Eulars Method

1) **By Hand:** Let \( y' = \frac{1}{4}y(y - 4) \), we will approximate a solution by successive tangent lines:

2) **Formalize** Eulars Method for \( y' = f(y) \), \( y_{n+1} = \)

For \( y' = f(y,t) \), \( y_{n+1} = \)

Write the Eulars Method update formula for the following differential equations:

\[
\frac{dy}{dt} = y(1 - y), \quad y_{n+1} = \\
\frac{dy}{dt} = \frac{t}{y}, \quad y_{n+1} =
\]

3) **Implement** Implementing Eulars method in Google Sheets: If \( y(x) \) is a solution to \( y' = y(y - 10)(y + 1) \) with \( y(0) = 3 \), what is \( y(5) \)?

First, write the Eulars Method update formula:

4) **Problem** If \( y(x) \) is a solution to \( y' = y - x \) with \( y(-2) = -1 \), what is \( y(2) \)? What is \( y(2) \) when \( y(-2) = -1.1 \)?

First, write the Eulars Method update formula:
5) **A model for the spread of disease** We want to model the spread of disease under the following assumptions:

1. Some number of members of the population are healthy (susceptible) $S(t)$, and some number is infected $I(t)$
2. The total population size $N(t) = S(t) + I(t) = N$ is fixed, no births, deaths or bifurcations.
3. The population mixes very well and contact is random.

Suppose the exchange of disease can be modeled by infection $S + I \rightarrow I + I$ and recovery $I \rightarrow S$, with infection rate $\beta$ and recovery rate $\mu$. Write a differential equation for the rate of change of $I(t)$, and another one for the rate of change of $S(t)$.

Use the fact that the population is constant to write $I'$ in terms of $I$ only.

5) **Model Domain** Draw a phase portrait for the equation in $I$ above both when $N - \mu/\beta > 0$ and $N - \mu/\beta < 0$. What is the physical interpretation of these different cases?

6) **Model Domain** Assume that $\beta$ and $\mu$ are fixed. How does the long term disease behavior depend on the size of the population $N$?