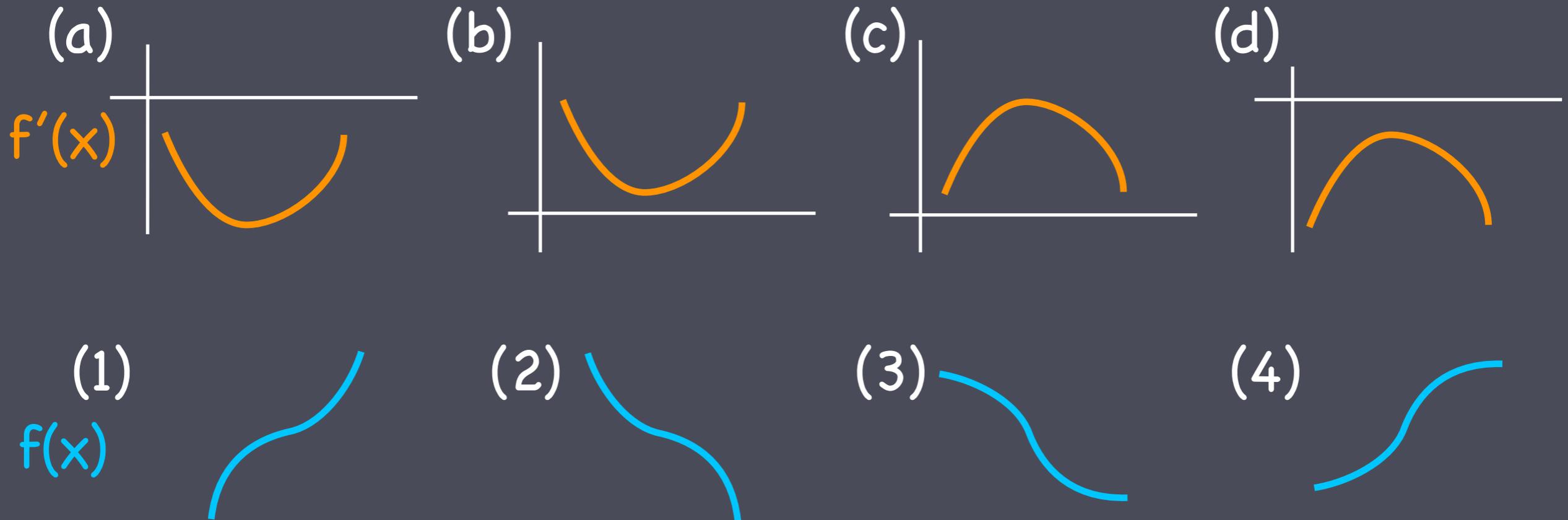


# Today

- Note about Hill functions:  $\frac{x^n}{k^n + x^n} = 1 - \frac{k^n}{k^n + x^n}$
- Shape of graphs using calculus

# Match $f'(x)$ to $f(x)$



(A) 1d, 2b, 3a, 4c

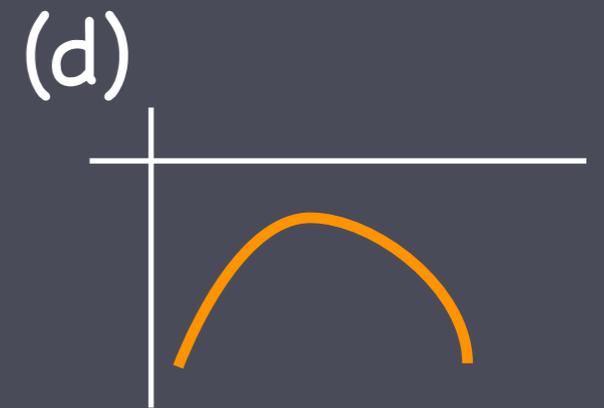
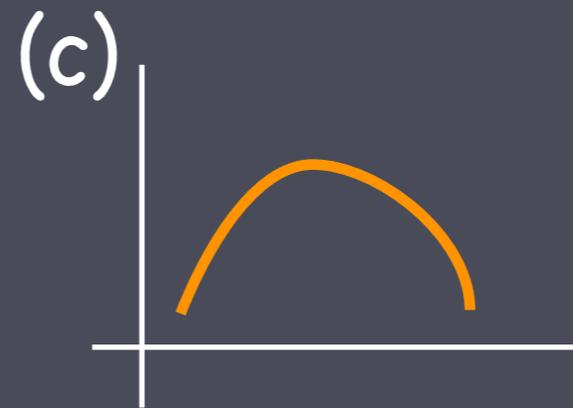
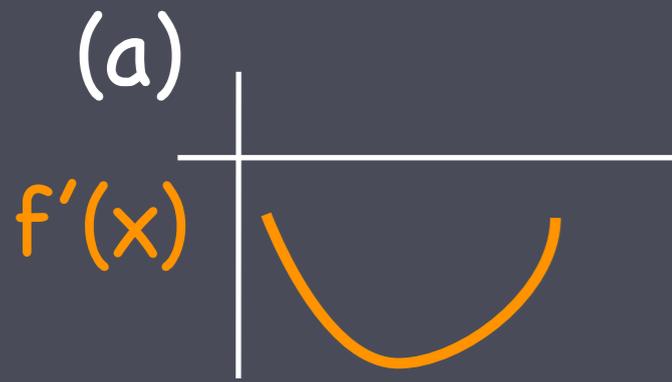
(C) 1b, 2d, 3c, 4a

(B) 1b, 2d, 3a, 4c

(D) 1c, 2a, 3d, 4b

(E) Don't know.

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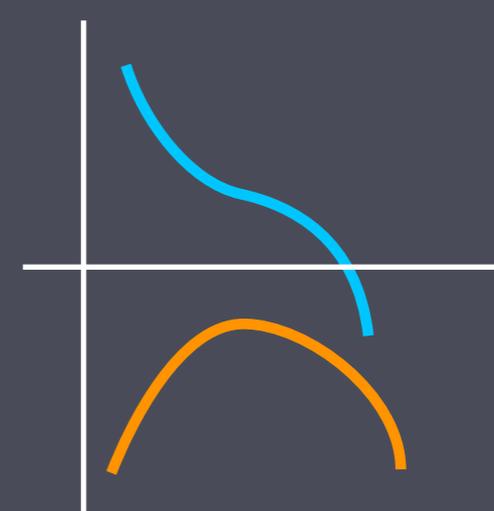
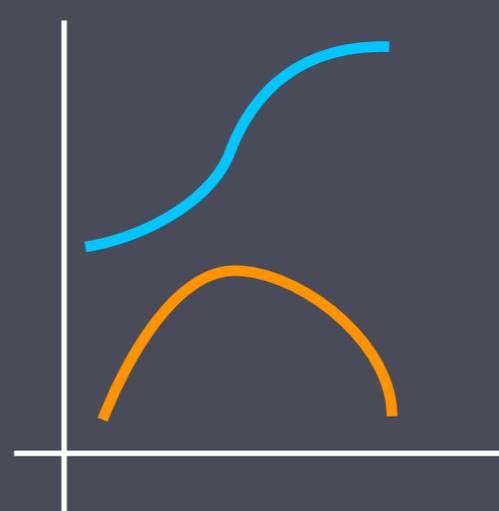
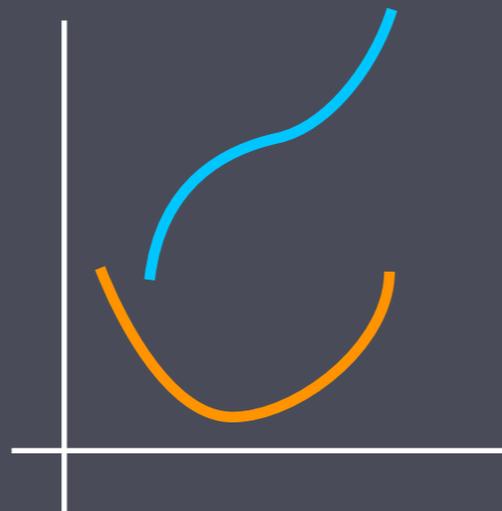
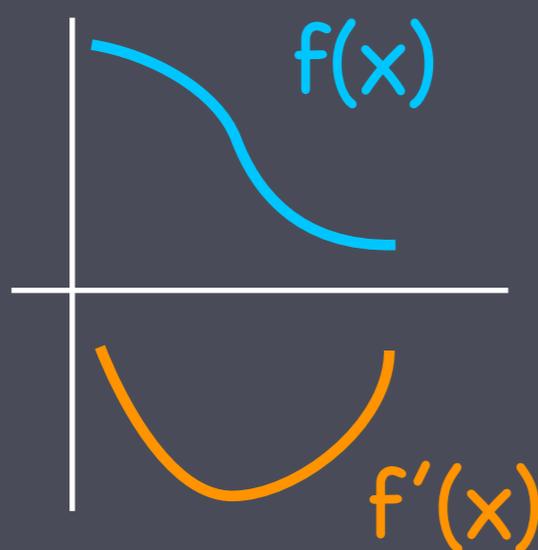
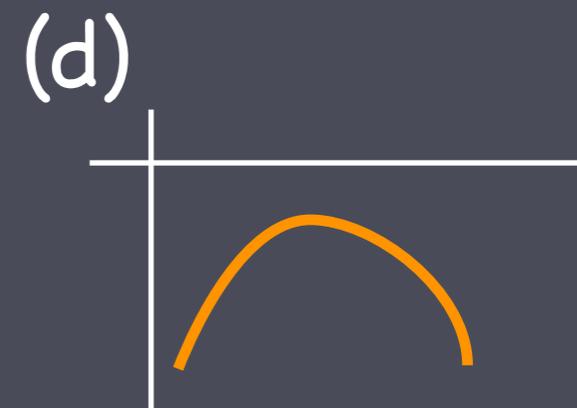
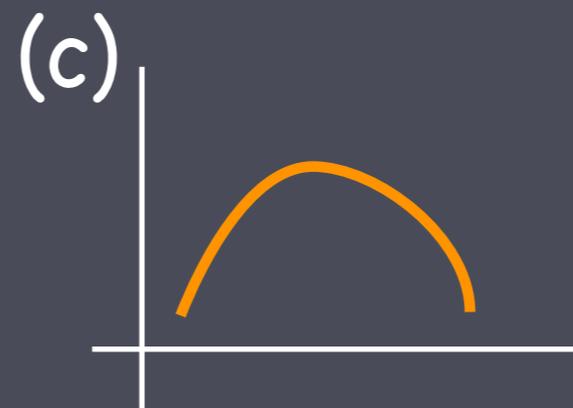
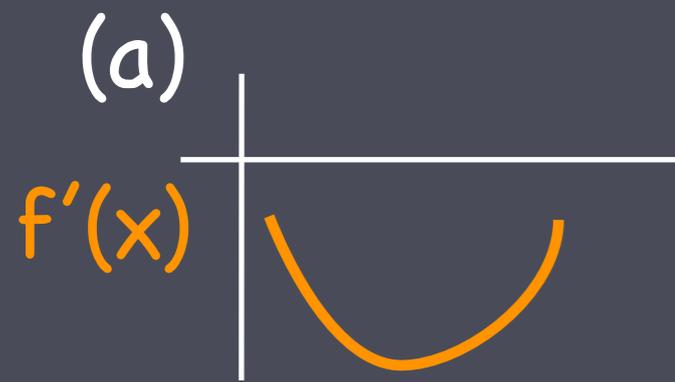
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(E) Don't know.

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If you want to find a min/max of  $f'(x)$ , look for points at which. . .

(A)  $f'(x) = 0$ .

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(C)  $f''(x) = 0$ .

(D)  $f''(x) = 0$  and  $f'''(x) \neq 0$ .

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This is "SDT" where the function considered is  $f'$  instead of  $f$ ! Would usually use "FDT".

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- If  $f''(x)$  does not change sign at a potential IP of  $f(x)$ , then the potential IP is not an IP of  $f(x)$ !

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Check that  $f'(x)$  **changes sign** at  $a$  (FDT) or that  $f''(a) \neq 0$  (SDT) to make sure.
- Use  $f''(x)$  to determine intervals of **concave up/down**.
- Solve  $f''(x)=0$  to find **potential inflection points** ( $x=a$ ). Check that  $f''(x)$  **changes sign** at  $a$  ("FDT") or that  $f'''(a) \neq 0$  ("SDT") to make sure.

Does  $f(x) = x^4$  have an inflection point?

(A)  $f'(0) = 0$  so yes.

(B)  $f''(0) = 0$  so yes.

(C)  $f'''(0) = 0$  so no.

(D)  $f''(0) = 0$  and  $f''(x) > 0$  for all  $x \neq 0$  so no.

(E) Don't know.

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- fails so no conclusion.

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(E) Don't know.

Not sure about (C)? Try this for  $f(x) = x^5$ .

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(D)  $f''(0) = 0$  and  $f''(x) > 0$  for all  $x \neq 0$  so no.

$$f''(x) = 12x^2$$

(E) Don't know.

Not sure about (C)? Try this for  $f(x) = x^5$ .