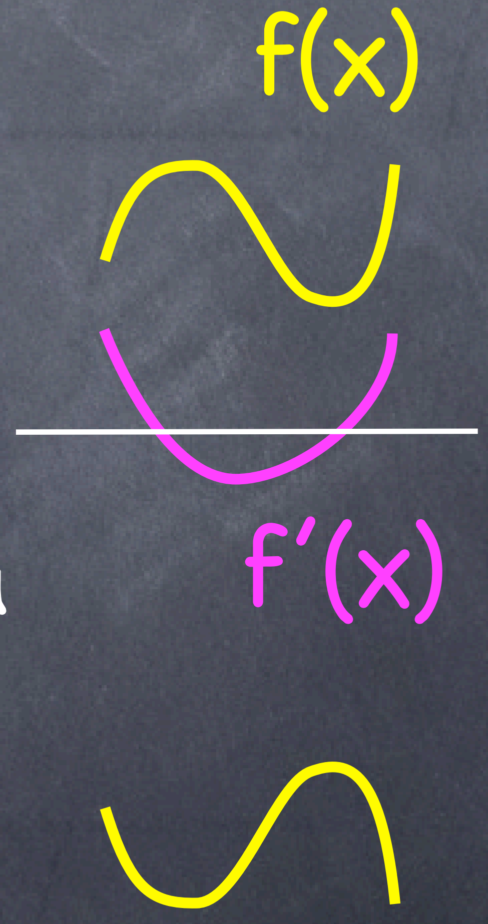


- A point **a** is an **s-like inflection point** of a function $f(x)$ provided that **a** is a **local maximum of $f'(x)$**.



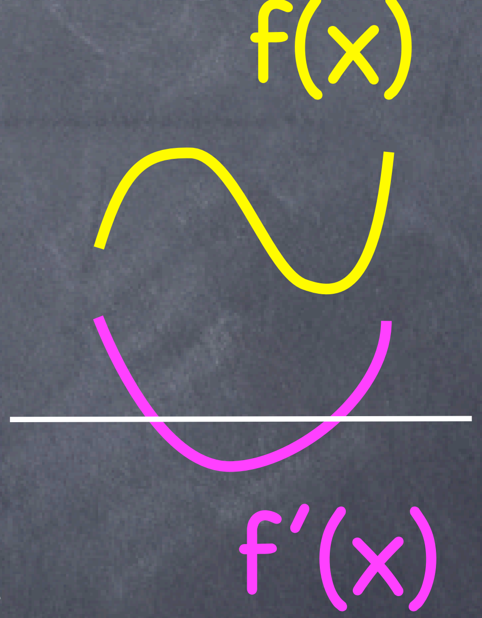
Inflection points (equivalent)

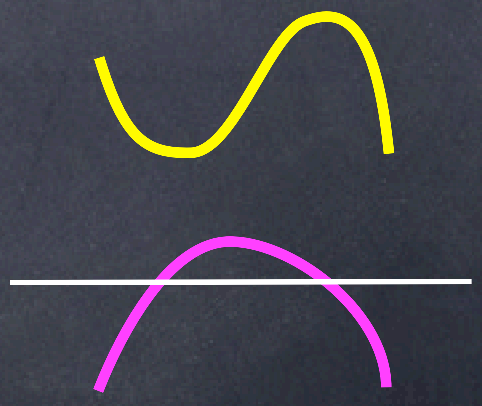
- A point **a** is a **2-like inflection point** of a function f(x) provided that **a** is a **local minimum of f′(x).**
- A point **a** is an **s-like inflection point** of a function f(x) provided that **a** is a **local maximum of f′(x)**.



Inflection points (equivalent)

- A point **a** is a **2-like inflection point** of a function f(x) provided that **a** is a **local minimum of f'(x).**
- A point **a** is an **s-like inflection point** of a function f(x) provided that **a** is a **local maximum of f'(x)**.



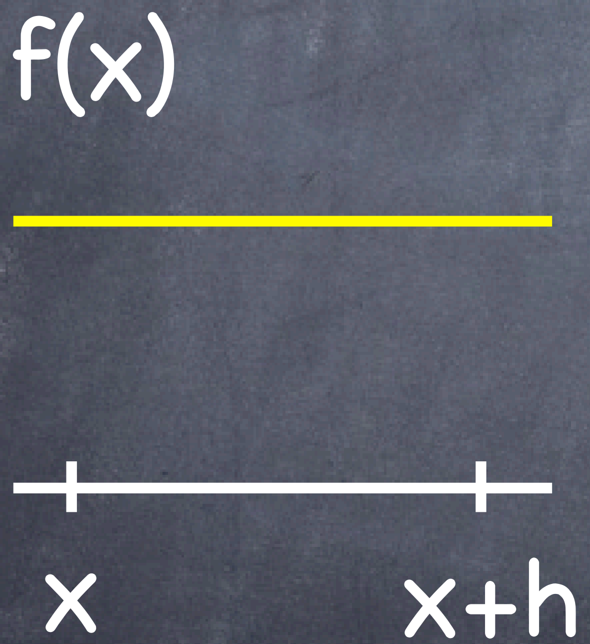


Tools

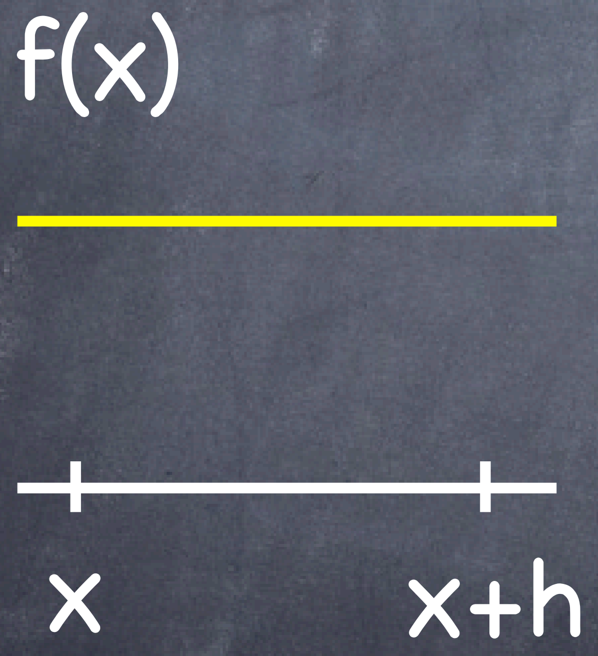
- Using $f'(x)$ to determine intervals of increase/decrease.
- Using $f'(x)$ to find extrema.
- Using $f''(x)$ to determine intervals of concave up/down.
- Using $f''(x)$ to find inflection points.

Link between increasing/ decreasing and f'

The function is constant.



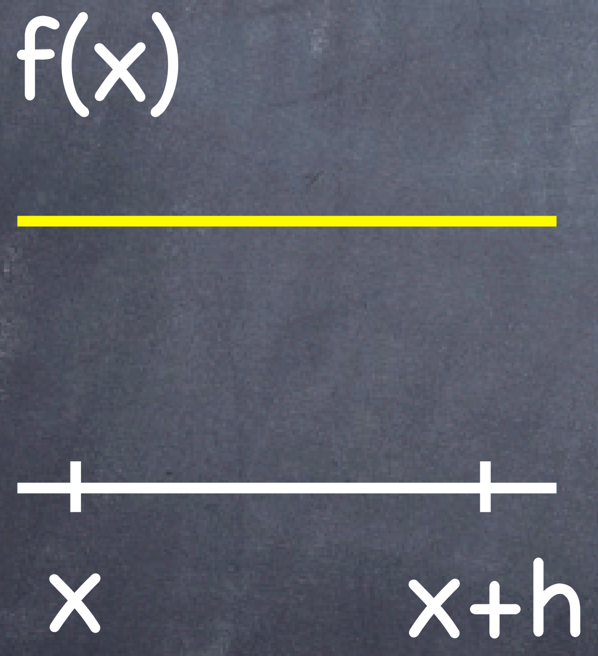
Link between increasing/ decreasing and f′



The function is constant.

➡ $f(x+h) = f(x)$

Link between increasing/ decreasing and f′

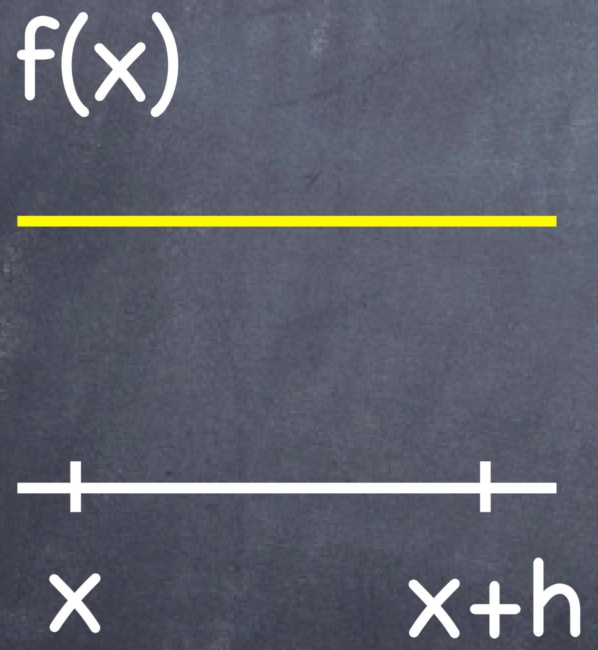


The function is constant.

➡ $f(x+h) = f(x)$

➡ $f(x+h) - f(x) = 0$

Link between increasing/ decreasing and f'



The function is constant.

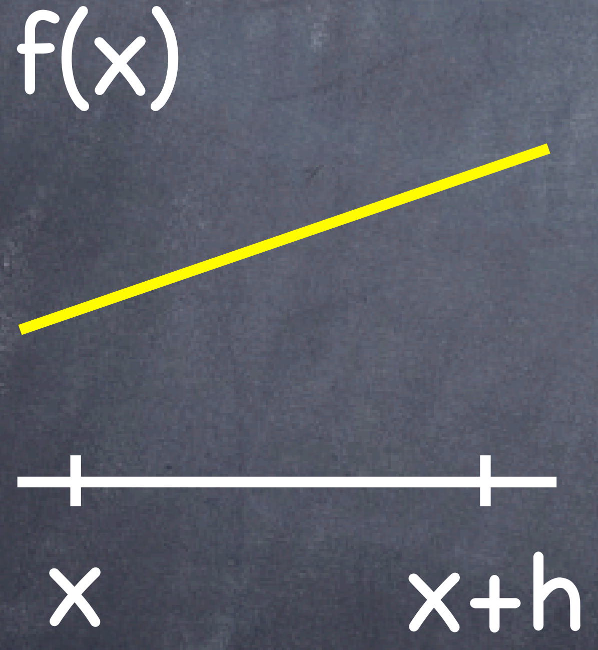
➡ $f(x+h) = f(x)$

➡ $f(x+h) - f(x) = 0$

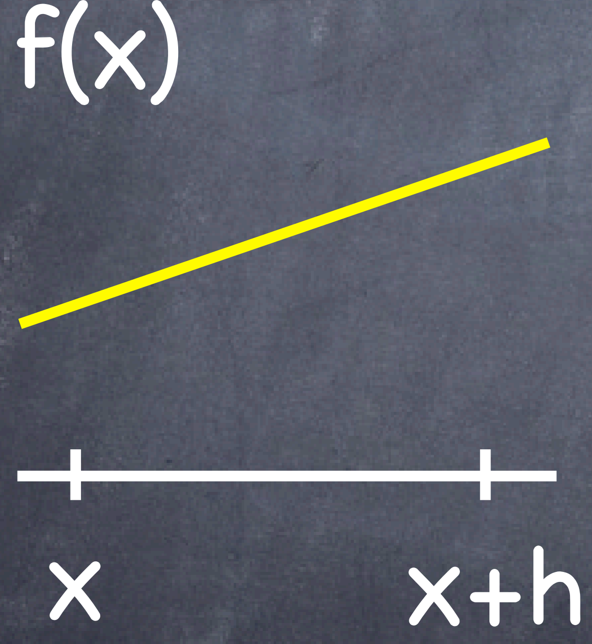
➡ $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0$$

Link between increasing/ decreasing and f'

The function is increasing.



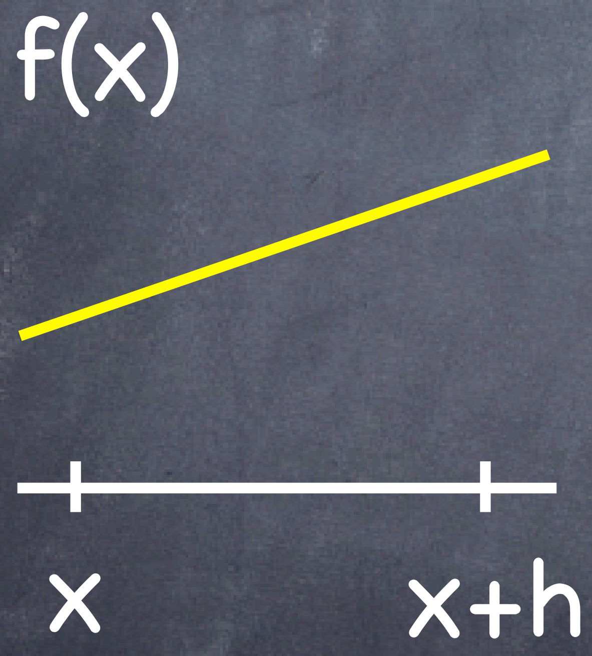
Link between increasing/ decreasing and f'



The function is increasing.

➡ $f(x+h) > f(x)$

Link between increasing/ decreasing and f'

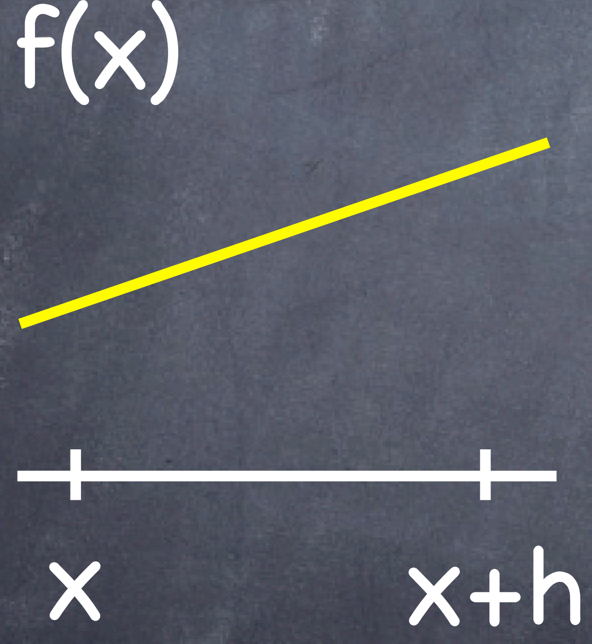


The function is increasing.

➡ $f(x+h) > f(x)$

➡ $f(x+h) - f(x) > 0$

Link between increasing/ decreasing and f'



The function is increasing.

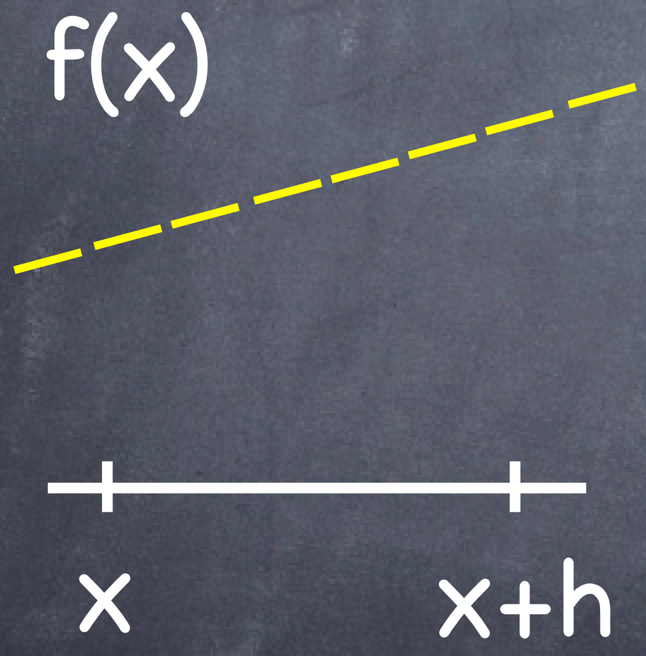
➡ $f(x+h) > f(x)$

➡ $f(x+h) - f(x) > 0$

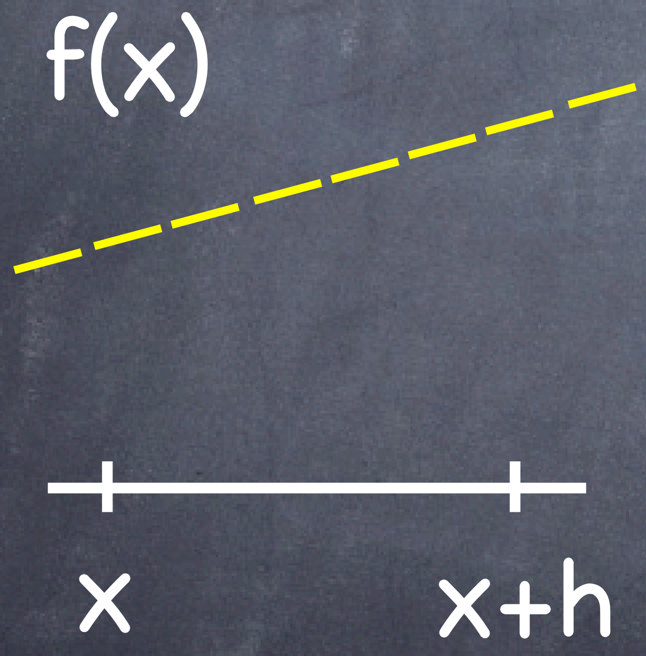
➡ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} > 0$

Link between concavity and f''

The slope is constant.



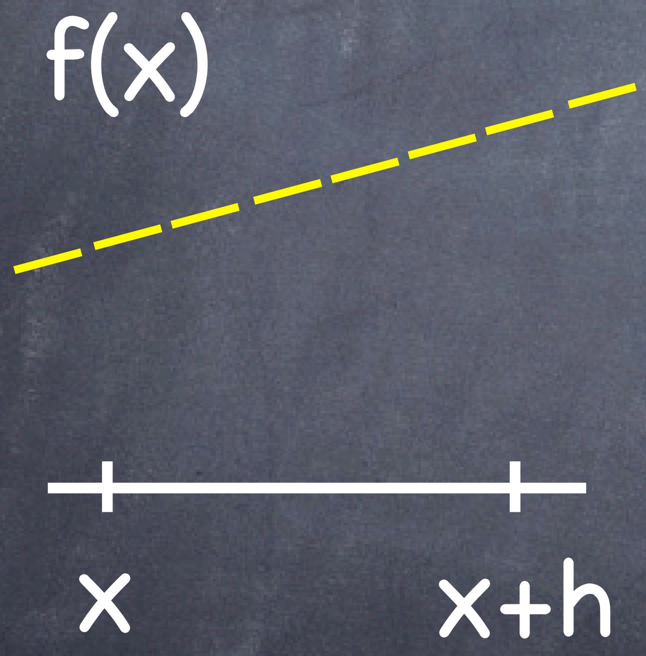
Link between concavity and f″



The slope is constant.

➡ $f'(x+h) = f'(x)$

Link between concavity and f''

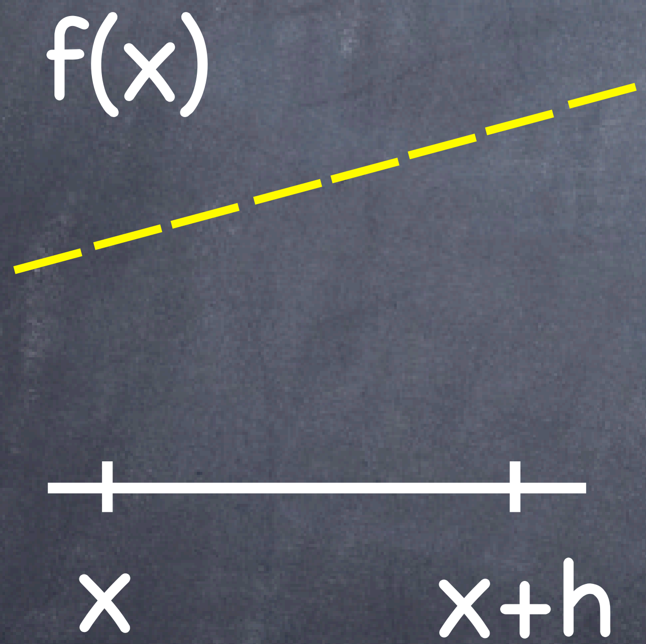


The slope is constant.

➡ $f'(x+h) = f'(x)$

➡ $f'(x+h) - f'(x) = 0$

Link between concavity and f″



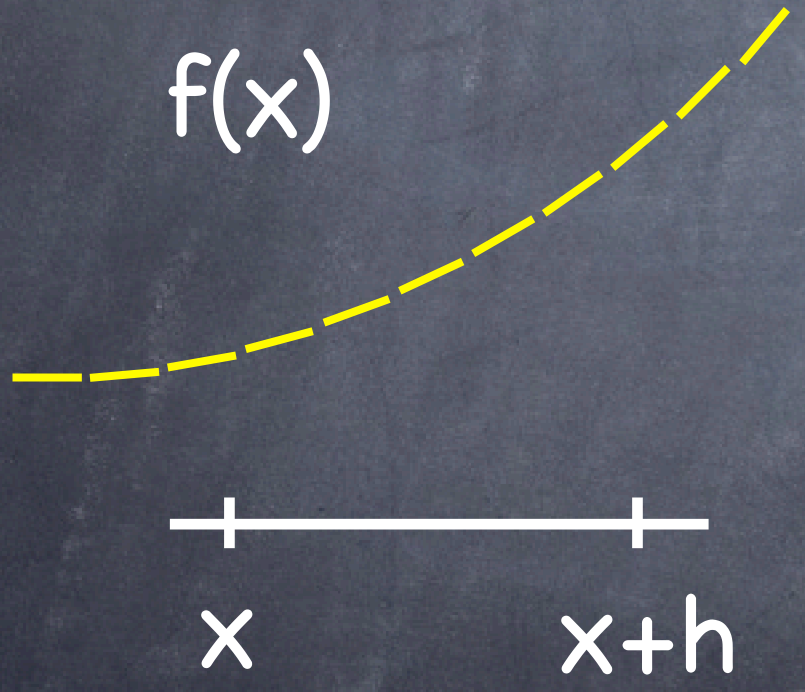
The slope is constant.

➡ $f'(x+h) = f'(x)$

➡ $f'(x+h) - f'(x) = 0$

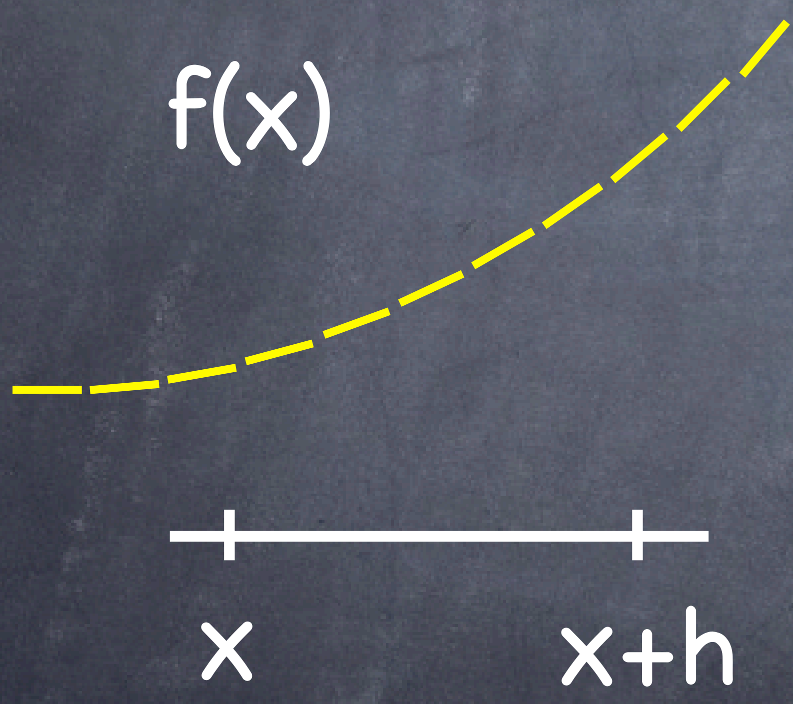
➡ $$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$
$$= 0$$

Link between concavity and f″



The slope is increasing.

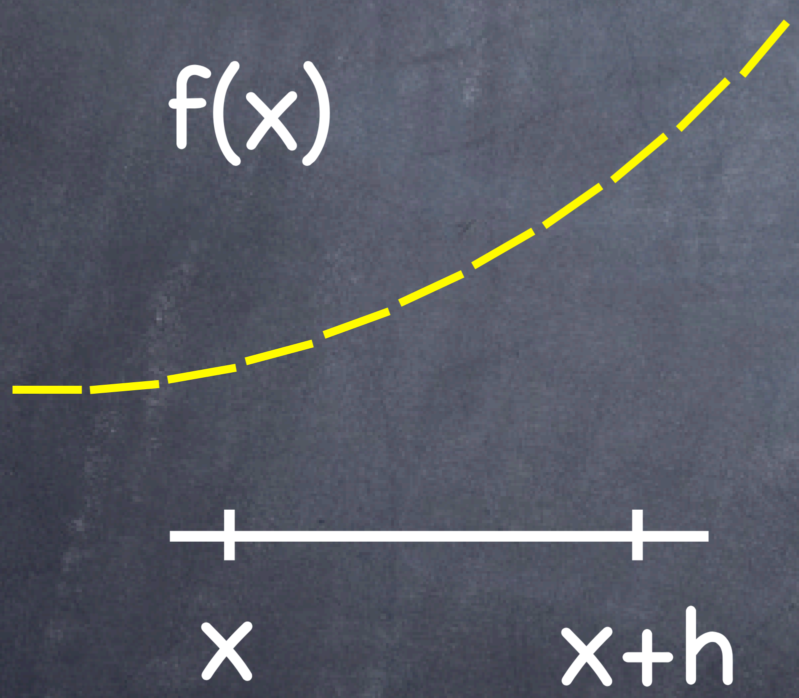
Link between concavity and f″



The slope is increasing.

➡ $f'(x+h) > f'(x)$

Link between concavity and f″

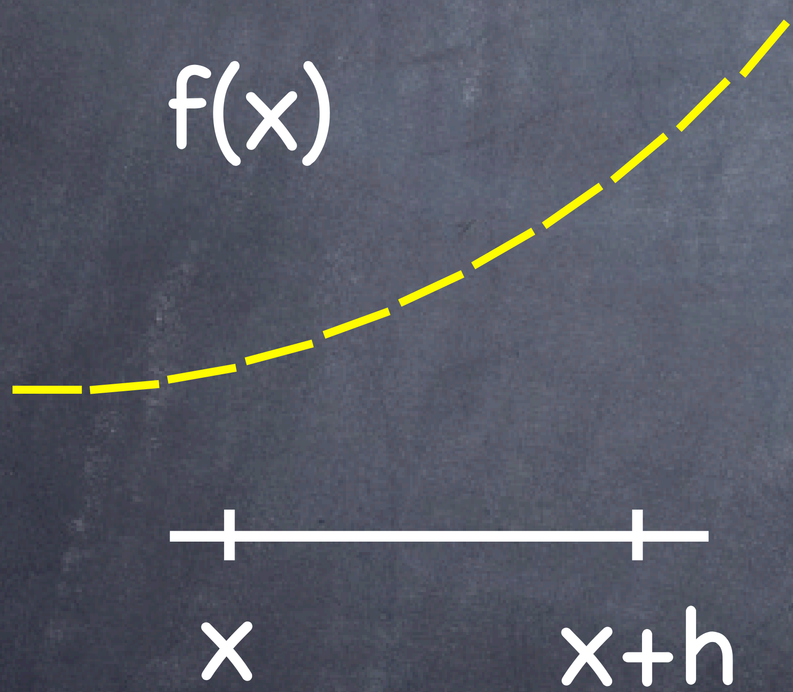


The slope is increasing.

➡ $f'(x+h) > f'(x)$

➡ $f'(x+h) - f'(x) > 0$

Link between concavity and f″



The slope is increasing.

➡ $f'(x+h) > f'(x)$

➡ $f'(x+h) - f'(x) > 0$

➡ $$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$> 0$$

Critical points

- A critical point of $f(x)$ is a point **a** at which **$f'(a)=0$** or **$f'(a)$ is not defined** even though $f(a)$ is defined.
- Use of critical points:
  - Critical points of $f(x)$ might be minima or maxima of $f(x)$. Not always though.
  - Critical points of $f'(x)$ might be minima or maxima of $f'(x)$ and hence inflection points of $f(x)$. Not always though.