Lecture 11 (Sept. 30, 2013)

Learning Goals: ① sketch a given function
② Global (absolute) extrema

Example 1: Sketch $f(x) = \frac{(x+1)^2}{x^2+1}$

① domain $(-\infty, +\infty)$

② $x \to +\infty \quad f(x) \to \frac{x^2}{x^2} = 1 \quad \Rightarrow$ horizontal asymptote

$x \to -\infty \quad f(x) \to \frac{x^2}{x^2} = 1 \qquad y=1$ (draw it before sketching)

③ $f'(x) = \frac{2(1-x^2)}{(1+x^2)^2} = \frac{2(1-x)(1+x)}{(1+x^2)^2}$

critical points: $x=1$ and $x=-1$

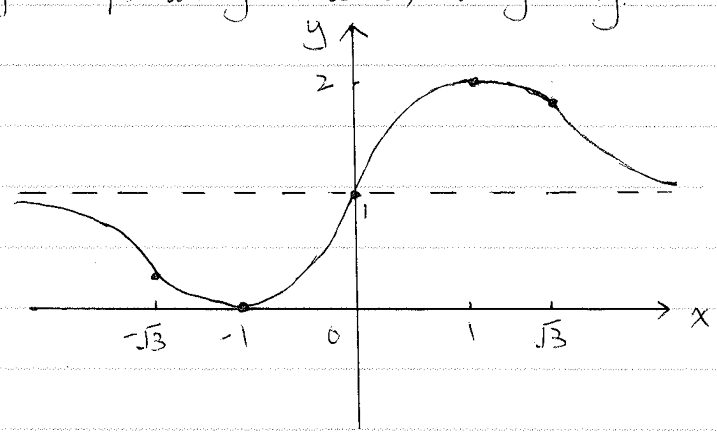
④ $f''(x) = \frac{4x(x^2-3)}{(1+x^2)^3} = \frac{4x(x+\sqrt{3})(x-\sqrt{3})}{(1+x^2)^3} = 0 \Rightarrow x=0;\ x=-\sqrt{3};\ x=\sqrt{3}$

* we can put the table together

	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,+\infty)$		
$f'(x)$	$-$	0	$+$	0	$-$		
$f(x)$	↘	0	↗	2	↘		
	$(-\infty,-\sqrt{3})$	$-\sqrt{3}$	$(-\sqrt{3},0)$	0	$(0,\sqrt{3})$	$\sqrt{3}$	$(\sqrt{3},+\infty)$
$f''(x)$	$-$	0	$+$	0	$-$	0	$+$
$f(x)$	$\cap$	0.134	$\cup$	1	$\cap$	1.866	$\cup$

⑤ x, y intercepts are given above, no symmetry

⑥



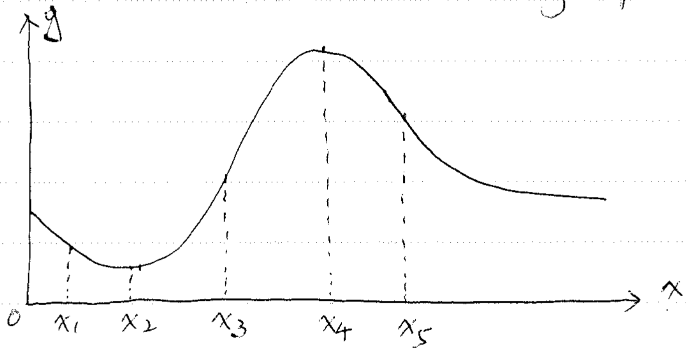
* mark the value of $f(x)$ at the points if possible

* be aware of the asymptote when it exists

• Global Extrema : Let $f(x)$ be a function with domain D
(absolute extrema) then $f(c)$ is a global maximum if $f(c) \geq f(x)$ for all the x in D
$f(c)$ is a global minimum if $f(c) \leq f(x)$ for all the x in D

Notice: (1) Global maximum/minimum can happen at multiple locations
(2) The domain D is very important

Example 2:



(1) given $x \in [x_1, x_5]$, $f(x_2)$ is a global minimum
$f(x_4)$ is a global maximum

(2) given $x \in (x_1, x_3)$, $f(x_2)$ is a global minimum
there's no global maximum, but still need to check the value of the function at the endpoint of the domain

Example 3: Find global extrema of $f(x) = x^3(x-5)^2$, $x \in [0, 4]$

$f'(x) = 5x^2(x-3)(x-5)$ $\Rightarrow$ critical points: $x=0$, $x=3$, $\cancel{x=5}$ out of the domain

* Not like sketching the function, we only need to plug in the value of x and find the largest/smallest one

$f(0) = 0$
$f(3) = 27 \times 4 = 108$
$f(4) = 64 \times 1 = 64$

$\Rightarrow$ $f(0) = 0$ is a global minimum
$f(3) = 108$ is a global maximum