Name: Solution

Quiz Score:____/10

Student Number:__________

Answer questions in the space provided. Show your work. No calculators or notes.

1. For $x > 0$, $x^n = e^{\ln(x^n)} = e^{n\ln(x)}$. We can use this identity to show that $\frac{d}{dx}(x^n) = nx^{n-1}$, for $x > 0$ with constant n.

   (a) (2 points) Determine $\frac{d}{dx}\left(e^{n\ln(x)}\right)$. [Do not convert back to x^n and use the power rule]

   set $u = n\ln(x)$, then $\frac{d}{dx}\left(e^{n\ln(x)}\right) = \frac{d}{du}(e^u)\cdot\frac{du}{dx}$

   by the chain rule.

   $\frac{d}{du}(e^u) = e^u$, $\frac{du}{dx} = n/x$

   $\Rightarrow \frac{d}{dx}\left(e^{n\ln(x)}\right) = e^u\left(\frac{n}{x}\right) = \frac{ne^{n\ln(x)}}{x}$

   (b) (1 point) Rewrite your answer from part (a) in a form that does not contain e or ln.

   $e^{n\ln(x)} = x^n$

   $\Rightarrow \frac{d}{dx}\left(e^{n\ln(x)}\right) = \frac{nx^n}{x} = nx^{n-1}$

2. (3 points) Is $y(t) = \left(\frac{2}{3}t + 1\right)^{\frac{3}{2}}$ a solution to the following differential equation? [You must show work to receive marks]

$$\frac{dy}{dt} = y^{\frac{1}{3}}, \quad y(0) = 1$$

$$\frac{dy}{dt} = 3/2\,(2/3\,t + 1)^{1/2} \cdot 2/3 = (2/3\,t + 1)^{1/2}$$

(by the chain rule)

$$(2/3\,t + 1)^{1/2} = \left((2/3\,t + 1)^{3/2}\right)^{1/3} = y^{1/3}.$$

$$y(0) = (2/3 \cdot 0 + 1)^{3/2} = 1.$$

So $y(t)$ satisfies the differential equation and the initial condition and is thus a solution.

3. For constant C, $y(t) = \sqrt{2t + C}$ is a solution to the differential equation

$$\frac{dy}{dt} = \frac{1}{y}$$

(a) (1 point) Determine the value of C so that $y(t)$ satisfies the initial condition $y(0) = y_0$, where $y(0) > 0$.

$y(0) = \sqrt{2 \cdot 0 + C} = y_0$

$\Rightarrow \sqrt{C} = y_0$

$\Rightarrow C = y_0^2$

$\Rightarrow y(t) = \sqrt{2t + y_0^2}$

(b) (1 point) For $y(0) > 0$, determine the time it takes for $y(t)$ to become three times its initial value.

Find t where $y(t) = 3y_0$.

$\Rightarrow 3y_0 = \sqrt{2t + y_0^2}$

$\Rightarrow 9y_0^2 = 2t + y_0^2$

$\Leftrightarrow 8y_0^2 = 2t$

$\Leftrightarrow t = 4y_0^2$.

4.

$$\frac{dy}{dt} = 3 - y, \quad y(0) = 2$$

(a) (1 point) Determine the solution to the differential equation.

A differential equation of the form

$\frac{dy}{dt} = a - by$, $y(0) = y_0$ has the

solution $y(t) = a/b - (\frac{a}{b} - y_0)e^{-bt}$

$\Rightarrow$ when $a = 3$, $b = 1$, and $y_0 = 2$

$y(t) = 3 - (3-2)e^{-t} = 3 - e^{-t}$

(b) (1 point) Determine $\lim_{t\to\infty} y(t)$.

$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} (3 - e^{-t}) = 3$,

as $\lim_{t\to\infty} (e^{-t}) = 0$.