Lecture 6 (Sept. 16, 2013)

Learning Goal: (1) the meaning of the derivative

(2) Power Rule, linear properties of the derivative

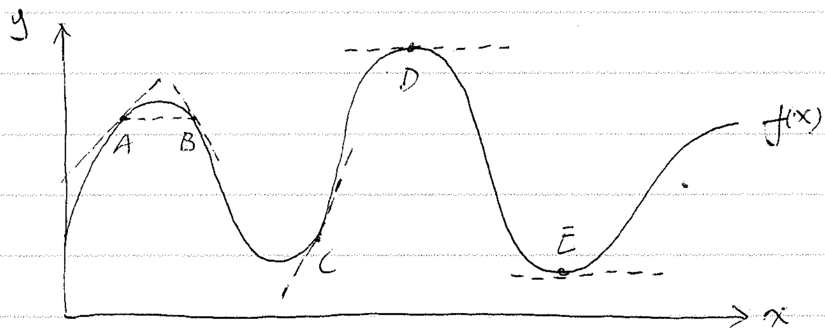
(3) tangent lines

- Derivative of $f(x)$ at $x=a$: $f'(x) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ by $h = x-a$

$\frac{f(a+h)-f(a)}{h}$ ~ average rate of the change of the function ~ slope of the secant line

$\downarrow$ take limit $\quad\quad \downarrow$ take limit $\quad\quad \downarrow$ take limit

$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ ~ instantaneous rate of the change of the function ~ slope of the tangent line (derivative)



* the average rate of change from A to B is 0, but the instantaneous rate at A, B isn't 0

* Positive/negative slope indicates the function has increasing/decreasing trend at that point

* At D, E, tangent line is horizontal, $\Rightarrow$ slope of the tangent line is 0

usually happens at local maximum/minimum

* Compare A and C, the steeper the tangent line is, the faster/larger the instantaneous rate is.

Find the derivative at every value of x in the domain. $\Rightarrow f(x)$ is differentiable, $\Rightarrow$ there exists $f'(x)$

- Power Rule:

Example 1: Find the derivative of $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{x^3}}{h} = \lim_{h \to 0} 3x^2 + 3hx + h^2$$

$$= 3x^2$$

In general, for $f(x) = x^n$, n - positive integer, we have

$$f'(x) = n \cdot x^{n-1}$$

• Linear Properties: assume $f(x), g(x)$ are differentiable, find the derivative of $H(x) = f(x) + g(x)$

Based on the definition, we have

$$H'(x) = \lim_{h \to 0} \frac{H(x+h) - H(x)}{h} = \lim_{h \to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} = f'(x) + g'(x)$$

Similarly, for $H(x) = k \cdot f(x)$, k - constant, we have $H'(x) = k \cdot f'(x)$

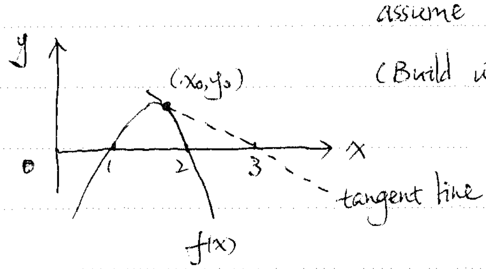
• Tangent line: $y = \underset{\substack{\downarrow \\ \text{slope} \to \text{derivative}}}{m}x + b \rightarrow$ y - intercept

In order to find the equation of the tangent line, we need at least two conditions to solve for two unknowns.

Example 2: Find the tangent line of $f(x) = -x^2 + 3x - 2$, which goes through the point $(3, 0)$

First of all, notice that $(3, 0)$ is not a point on $f(x)$

assume the intersection of the tangent line and $f(x)$ is (x_0, y_0)



(Build up two equations based on the problem)

(i) slope $m = \underset{\substack{\downarrow \\ \text{derivative viewpoint}}}{-2x_0 + 3} = \frac{0 - y_0}{3 - x_0} \rightarrow$ straight line viewpoint

(ii) (x_0, y_0) on the curve $f(x)$: $y_0 = -x_0^2 + 3x_0 - 2$

(Solve for x_0, y_0) $\quad (-2x_0 + 3)(3 - x_0) = x_0^2 - 3x_0 + 2$

$\Rightarrow x_0^2 - 6x_0 + 7 = 0$

$\Rightarrow x_0 = 3 \pm \sqrt{2}$ (quadratic formula)

$\Rightarrow m = -2(3 \pm \sqrt{2}) + 3$, indicates two tangent lines