Today

- Survey - check.
- Annual sunlight (continued)
- Derivatives of sin(x), cos(x)
- Related rates with trig (if time)
- Reminders:
  - Teaching evals

Annual variation in daylight per day in Vancouver (Jan 1 --> t=0)

(A) $$L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t-172)\right)$$

(B) $$L(t) = 12 + 4\sin\left(\frac{2\pi}{365}(t+80)\right)$$

(C) $$L(t) = 12 + 4\cos\left(\frac{2\pi}{365}(t-172)\right)$$

(D) $$L(t) = 12 - 4\sin\left(\frac{2\pi}{365}(t-80)\right)$$

Note: t=172 is June 21; t=80 is March 21.

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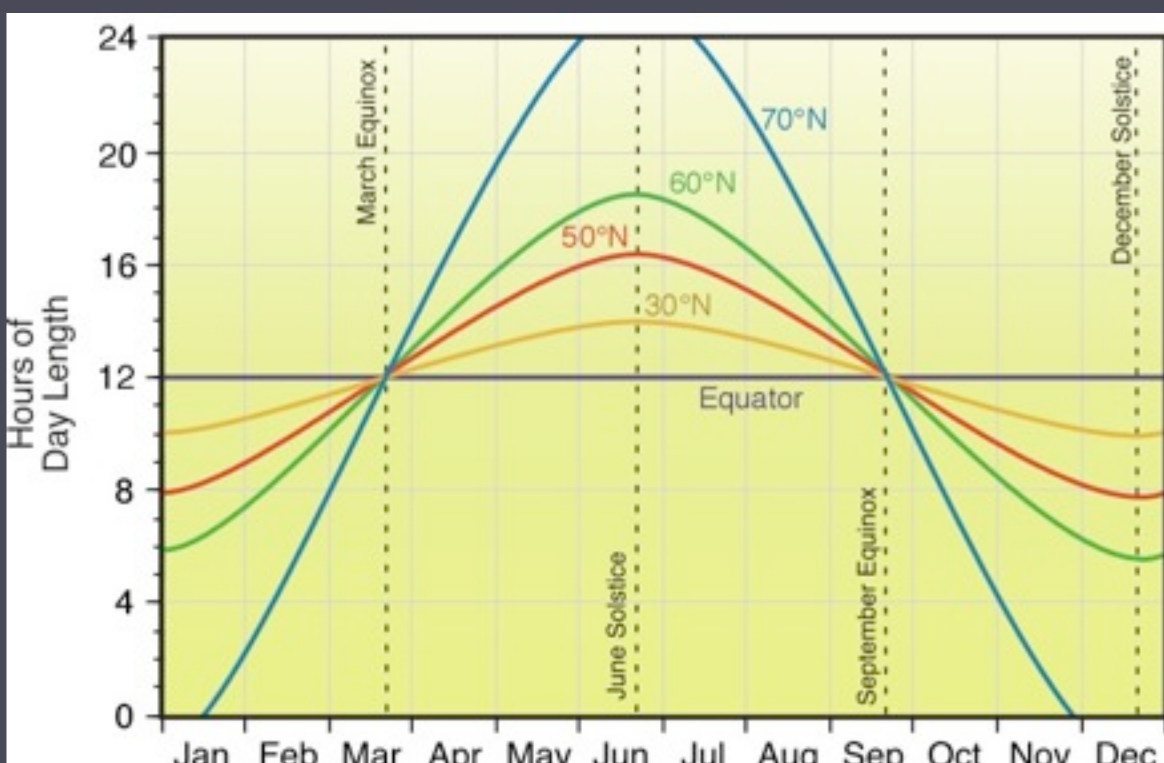

$+ 4\sin\left(\frac{2\pi}{365}(t+80)\right)$

$+ 4\cos\left(\frac{2\pi}{365}(t-172)\right)$

$- 4\sin\left(\frac{2\pi}{365}(t-80)\right)$

Note: t=172 is June 21; t=80 is March 21.

Derivative of $f(x)=\sin(x)$

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See what h=0.0001 gives...

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$= \sin(x) \times 0 + \cos(x) \times 1 = \cos(x).$

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Note: this last step requires a bunch of work to show.

More details on that last step (not shown in class)

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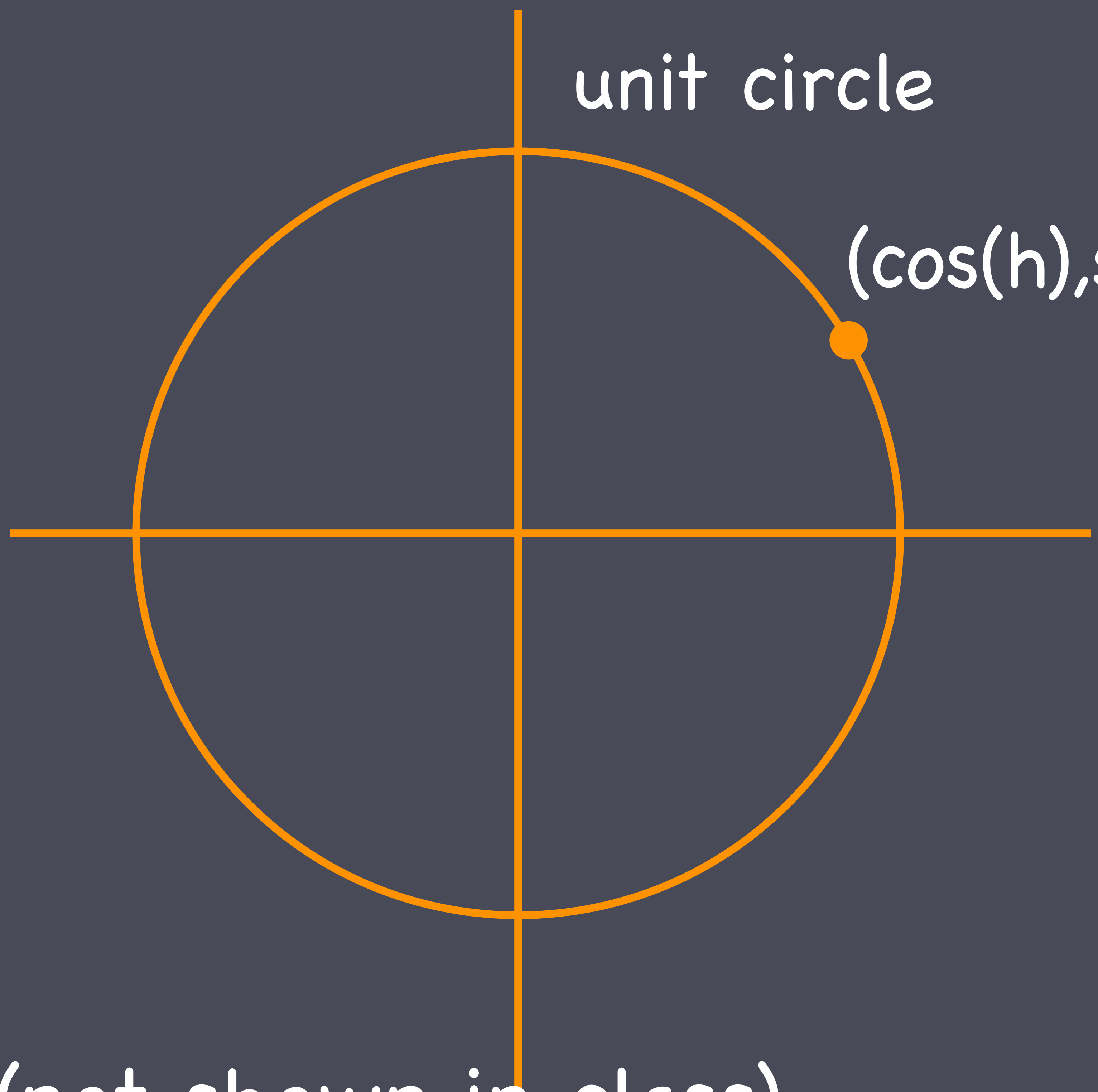
$= \lim_{h\to 0} -\sin(h)/(\cos(h)+1) \times \lim_{h\to 0} \sin(h)/h$

$= \quad 0 \quad \times \quad 1$

Why is $\lim_{h \to 0} \sin(h)/h = 1$?



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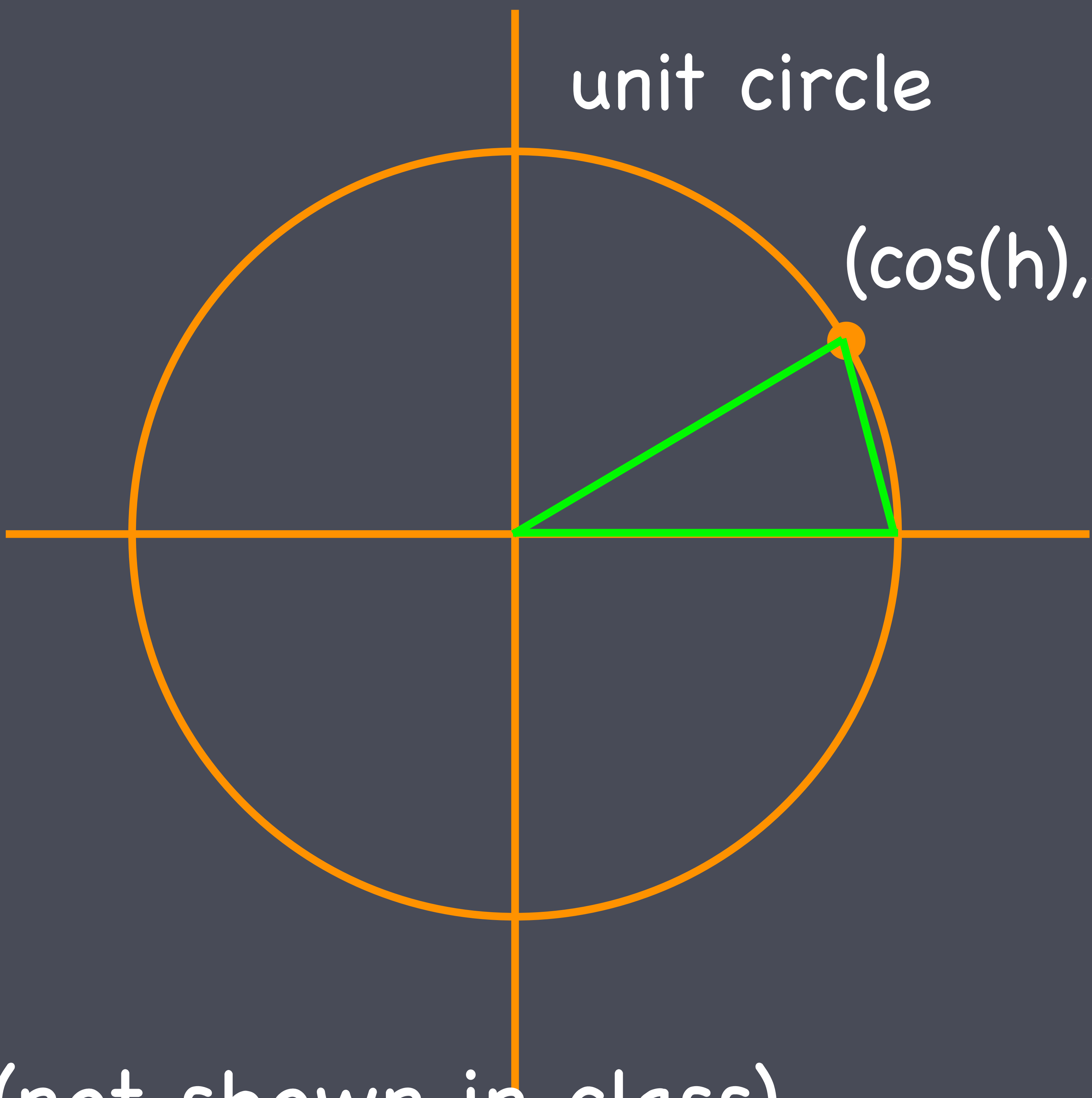

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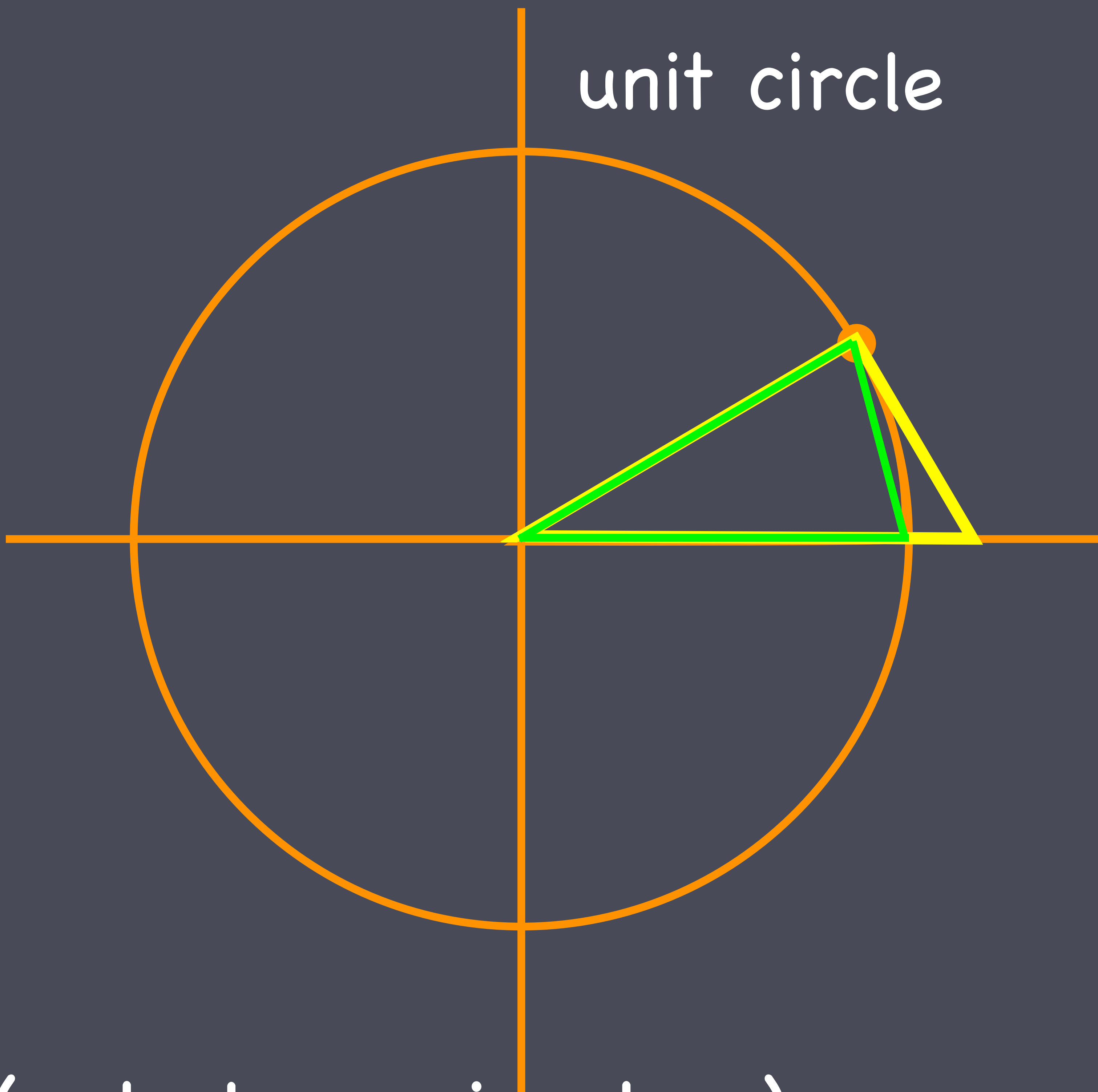

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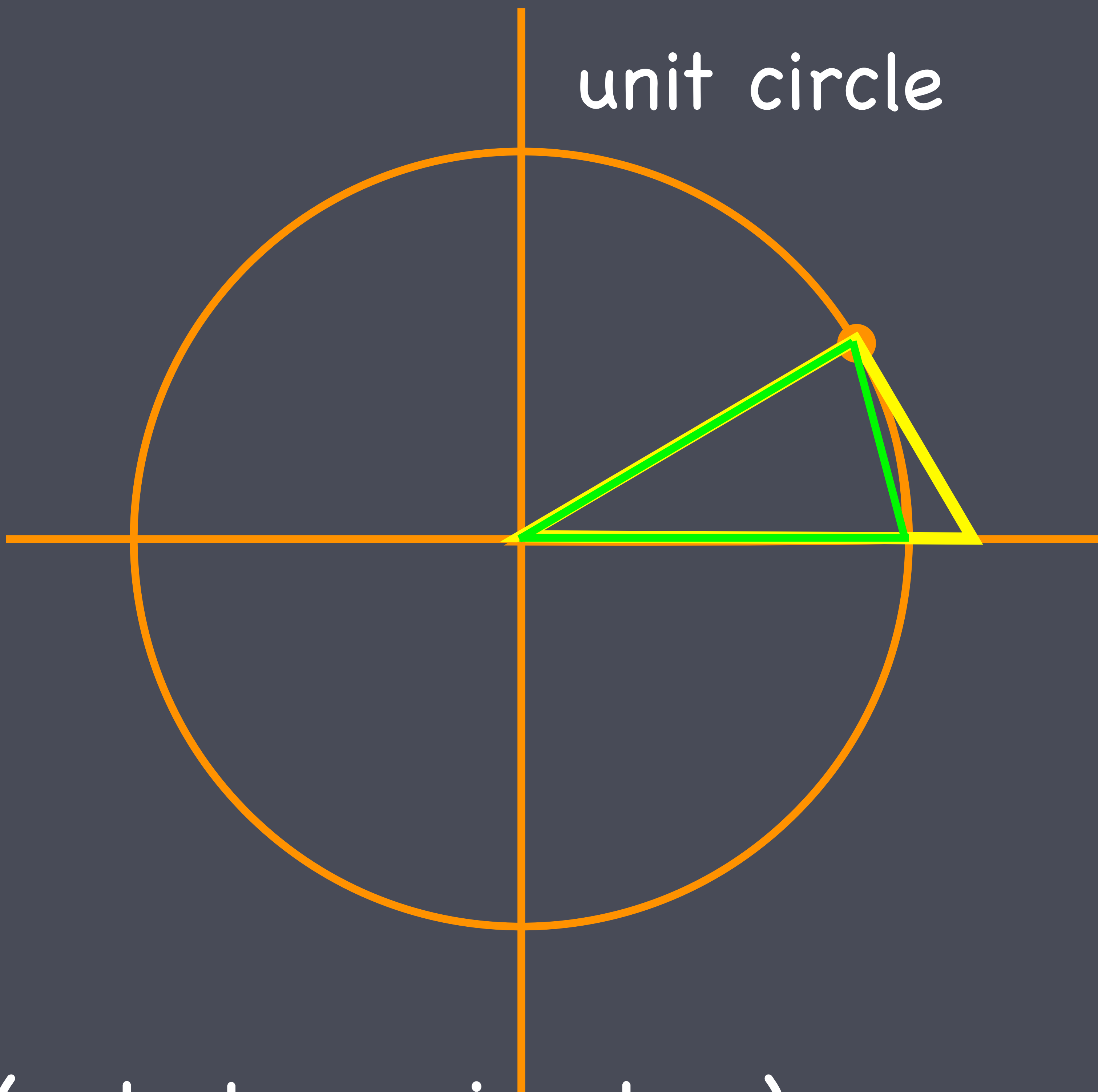

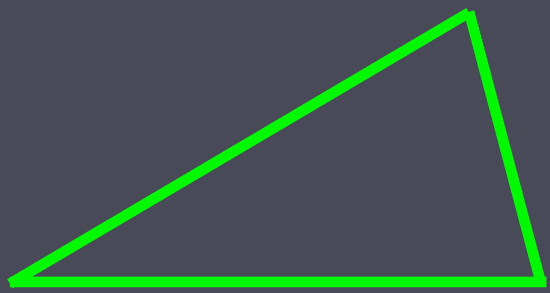

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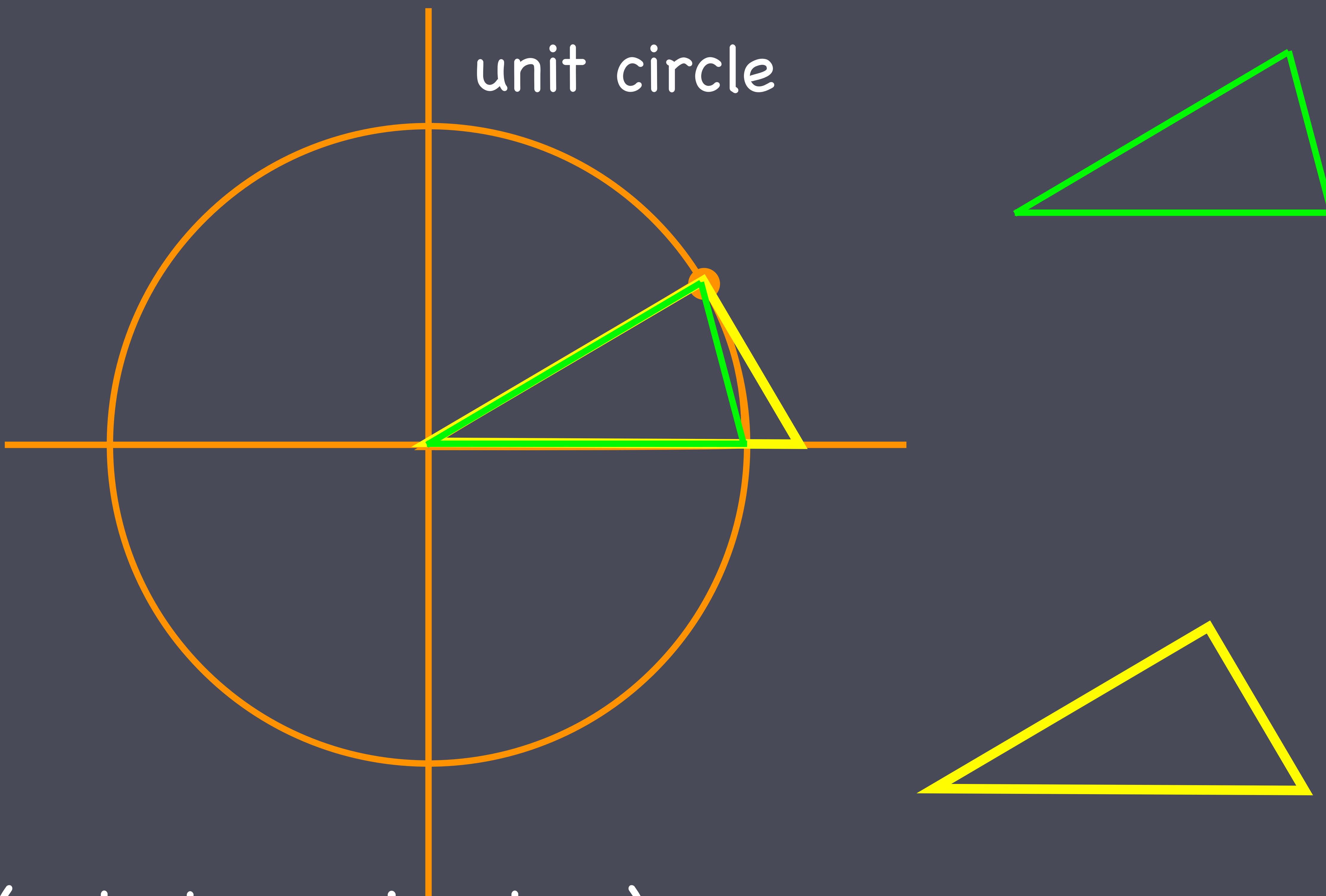

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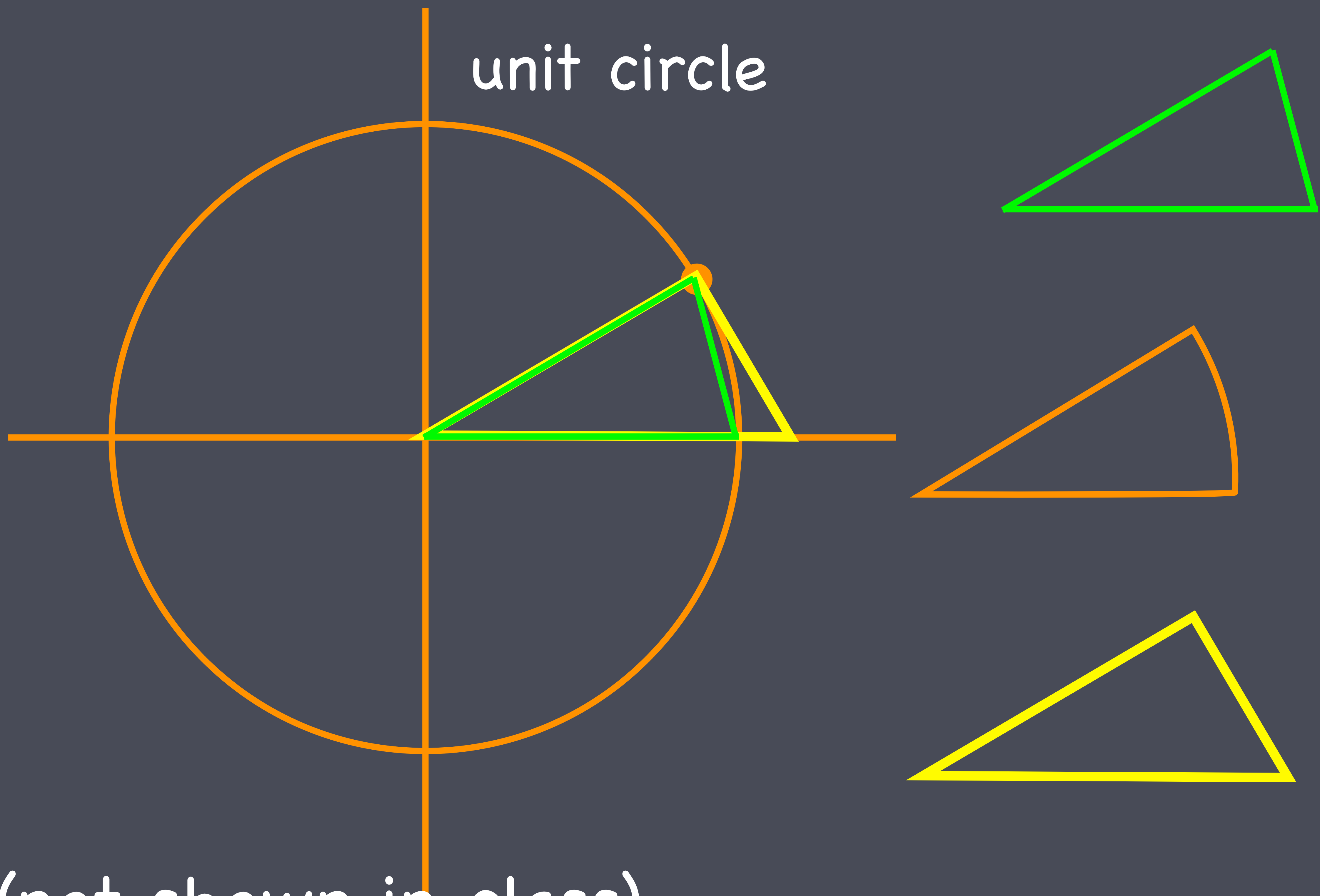

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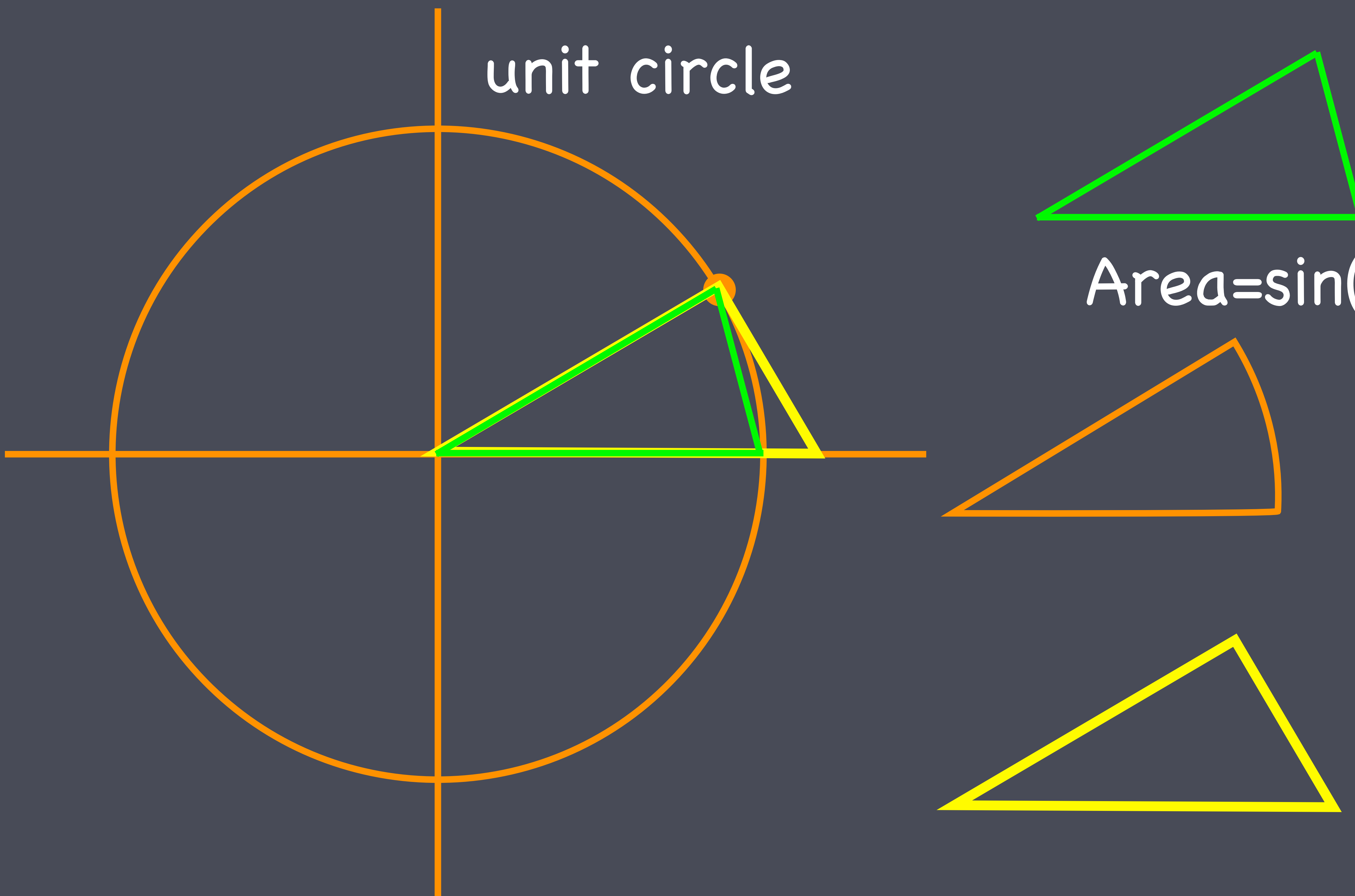

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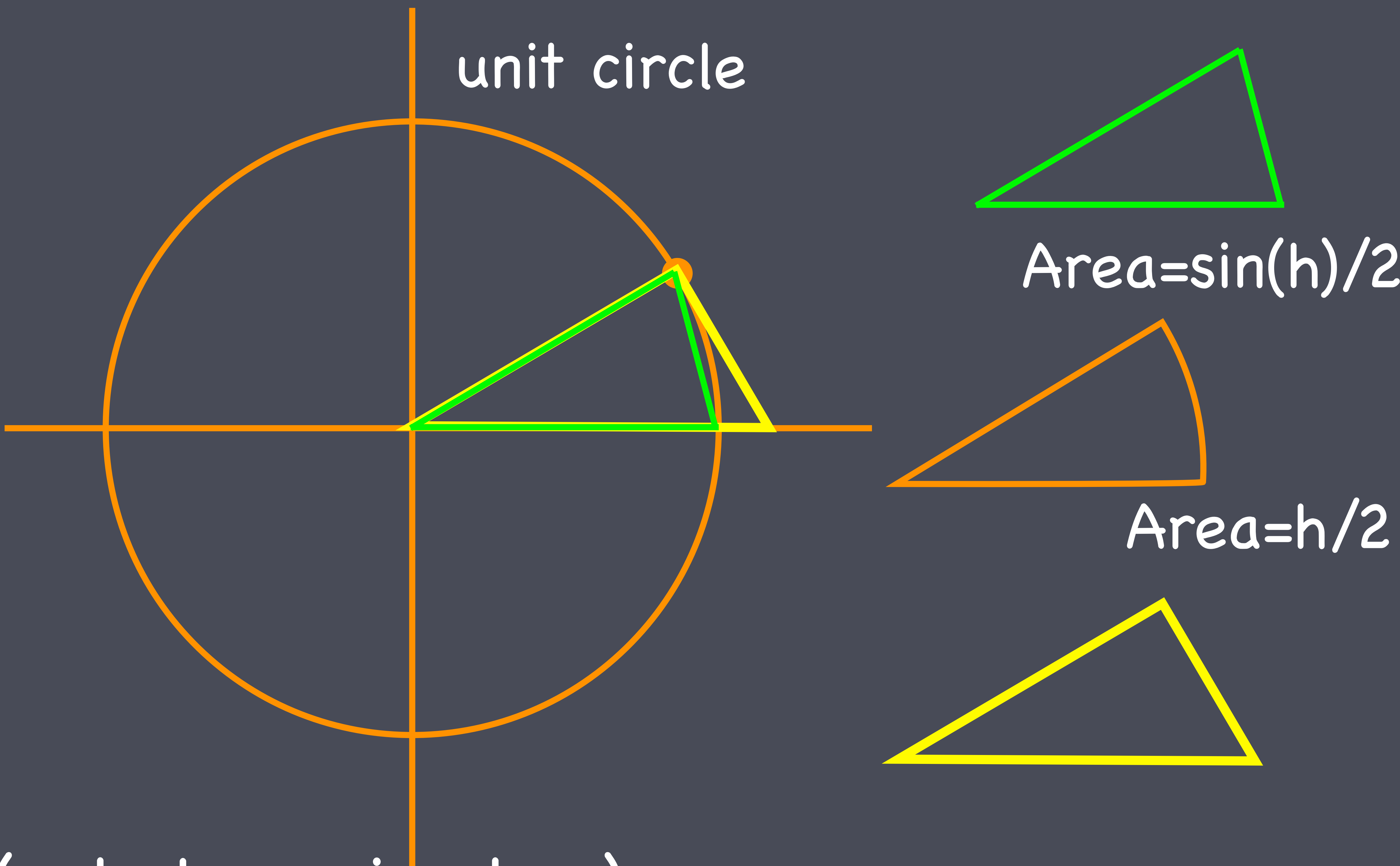

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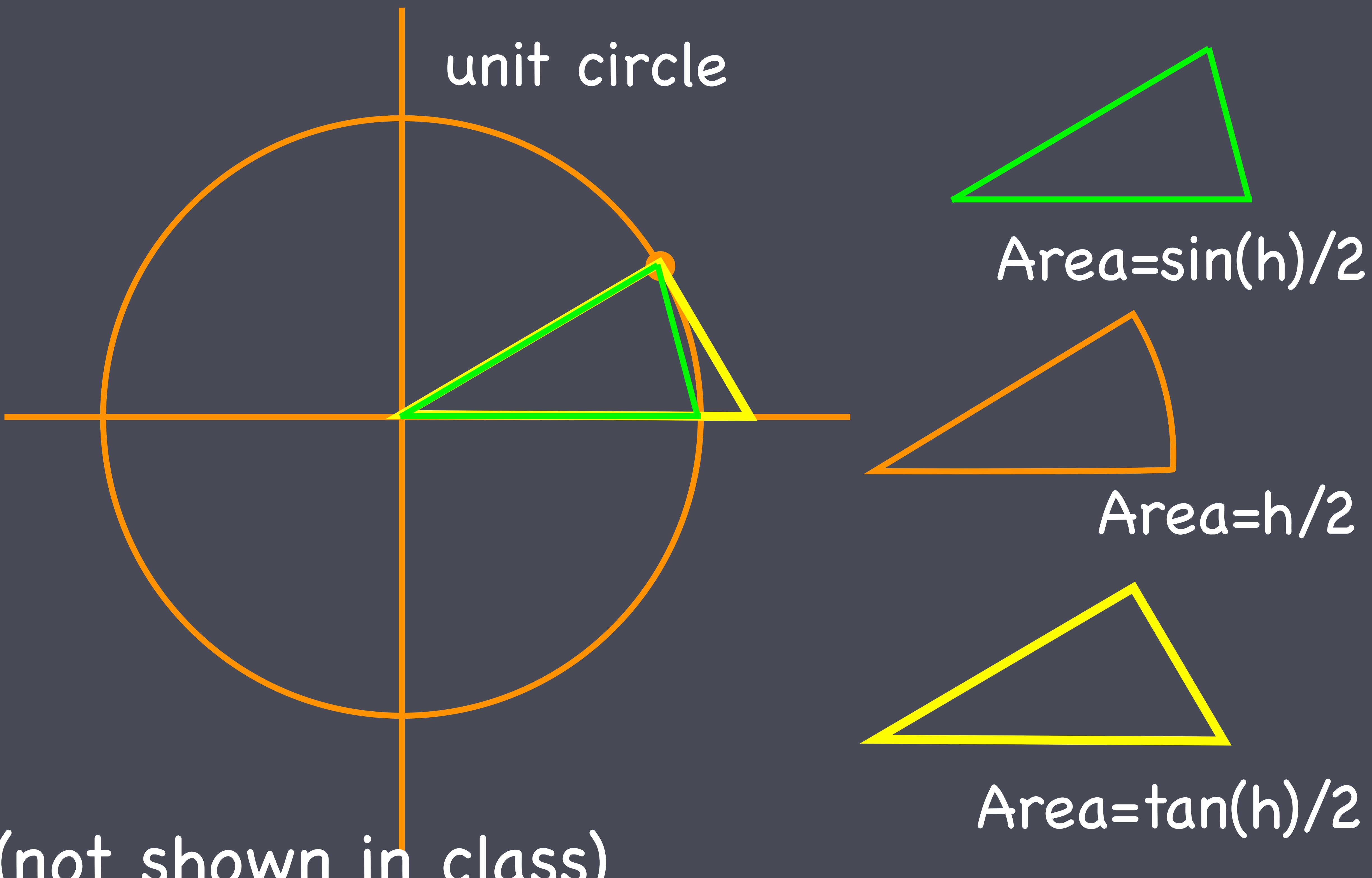

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Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$\sin(h)/2 < h/2 < \tan(h)/2$

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$$\sin(h)/\sin(h) < h/\sin(h) < \tan(h)/\sin(h)$$

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Take $\lim_{h \to 0}$:

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1

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Take $\lim_{h \to 0}$: $\quad \cos(h) \downarrow 1 \qquad 1 \downarrow 1$

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$$\cos(h) < \sin(h)/h < 1$$

Take $\lim_{h \to 0}$: $\cos(h) \to 1$ and $1 \to 1$

sin(h)/h is stuck between!

Why is $\lim_{h \to 0} \sin(h)/h = 1$?

$$\sin(h)/2 < h/2 < \tan(h)/2$$

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Take $\lim_{h \to 0}$: ↓ ↓ ↓

1 1 1

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Derivative of $g(x)=\cos(x)$.
Rewrite $\cos(x)$ as...

(A) $g(x) = \cos(x) = \sin(x-\pi/2)$

(B) $g(x) = \cos(x) = \sin(x+\pi/2)$

(C) $g(x) = \cos(x) = \sin(x+\pi)$

(D) $g(x) = \cos(x) = \sin(x-\pi)$

(E) $g(x) = \cos(x) = \sin(x+3\pi/2)$

Derivative of g(x)=cos(x). Rewrite cos(x) as...

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Derivative of $g(x)=\sin(x+\pi/2)$

(A) $g'(x) = \cos(x+\pi/2) = \sin(x)$

(B) $g'(x) = \cos(x+\pi/2) = -\sin(x)$

(C) $g'(x) = \cos(x+\pi/2) = \sin(x-\pi/2)$

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