Today

• Spreadsheets
• Differentiation rules
• Tangent lines
• Reminders:
  • PL4.2 Wednesday 7am,
  • Assignment 3 Thursday 7am,
  • OSH 2 Friday 11:59 pm.
  • Sign up for midterm time/room.
How to graph \( f'(x) \) using a spreadsheet

• Sketch \( f'(x) \) for the following functions:

\[
f(x) = |\sin(x)|
\]

\[
f(x) = e^{-x^2} \sin(5x)
\]

• Zooming in on a specific region...

Demo in Google sheets or Excel.
A comment on derivative notation

Leibniz: \[ y = f(x) \]
\[ \frac{dy}{dx} = f'(x) \]

Newton: \[ \left. \frac{dy}{dx} \right|_{x=2} = f'(2) \]
Power rule

\[ f(x) = x^2 \]
Power rule

\[ f(x) = x^2 \]

Find \( f' \) at \( x=2 \) (using the definition of the derivative).
Power rule

\[ f(x) = x^2 \]

\[ f'(2) = \lim_{h \to 0} \frac{(2 + h)^2 - 2^2}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(2) = \lim_{h \to 0} \frac{(2 + h)^2 - 2^2}{h} \]

\[ = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(2) = \lim_{h \to 0} \frac{(2 + h)^2 - 2^2}{h} \]

\[ = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \]

\[ = \lim_{h \to 0} \frac{4h + h^2}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(2) = \lim_{h \to 0} \frac{(2 + h)^2 - 2^2}{h} \]

\[ = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \]

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Power rule

\[ f(x) = x^2 \]

\[ f'(2) = \lim_{h \to 0} \frac{(2 + h)^2 - 2^2}{h} \]

\[ = \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \]

\[ = \lim_{h \to 0} \frac{4h + h^2}{h} = 4 \]
Power rule

\[ f(x) = x^2 \]
Power rule

\[ f(x) = x^2 \]

Find \( f' \) at all points \( x \) at the same time
Power rule

\[ f(x) = x^2 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \]

\[ = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \]

\[ = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} \]

\[ = \lim_{h \to 0} \frac{2hx + h^2}{h} \]
Power rule

\[ f(x) = x^2 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \]

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Power rule

\[ f(x) = x^2 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} \]

\[ = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h} \]

\[ = \lim_{h \to 0} \frac{2hx + h^2}{h} = 2x \]
Power rule

\[ f(x) = x^3 \]
Power rule

\[ f(x) = x^3 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} \]
Power rule

\[ f(x) = x^3 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} \]

\[ = \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h} \]
Power rule

\[ f(x) = x^3 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} \]

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Power rule

\[ f(x) = x^3 \]

\[ f'(x) = \lim_{h \to 0} \frac{(x + h)^3 - x^3}{h} \]

\[ = \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h} \]

\[ = 3x^2 \]
Power rule

\[ f(x) = x^n \]

\[ f'(x) = nx^{n-1} \]

So far, we have justified this rule ONLY when \( n \) is positive integer!
Rules for differentiation

- Addition rule
- Product rule
- Chain rule (for composition of functions)
- Quotient rule
Suppose \( f(x) = g(x) + k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

(A) 4

(B) 7

(C) 10

(D) 11
Suppose \( f(x) = g(x) + k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

- What is \( f'(2) \)?
  
  (A) 4
  
  (B) 7
  
  (C) 10
  
  (D) 11
Suppose $f(x) = g(x)k(x)$ and that $g(2) = 3$, $k(2) = 1$, $g'(2) = 2$, $k'(2) = 5$.

• What is $f'(2)$?

(A) 3

(B) 10

(C) 11

(D) 17
Suppose \( f(x) = g(x)k(x) \) and that  
\[ g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5. \]

- What is \( f'(2) \)?

(A) 3

(B) 10

(C) 11

(D) 17
Suppose $f(x) = g(x)k(x)$ and that $g(2) = 3, \ k(2) = 1, \ g'(2) = 2, \ k'(2) = 5$.

- What is $f'(2)$?

(A) 3
(B) 10
(C) 11
(D) 17

Try $g(x)=x$ and $k(x)=x^2$. (if many choose B)
Suppose \( f(x) = g(x)k(x) \) and that
\[
g(2) = 3, \quad k(2) = 1, \quad g'(2) = 2, \quad k'(2) = 5.
\]

• What is \( f'(2) \)?

(A) 3 
Try \( g(x) = x \) and \( k(x) = x^2 \).  
(if many choose B)

(B) 10 
If \( f'(x) = g'(x)k'(x) \) then
\[
f(x) = (x)(x^2) \text{ so } f'(x) = (1)(2x) = 2x.
\]

(C) 11 
But \( f(x) = x^3 \) and power rule says
\[
f'(x) = 3x^2.
\]

(D) 17 
So \( g'(x)k'(x) \) can’t be right.
What is the correct derivative for $f(x)=g(x)k(x)$?

$$A(t) = d(t)w(t)$$
What is the correct derivative for \( f(x) = g(x)k(x) \)?

\[
A(t) = d(t)w(t)
\]
What is the correct derivative for $f(x) = g(x)k(x)$?

$$A(t) = d(t)w(t)$$
What is the correct derivative for \( f(x) = g(x)k(x) \)?

\[
A(t) = d(t)w(t)
\]

\[
(d(t + h) - d(t)) \cdot w(t)
\]
What is the correct derivative for \( f(x) = g(x)k(x) \)?

\[
A(t) = d(t)w(t)
\]

\[
(d(t + h) - d(t)) \cdot w(t)
\]

\[
d(t) \cdot (w(t + h) - w(t))
\]
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$$A(t) = d(t)w(t)$$

$$(d(t + h) - d(t)) \cdot w(t)$$

$$d(t) \cdot (w(t + h) - w(t))$$
What is the correct derivative for $f(x) = g(x)k(x)$?

\[
A(t) = d(t)w(t)
\]

\[
A(t + h) = A(t) + (d(t + h) - d(t)) \cdot w(t) + d(t) \cdot (w(t + h) - w(t)) + \text{small corner}
\]

\[
\frac{A(t + h) - A(t)}{h} \approx \frac{(d(t + h) - d(t)) \cdot w(t)}{h} + \frac{d(t) \cdot (w(t + h) - w(t))}{h}
\]

\[
A'(t) = d'(t)w(t) + d(t)w'(t)
\]