Lecture 2 (Sept. 05 2013)

Important Information: OSH 1 due Monday Sept. 09
Lectures Sept. 09 - Sept. 13 are given by Maxim

Learning Goal: Properties of functions (Power functions, Polynomials, Hill functions)

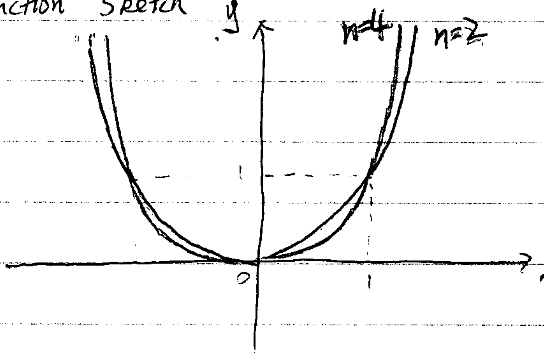
2 Power functions: $y = k \cdot x^n$

② Symmetry: when n is even, $f(-x) = f(x)$ → even function, symmetrical to the line $x=0$
when n is odd, $f(-x) = -f(x)$ ⇒ odd function, symmetrical to the point $(0,0)$

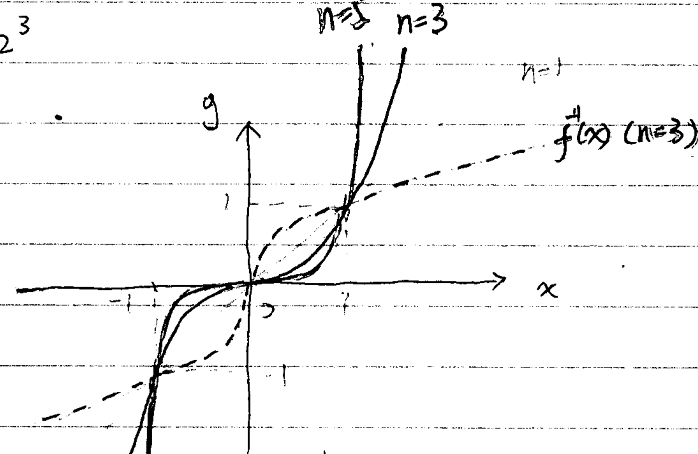
③ Intersections: Given function $g(x)$, set equation $g(x) = k \cdot x^n$, solve for x
(Appendix C.D.1 in Leah's notes)

④ Behaviour of the function for small / large x (set $k=1$)
(1) $x \in (0,1)$, $n > m$, $x^n < x^m$ e.g. $0.1^2 < 0.1^3$
(2) $x \in (1, +\infty)$, $n > m$, $x^n > x^m$ e.g. $2^2 < 2^3$

⑤ Function Sketch



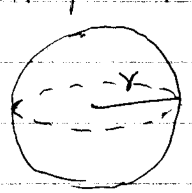
n is even



n is odd

⑥ Inverse function: given $y = f(x)$, can we write x as a function of y? $y = f^{-1}(x)$
n is odd $k \cdot x^n = y$ ⇒ $\frac{y}{k} = x^n$ ⇒ $(\frac{y}{k})^{\frac{1}{n}} = x$ ⇒ $y = f^{-1}(x) = (\frac{x}{k})^{\frac{1}{n}}$
n is even for $x \geq 0$ we have $y = f^{-1}(x) = (\frac{x}{k})^{\frac{1}{n}}$
(Question: why can't we find the inverse function for $x \in (-\infty, +\infty)$?
* The graph of $f^{-1}(x)$ is a reflection of $f(x)$ about the line $y = x$

Example: A spherical cell, what's the proper size for a cell to survive?



Recall the absorbtion rate: $A = k_1 S = k_1 \cdot 4\pi r^2$
consuming rate: $C = k_2 V = k_2 \cdot \frac{4}{3}\pi r^3$
Build up a relationship between A and C: absorbtion rate ≥ consuming rate

$\Rightarrow k_1 \cdot 4\pi r^2 \geqslant k_2 \cdot \frac{4}{3}\pi r^3$

(Solve the inequality for unknown): $r \leqslant \frac{3k_1}{k_2}$

3. Polynomials: $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

$a_0, a_1, \cdots, a_n$ - constants, n - positive integer

(1) $x \ll 1$, $y \approx a_1 x + a_0$

(2) $x \gg 1$, $y \approx a_n x^n$

(3) intersections

4 Rational functions: $y = \frac{p(x)}{q(x)}$, $p(x), q(x)$ - polynomials

Example: Michaelis - Menten kinetics (§1.6 of Leah's notes)

$v = \frac{Kx}{k_n + x}$ v - the speed of reaction K, k_n - positive constants

x - the concentration of substrate

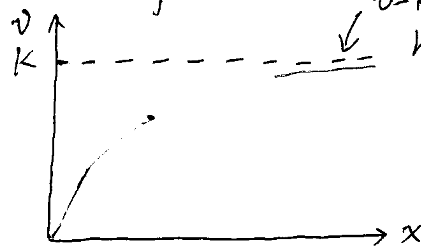
$v = K$ is called a horizontal asymptote

(1) $x \geqslant 0$, $v \geqslant 0$

(2) $x \ll k_n$, $v \approx \frac{Kx}{k_n}$

$x \gg k_n$, $v \approx \frac{Kx}{x} = K$

(3) Sketch the function



Example: Hill functions (not mention in class, more details are given on Monday)

$y = \frac{Ax^n}{a^n + x^n}$, A, a - positive constant

for $n=1$, we have the same expression as Michaelis-Menten

(1) $x \ll a$, $y \approx \frac{A}{a^n} \cdot x^n$ (Compare this with the property of Michaelis-Menten)

$x \gg a$, $y \approx \frac{Ax^n}{x^n} = A$

(2) Sketch function (See Figure 1.6 of Leah's notes)

(3) Question: At what value of x, y reaches $\frac{A}{2}$?

Solution: $\frac{Ax^n}{a^n + x^n} = \frac{A}{2} \Rightarrow 2x^n = a^n + x^n \Rightarrow x^n = a^n \Rightarrow x = a$

(4) Question: At what value of x, all Hill functions with the same A and a intersect?

$x = a \Leftarrow$ Try get this by (3)