Today

- Brief midterm “discussion”
- Euler’s method
- Trig review

Midterm 2...

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- I expect I'll get (A) (B) (C) (D) (E=F).

Midterm 2

The hardest part was

(A) The multiple choice.

(B) The short answer problems.

(C) Long answer problem 1.

(D) Long answer problem 2.

Differential equation notation

- Some equations we've seen:
  - $y' = ky$
  - $y' = a-by$
  - $y' = -6 \text{ sqrt}(y)$
  - $y' = y^2$
  - $y' = -y^2$

General form: $y'=f(y)$ where $f(y)$ is just a way to represent all of these RHSs.

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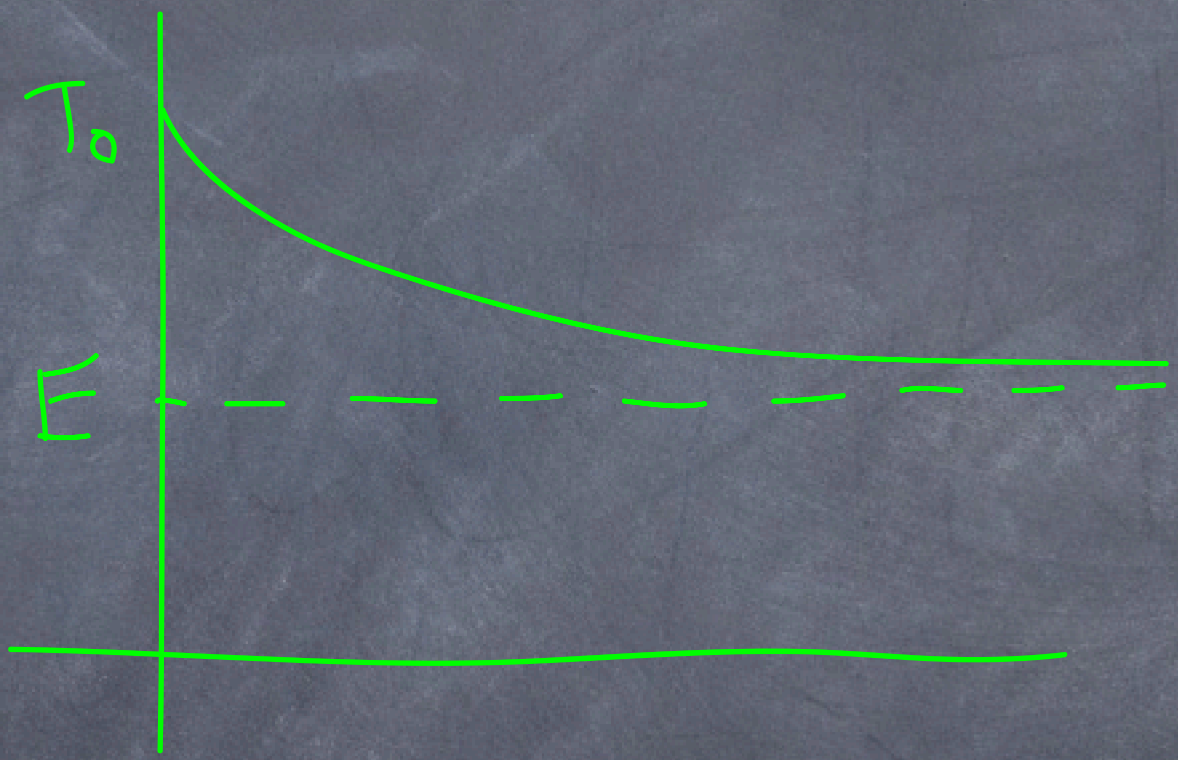
$$T(t) = E + (T_0 - E)e^{-kt},$$

we find T_1, T_2, T_3,... which approximate T(0.1), T(0.2), T(0.3) ...

Euler's method for $T'(t)=k(E-T(t))$ with $T(0)=T_0$.



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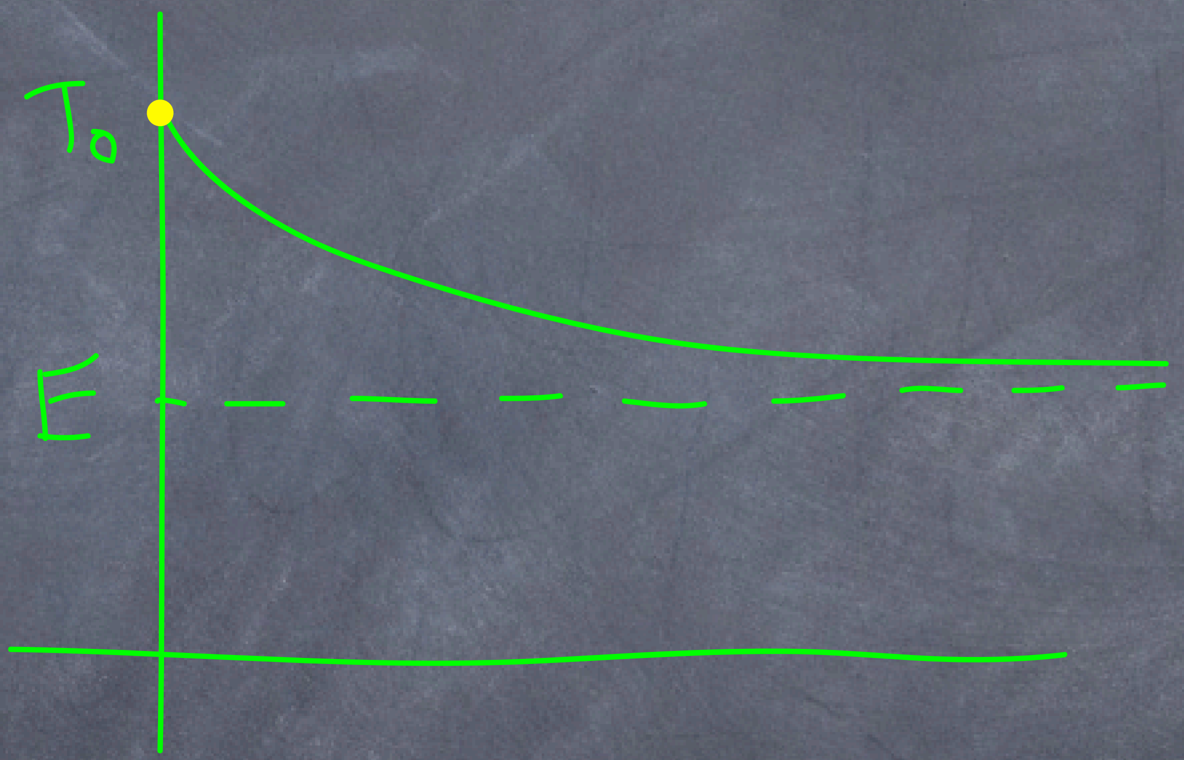

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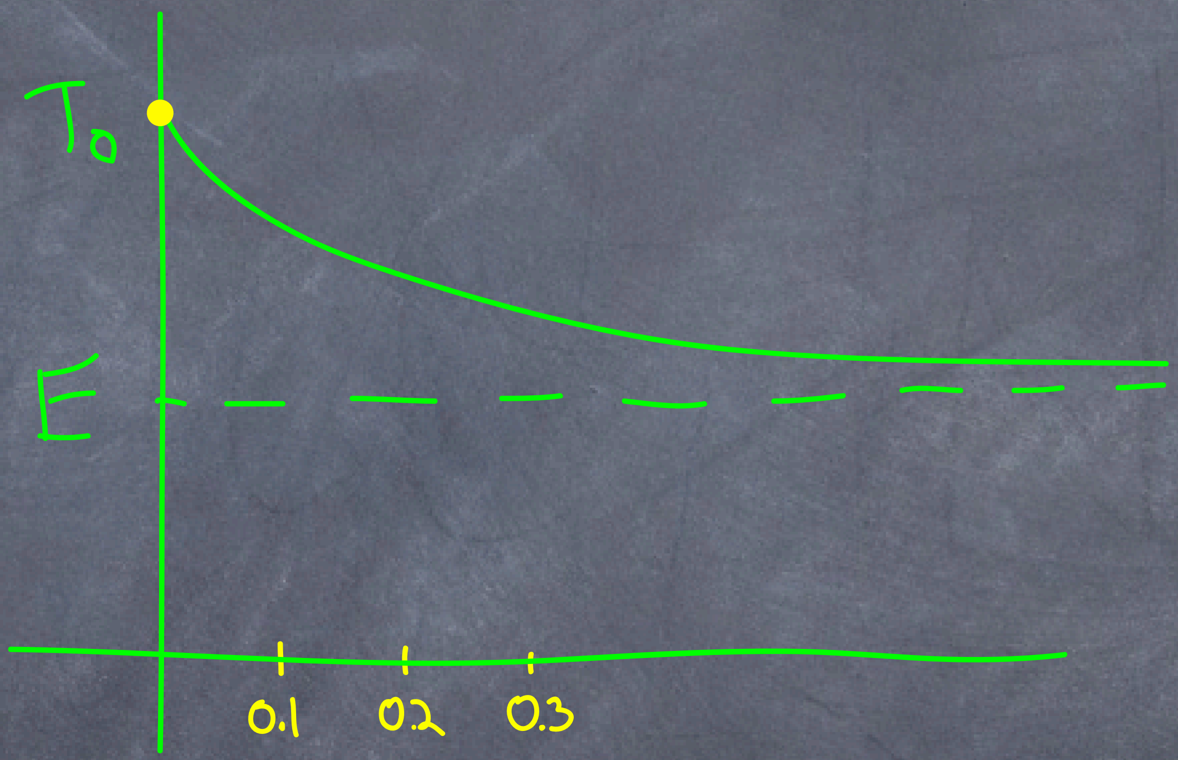

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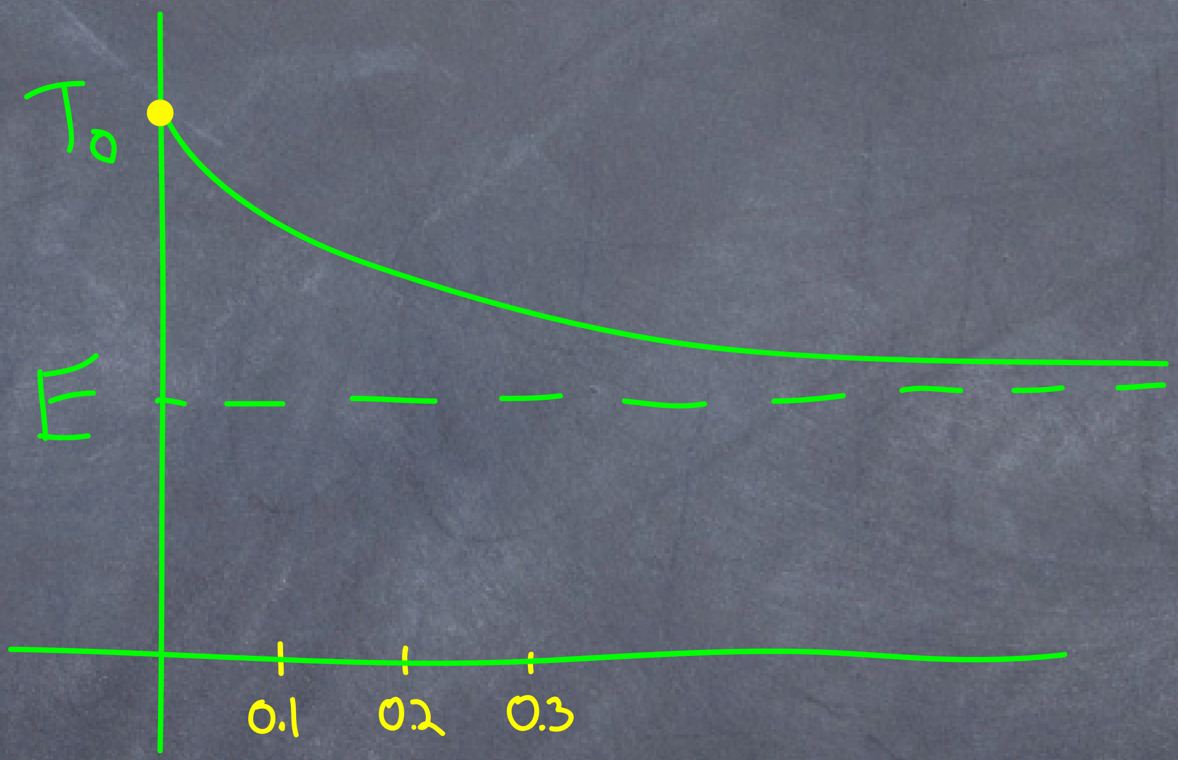

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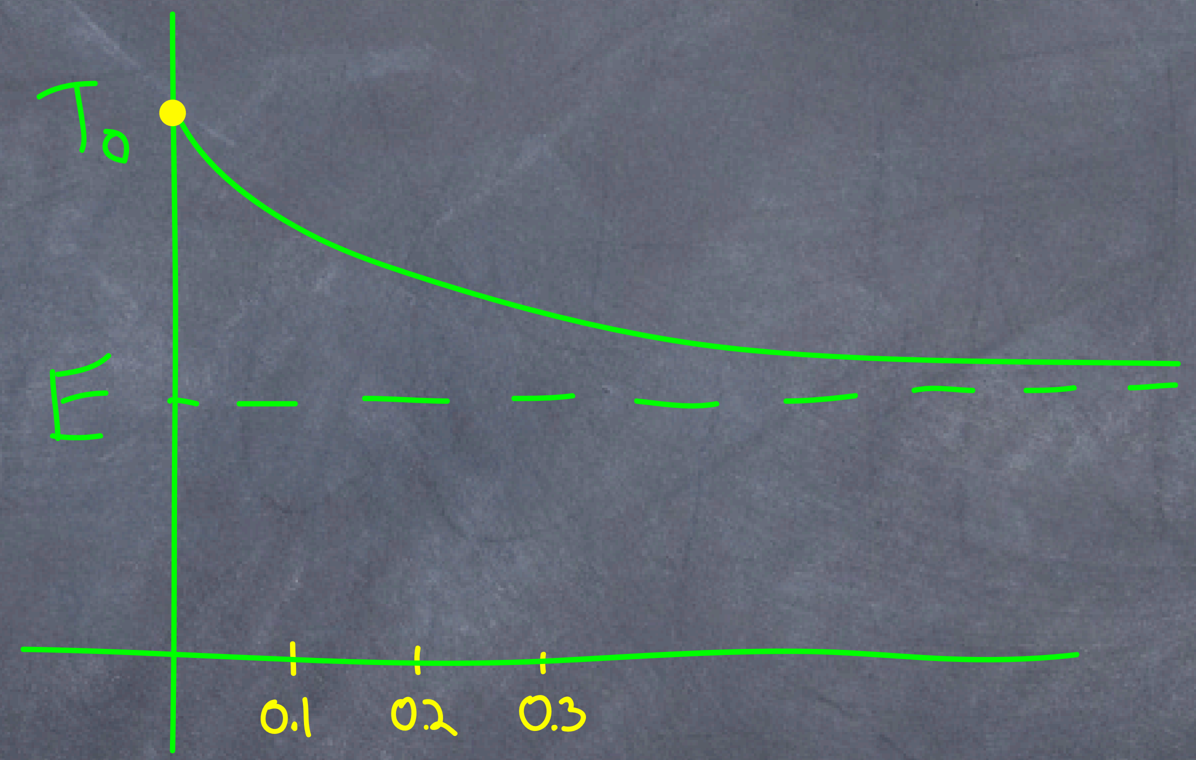

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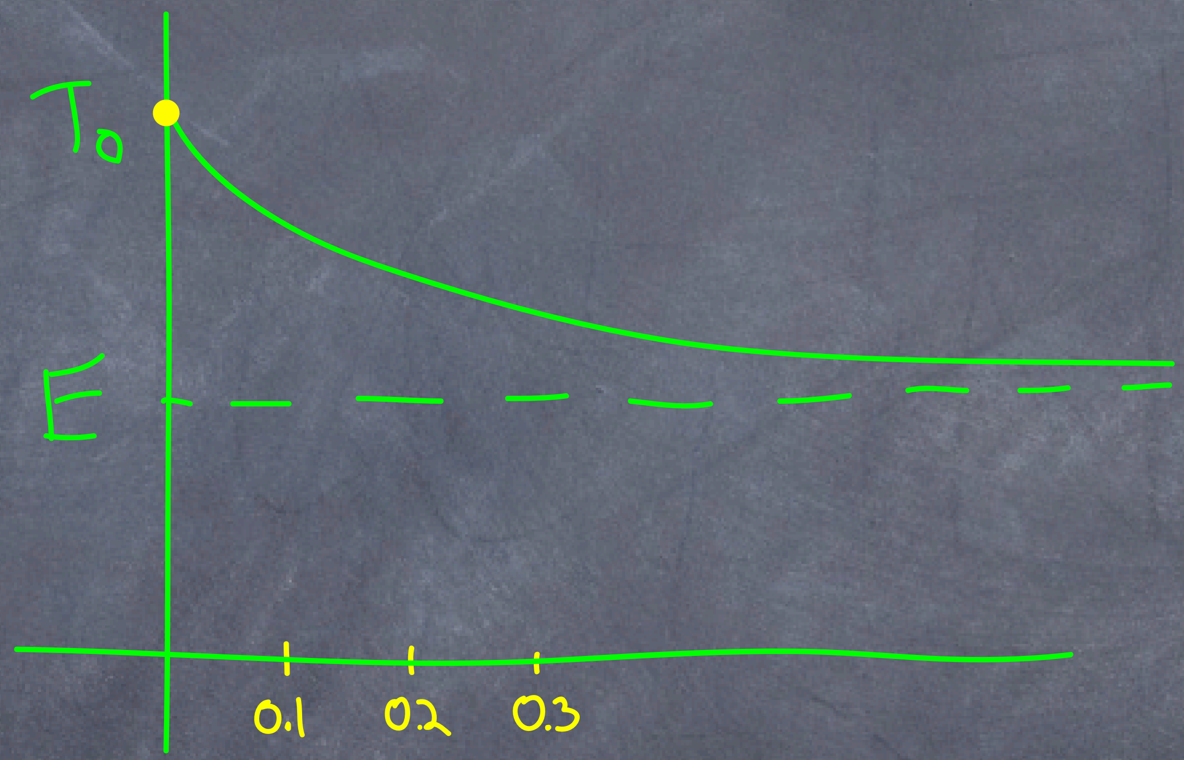

$$(T(0.1)-T(0)) / 0.1 \approx T'(0) = k(E-T(0)) = k(E-T_0)$$

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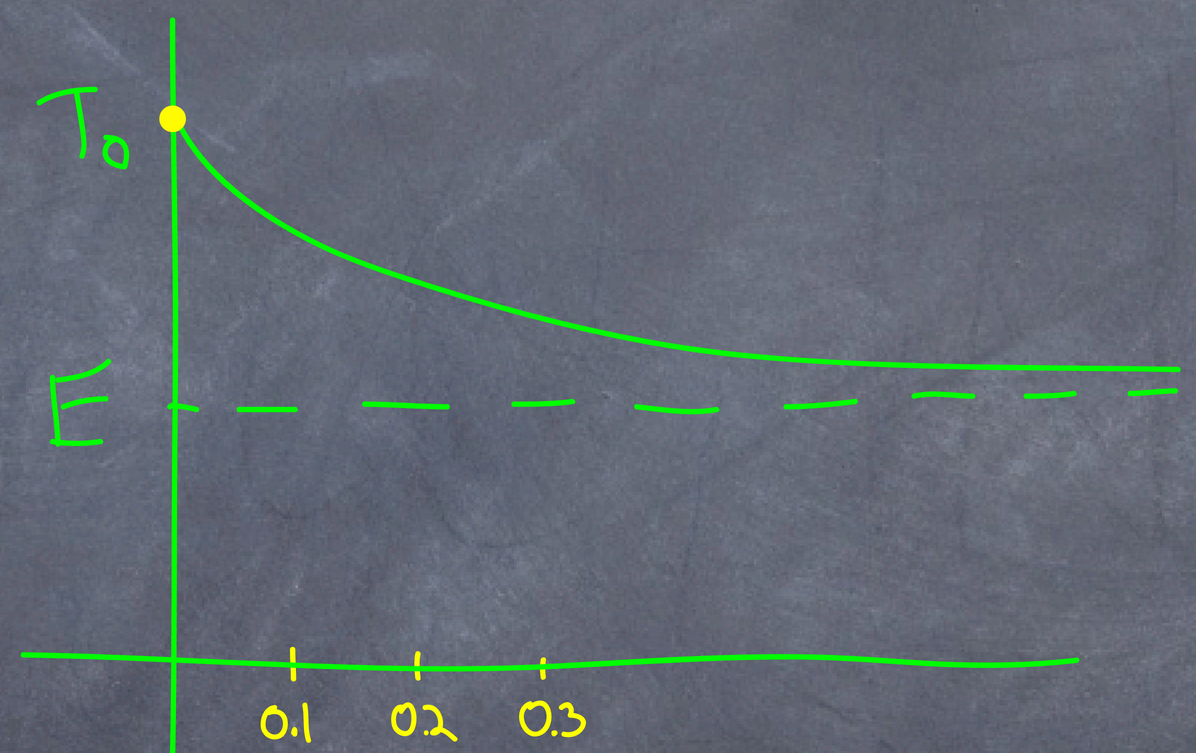

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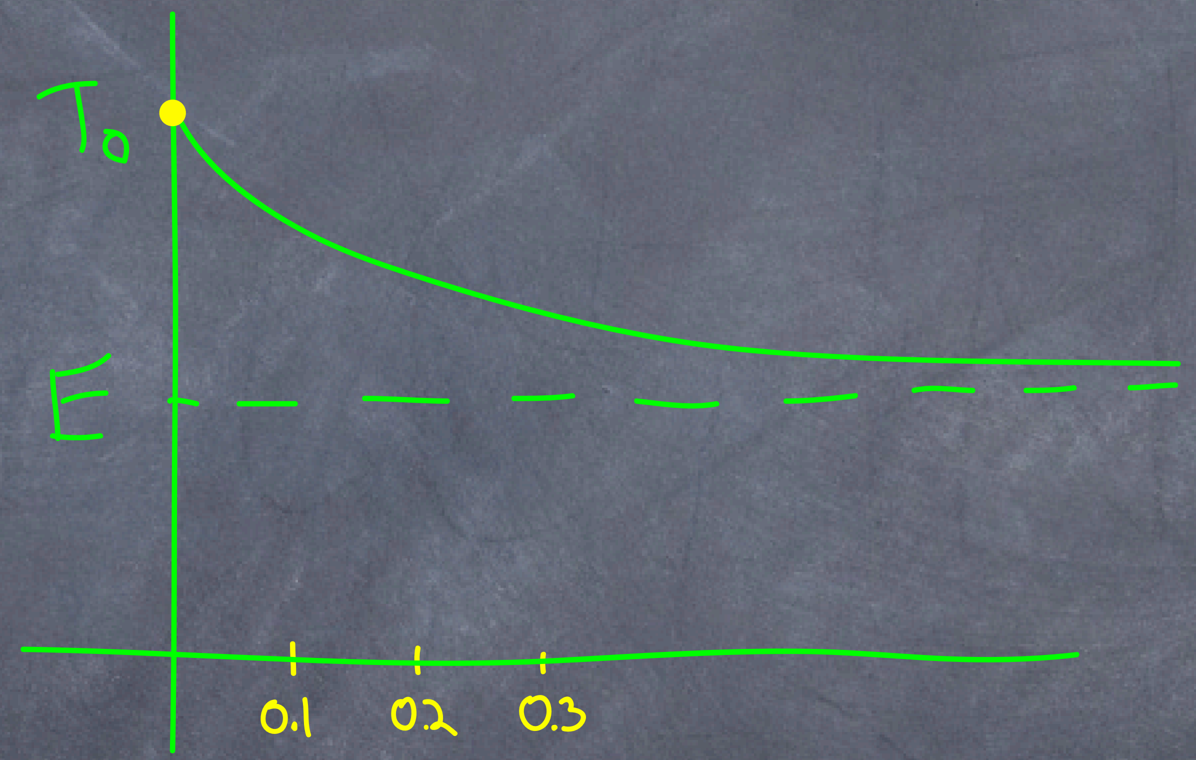

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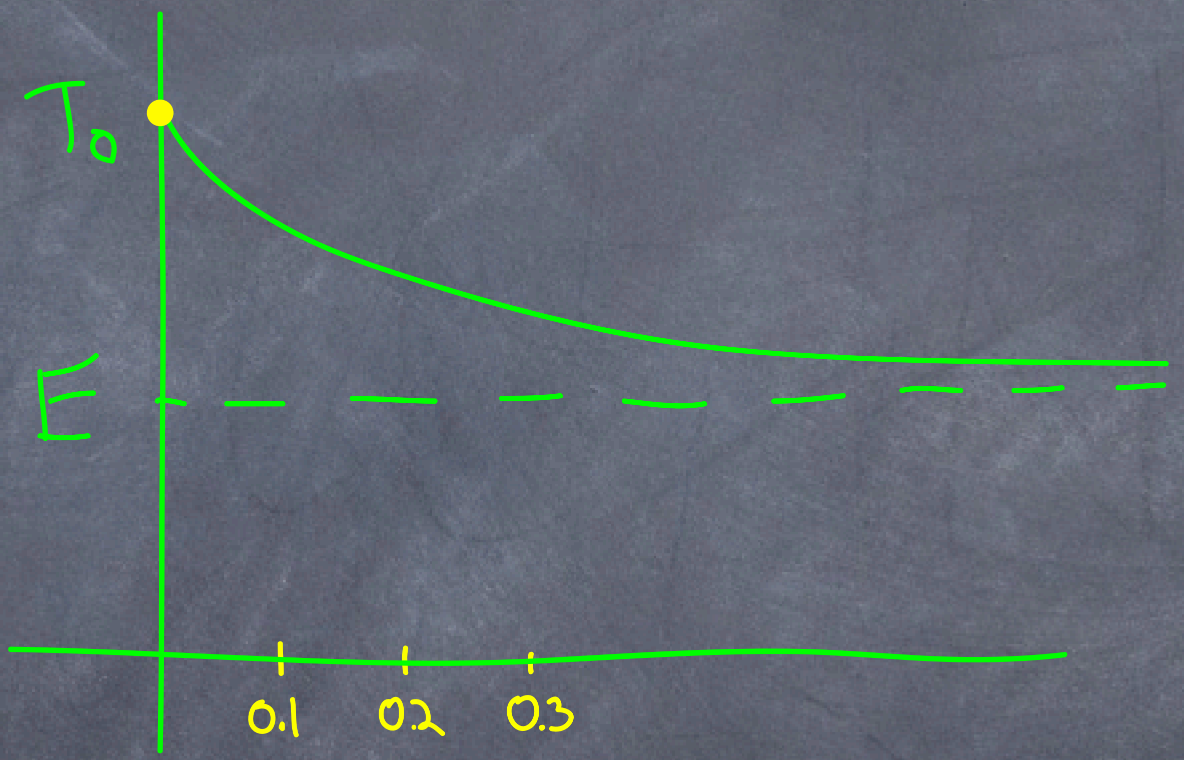

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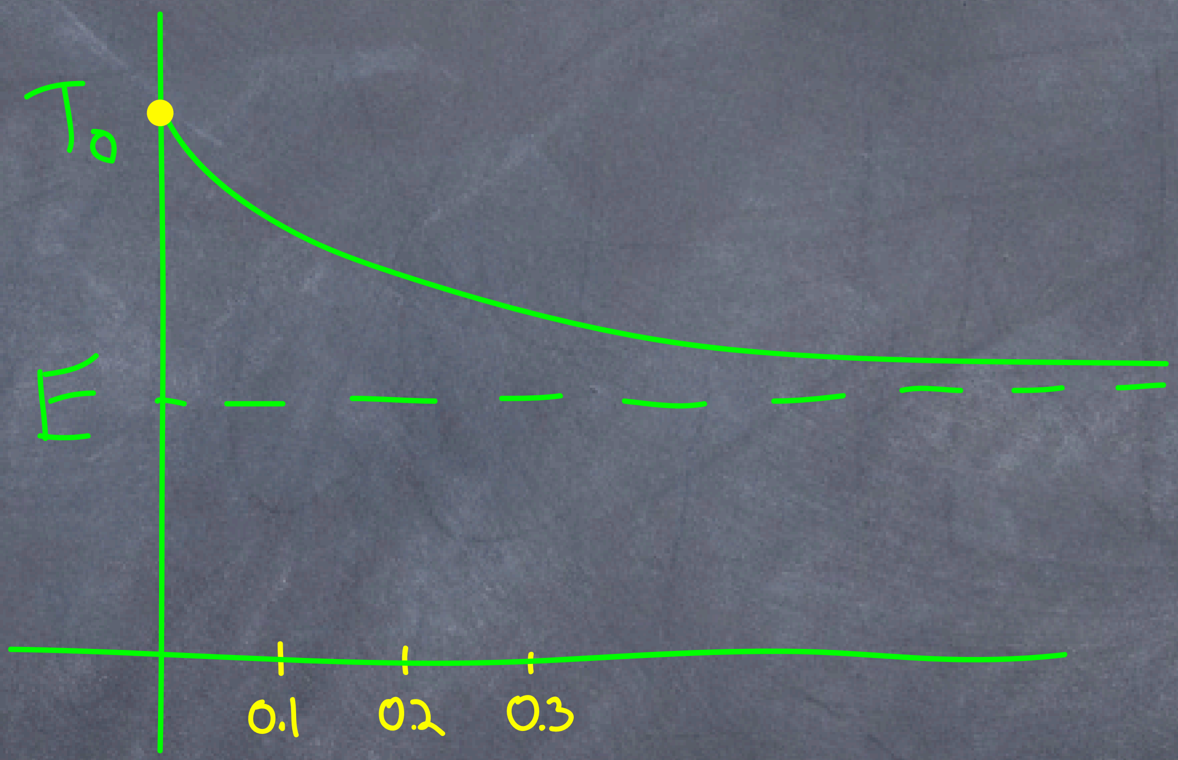

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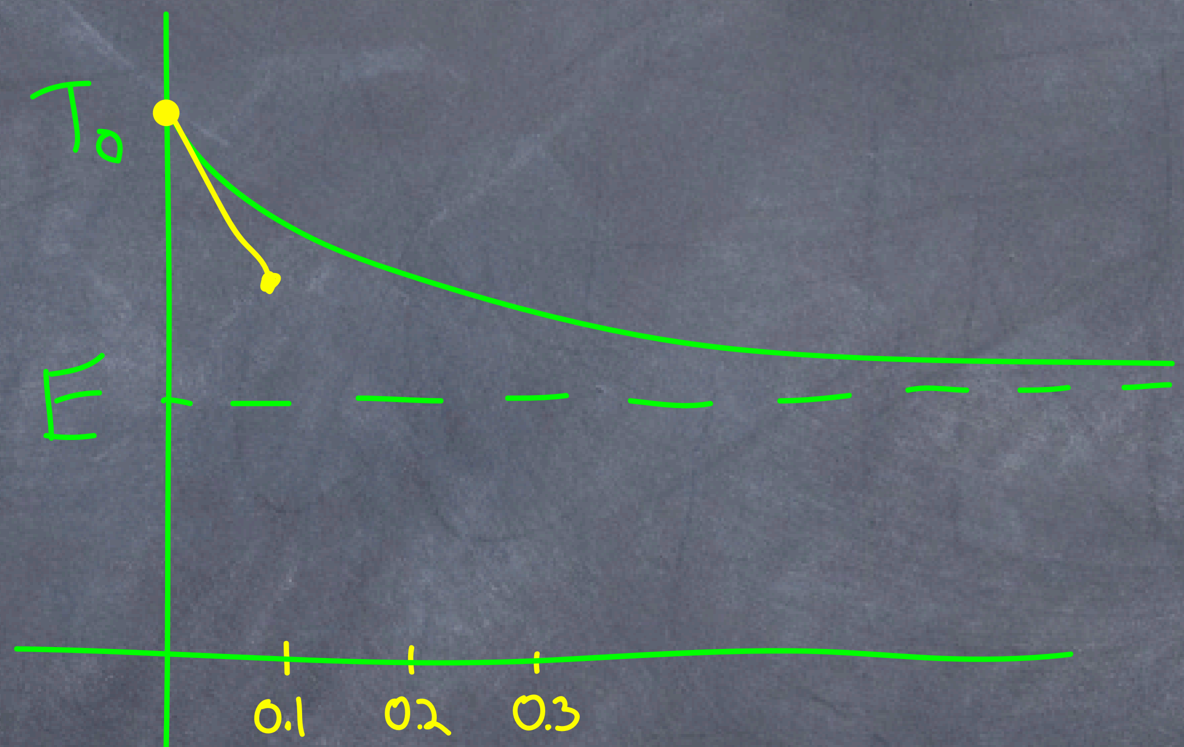

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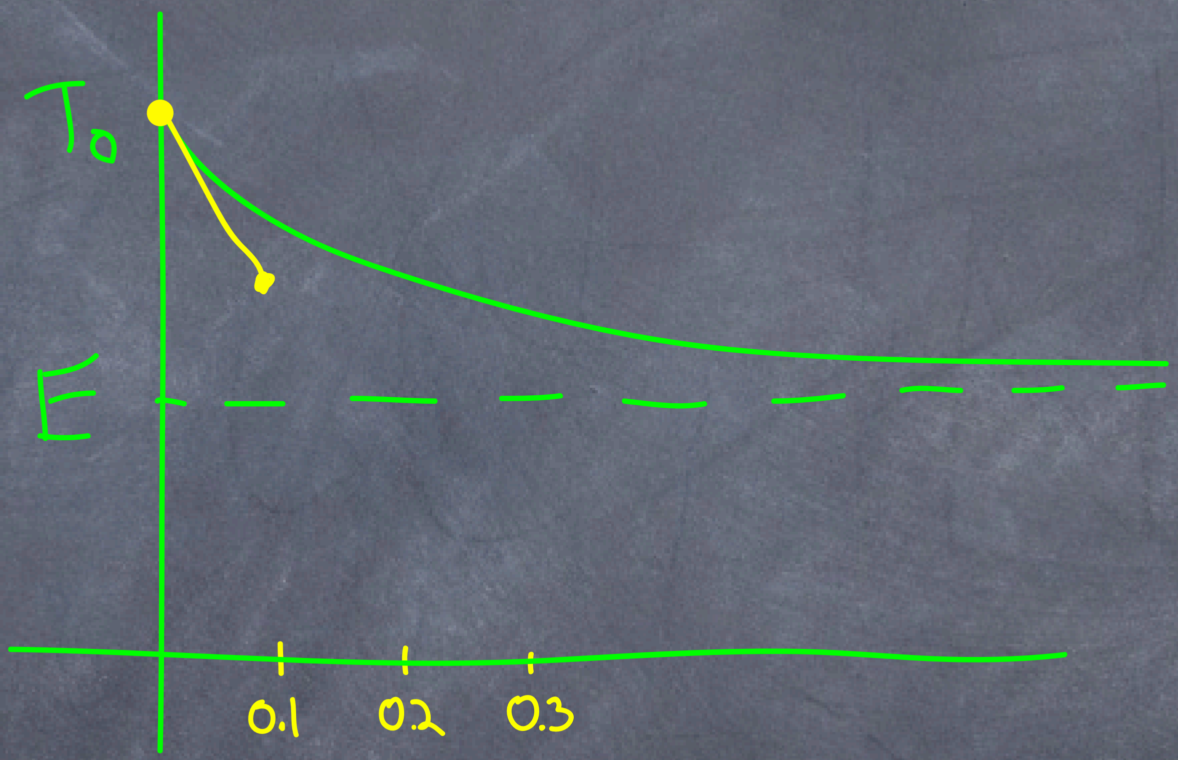

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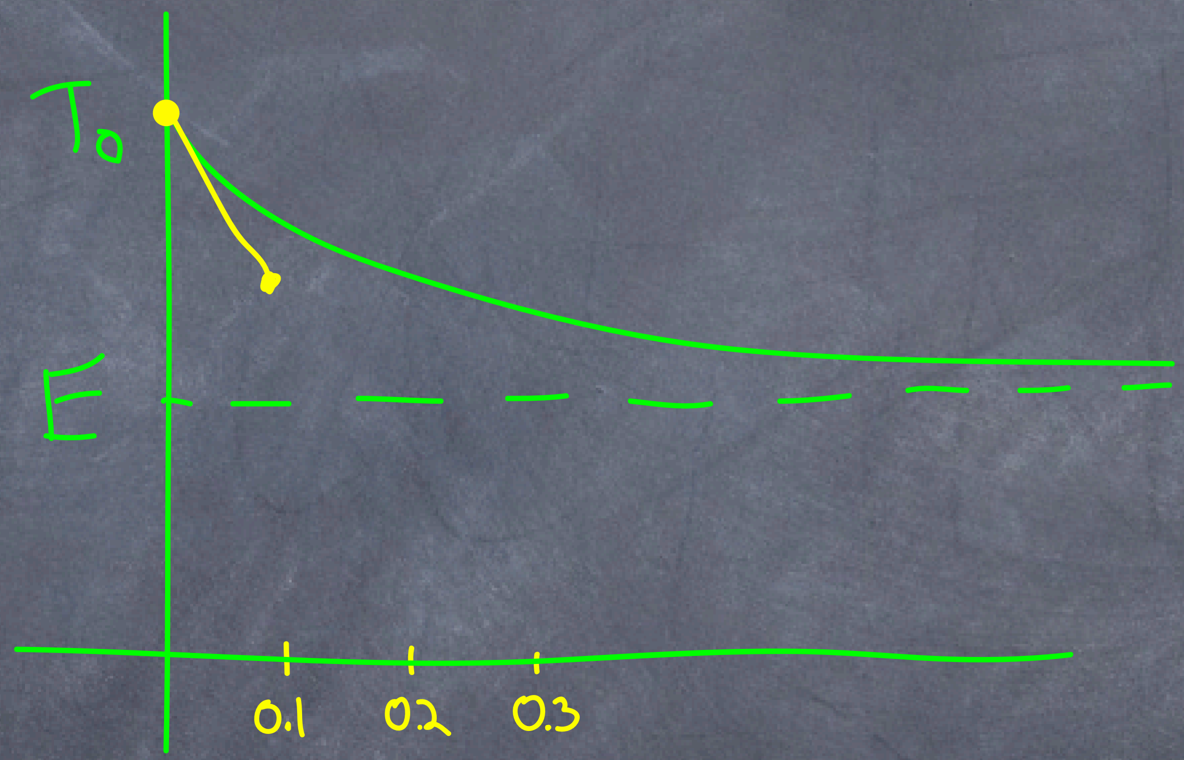

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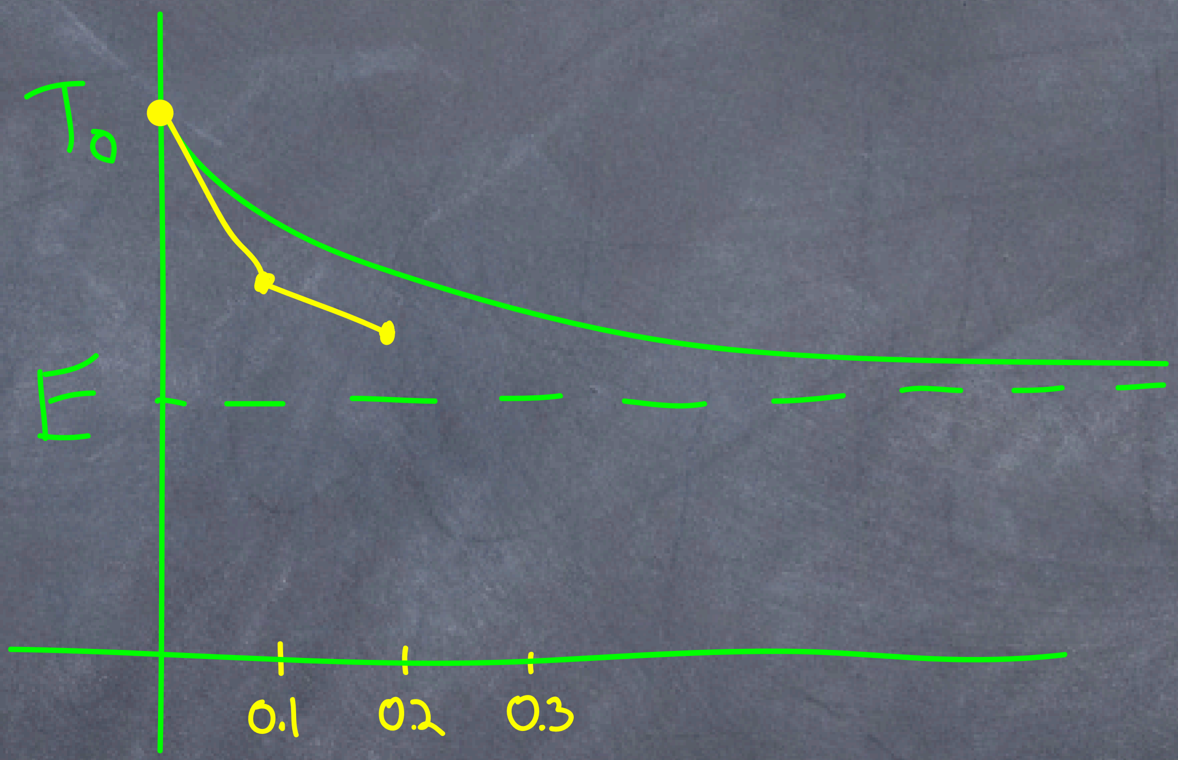

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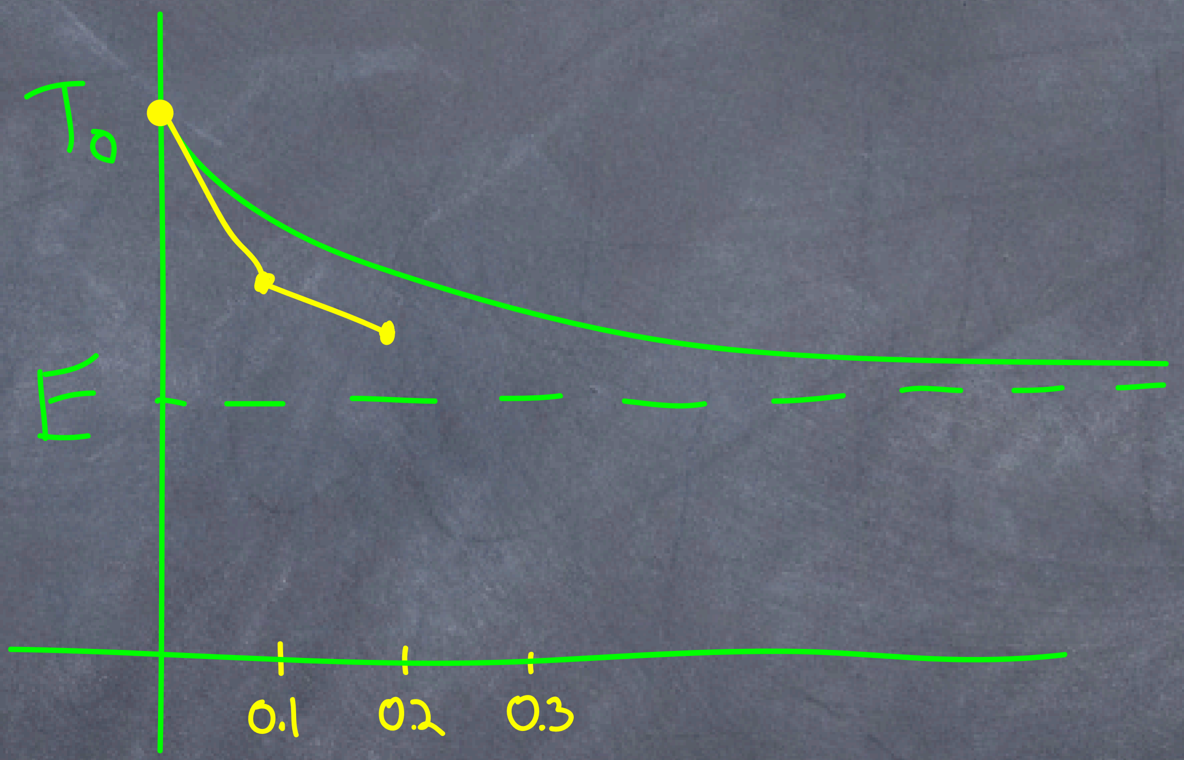

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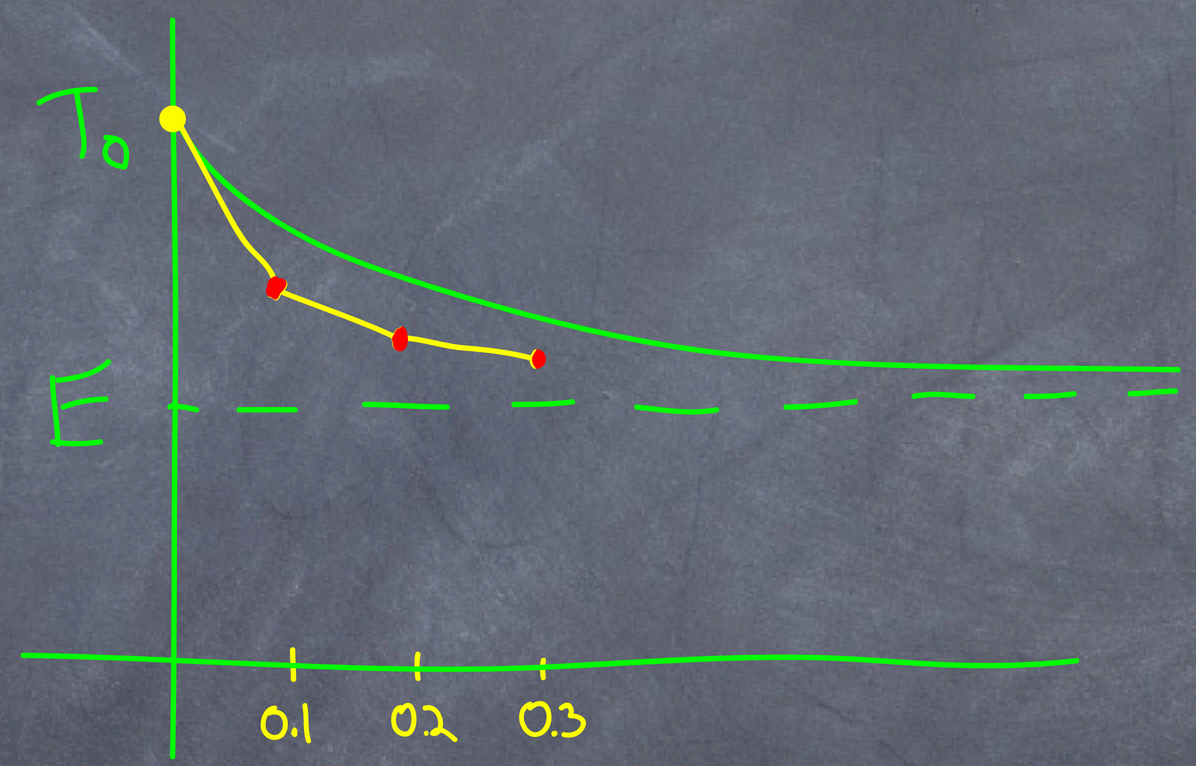

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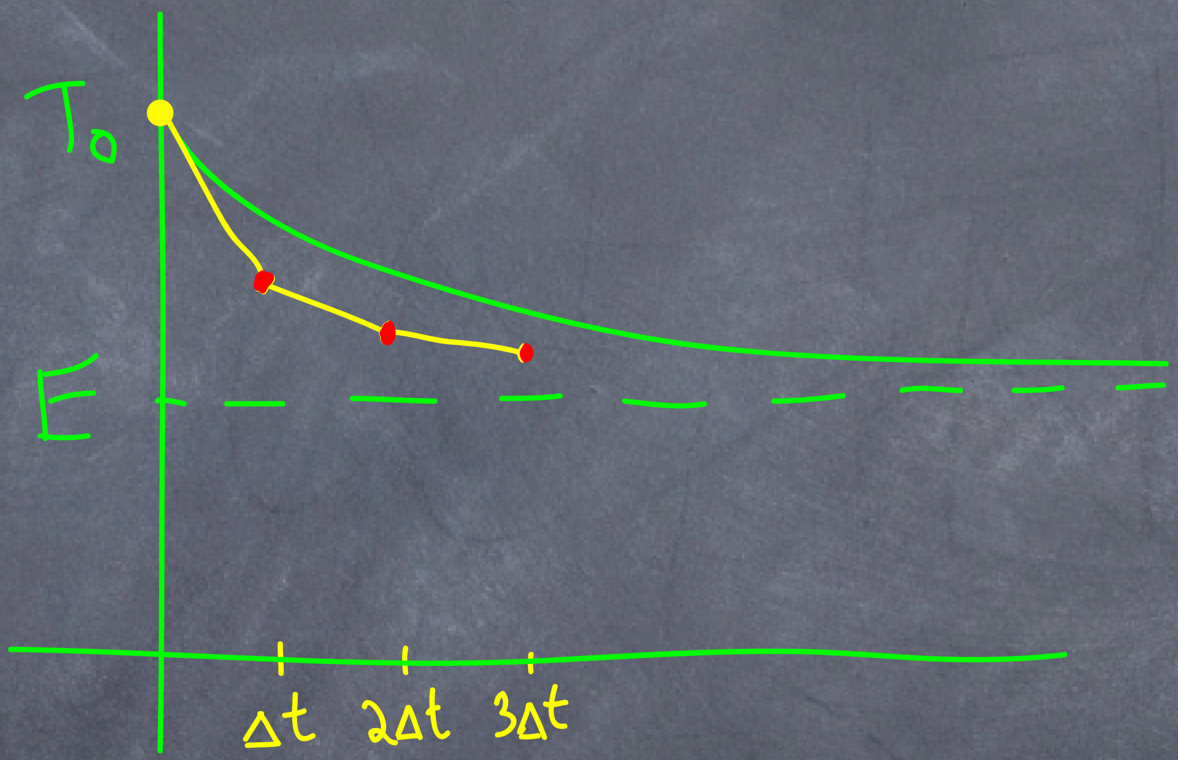

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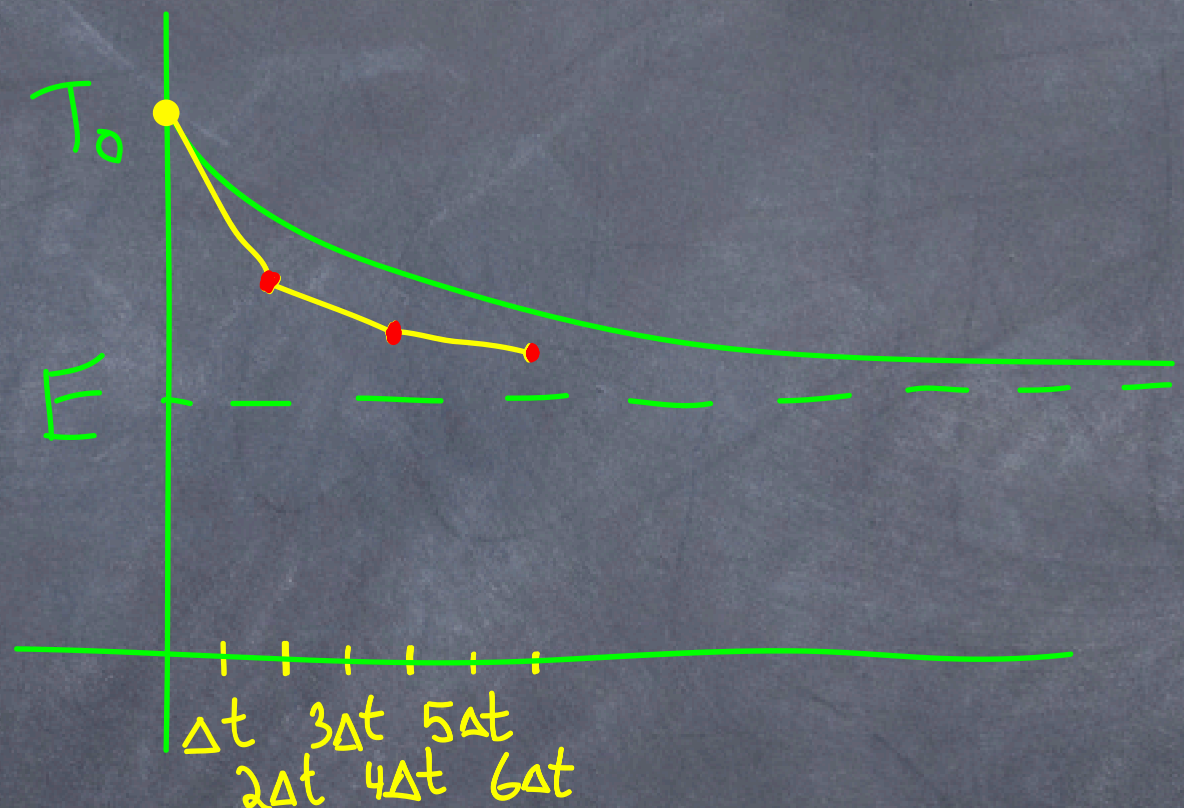

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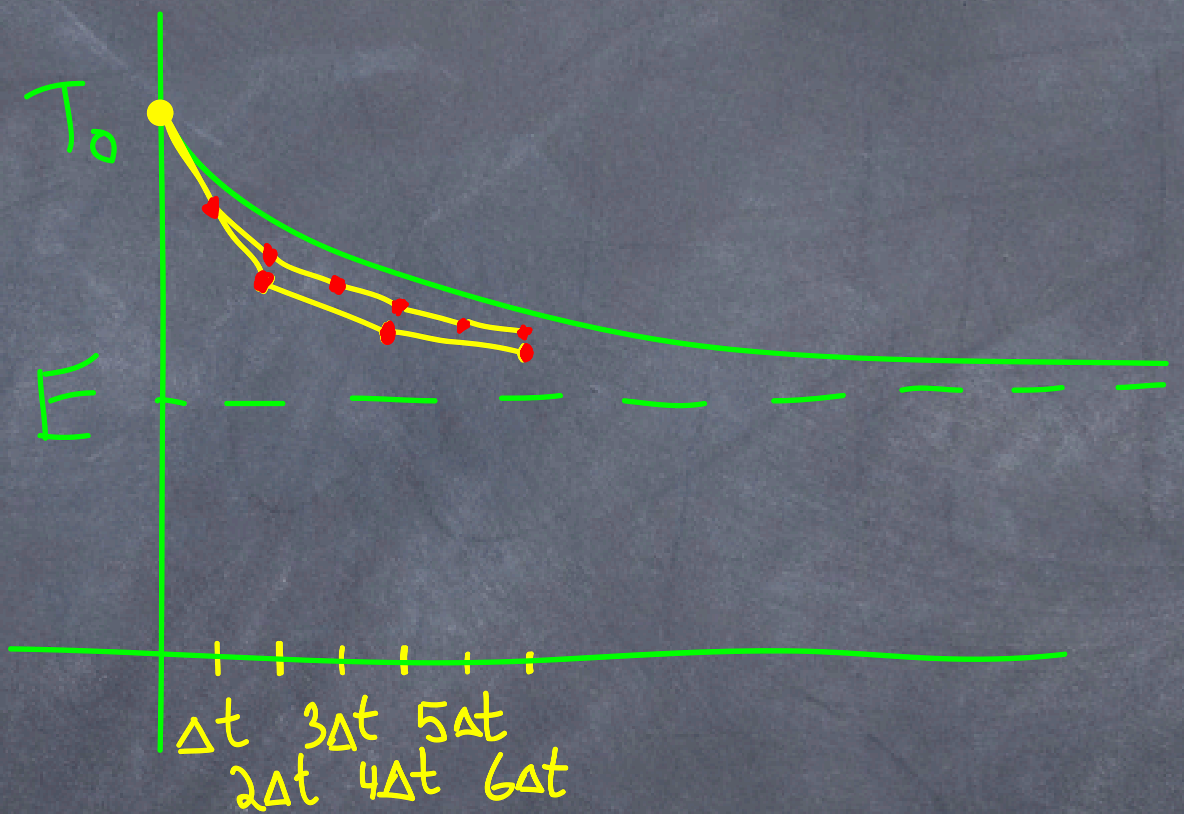

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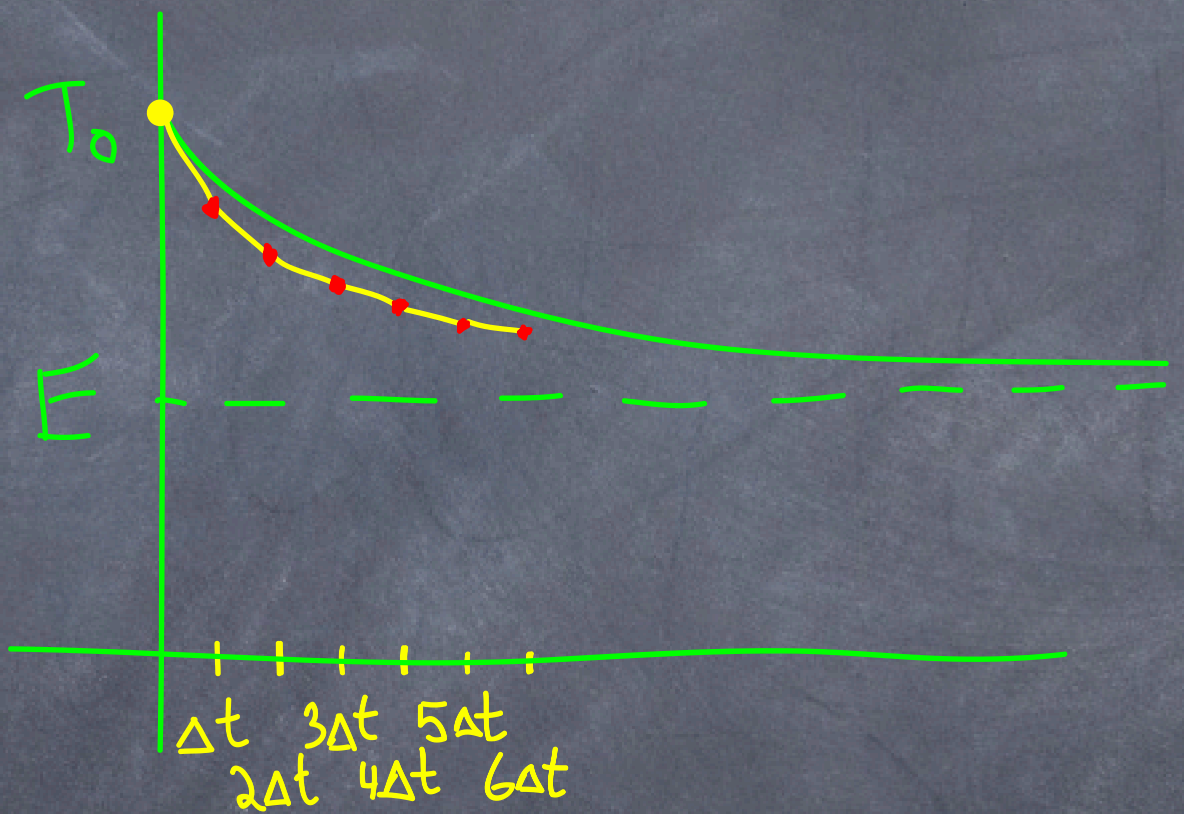

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When will Euler's method underestimate the true solution?

(A) When the derivative of the true solution is positive.

(B) When the derivative of the true solution is negative.

(C) When the second derivative of the true solution is positive.

(D) When the second derivative of the true solution is negative.

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  - $\cot\theta = 1 / \cot\theta$

Trig review

- If θ is measured counterclockwise from the positive x axis,

(A) $x=\sin(\theta)$, $y=\tan(\theta)$.

(B) $x=\tan(\theta)$, $y=\sin(\theta)$.

(C) $x=\sin(\theta)$, $y=\cos(\theta)$.

(D) $x=\cos(\theta)$, $y=\sin(\theta)$.

(E) $x=\cos(\theta)$, $y=\tan(\theta)$.

Trig review

- Which of the following is not a trig identity?

(A) $1 + \cot^2\theta = \csc^2\theta$

(B) $\tan^2\theta + 1 = \sec^2\theta$

(C) $\sin(2\theta) = 2 \sin\theta \cos\theta$

(D) $\cos(\theta) = \sin(\theta - \pi/2)$

(E) $\cos(\theta) = -\sin(\theta - \pi/2)$