Today...

- Demo WeBWorK tricks
- A Hill function clicker question.
- From secant line to tangent line.
- The Definition of the Derivative.
• uM and um

• How much to write for "Describe in words".

• I read through the example solutions carefully:

  (A) Yes

  (B) No
What if you want the rate of change AT $x_1$? (instantaneous instead of average)

Take a point $x_2$ so that the secant line is closer to the “secant line” AT $x_1$.

Alternate notation: let $x_2 = x_1 + h$ so that

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{x_1 + h - x_1}$$
If we take $h$ values closer and closer to 0...

- The average slope becomes the instantaneous slope.
- The secant line approaches the tangent line.
- The slope of the secant line approaches the slope of the tangent line.
- We call the slope of the tangent line the derivative at $x_1$.
- We now have to learn how to take limits!

\[
slope \text{ at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}
\]
Another example

\[
f'(x_1) = 0 \, ? \quad f'(x_1) = m > 0 \, ?
\]

(A) \[\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0\]

(B) \[\lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0\]

(C) Both (A) and (B)

(D) The limit does not exist.
Another example

\[ f'(x_1) = 0 ? \]

Limits from left and right must agree for the limit to exist.

(A) \[ \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = m > 0 \]

(B) \[ \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h} = 0 \]

(C) Both (A) and (B)

(D) The limit does not exist.
The derivative of $f(x)$ at $x=a$...

(A) ...touches the function at $x=a$ but does not cross it.

(B) ...looks more and more like the function as you zoom in around $x=a$.

(C) ... only exists when the function looks like a straight line close to $x=a$.

(D) All of the above.
The derivative of $f(x)$ at $x=a$...

(A) ...touches the function at $x=a$ but does not cross it.

(B) ...looks more and more like the function as you zoom in around $x=a$.

(C) ... only exists when the function looks like a straight line close to $x=a$.

(D) All of the above.
The derivative of $f(x)$ at $x=a$...

(A) ... touches the function at $x=a$ but does not cross it.

(B) ... looks more and more like the function as you zoom in around $x=a$.

(C) ... only exists when the function looks like a straight line close to $x=a$.

(D) All of the above.
To evaluate a limit

To evaluate \( \lim_{x \to a} f(x) \), you plug in values closer and closer to \( a \) but you never get to \( a \). In fact, \( f(a) \) may not even be defined. If you always get the same number no matter how you approach \( a \), then the limit exists.

Note: the limit involved in the derivative is only one special case. The limit above is concerned with the value of the function. When a limit has the form

\[
\lim_{h \to 0} \frac{f(2 + h) - f(2)}{h}
\]

we’re talking about the slope of \( f \) (in this case, at \( x=2 \)).
A WeBWorK limit example

Guess the value of the limit (if it exists) by evaluating the function at values close to where the limit is to be done. If it does not exist, enter DNE below.

\[ \lim_{{h \to 0}} \frac{\sin\left(\frac{\pi}{4} + h\right) - \sin\left(\frac{\pi}{4}\right)}{h} \]

Limit: 

Go over \( f'(2) \) where \( f(x) = \frac{1}{x} \) on the board.

Do not drop the “lim” along the way!