Today

- log graph and semi-log plots
- Exponential derivative (what is C_a?)
- Bacterial growth example
- Doubling time, half life, characteristic time
- Exponential behaviour as solution to DE

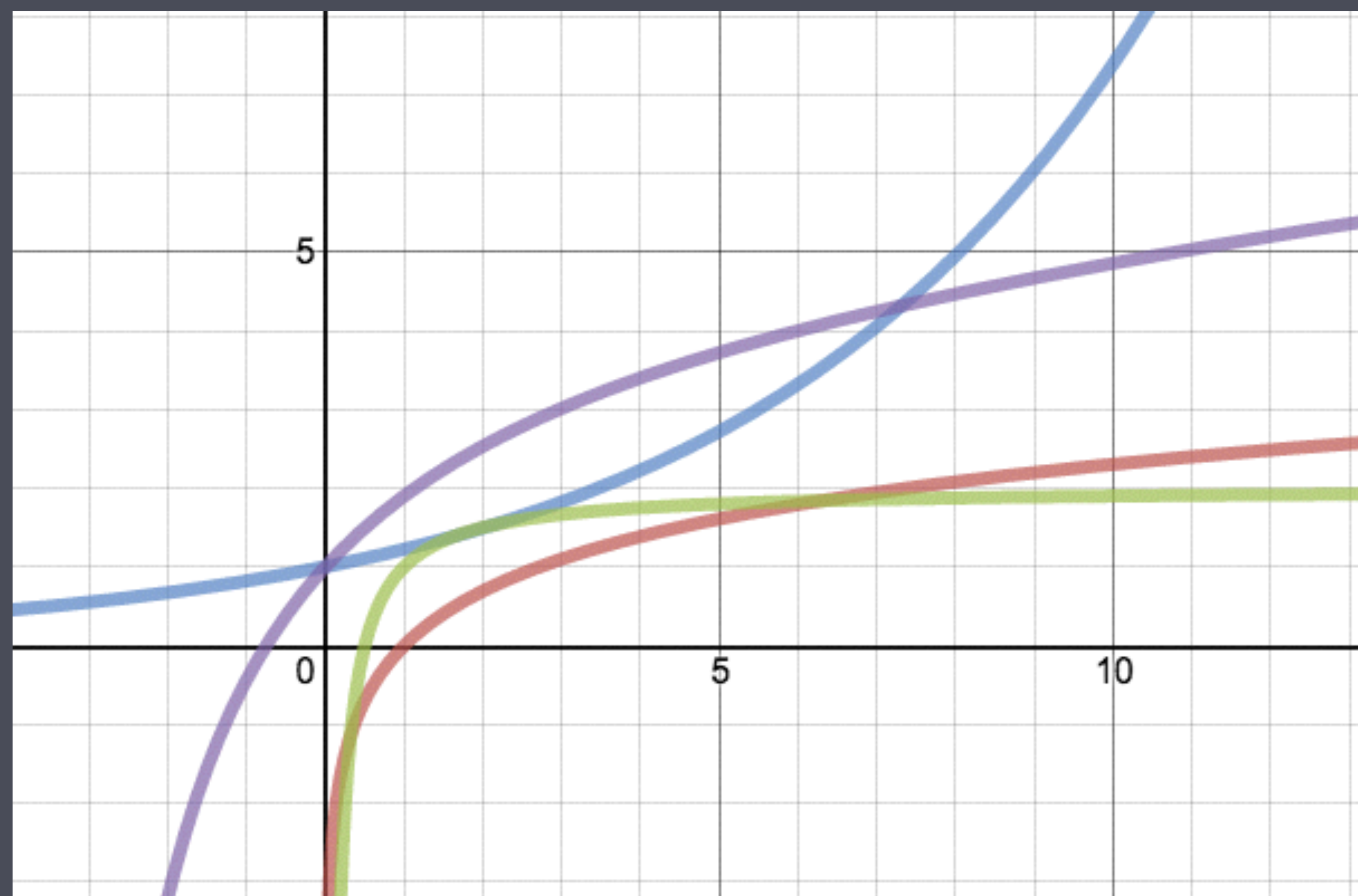
Which of following is the graph of ln(x)?

(A)

(B)

(C)

(D)



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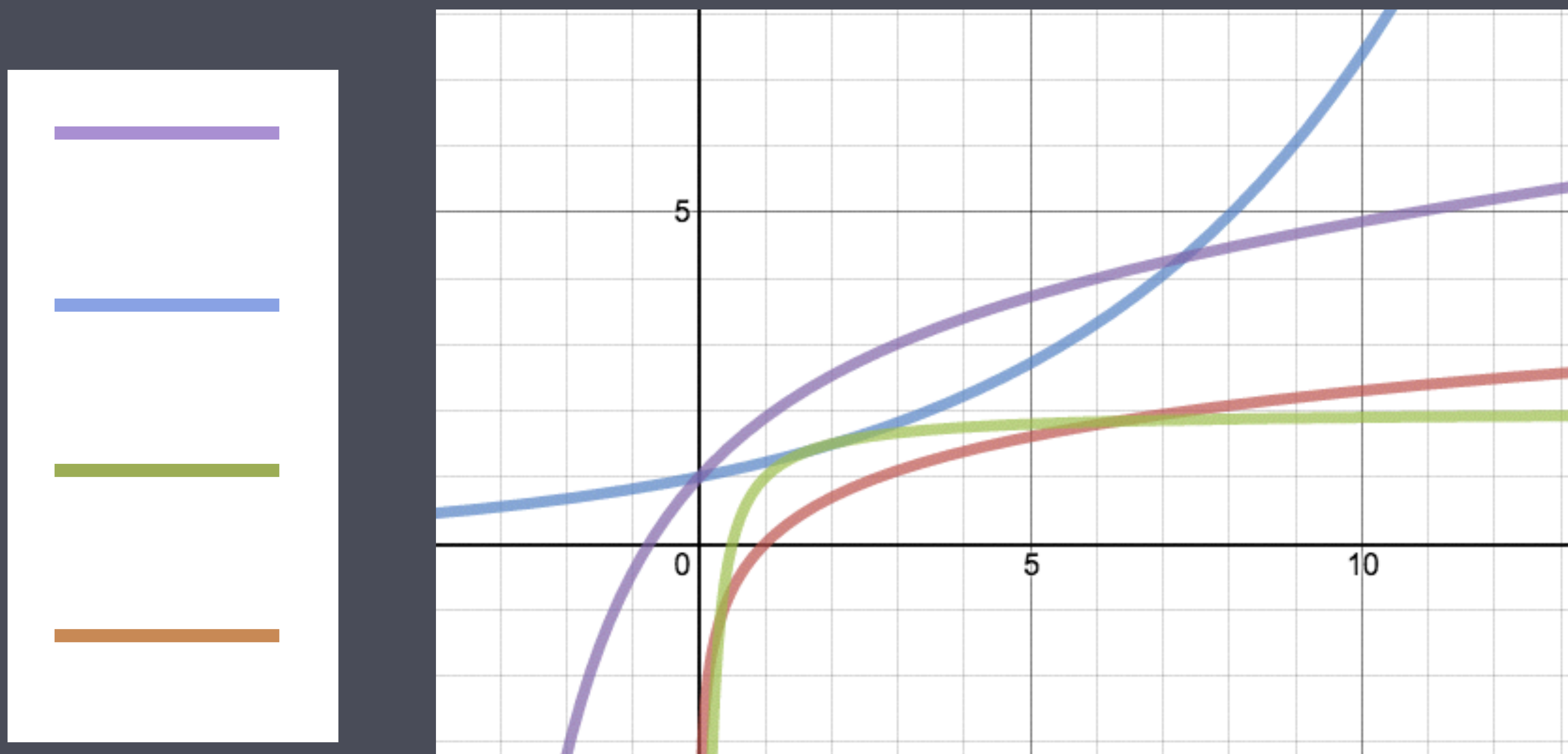

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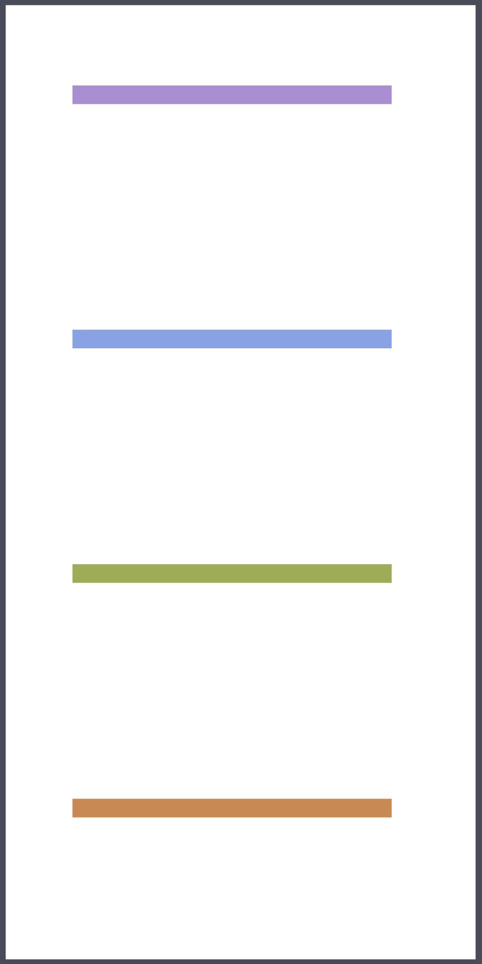
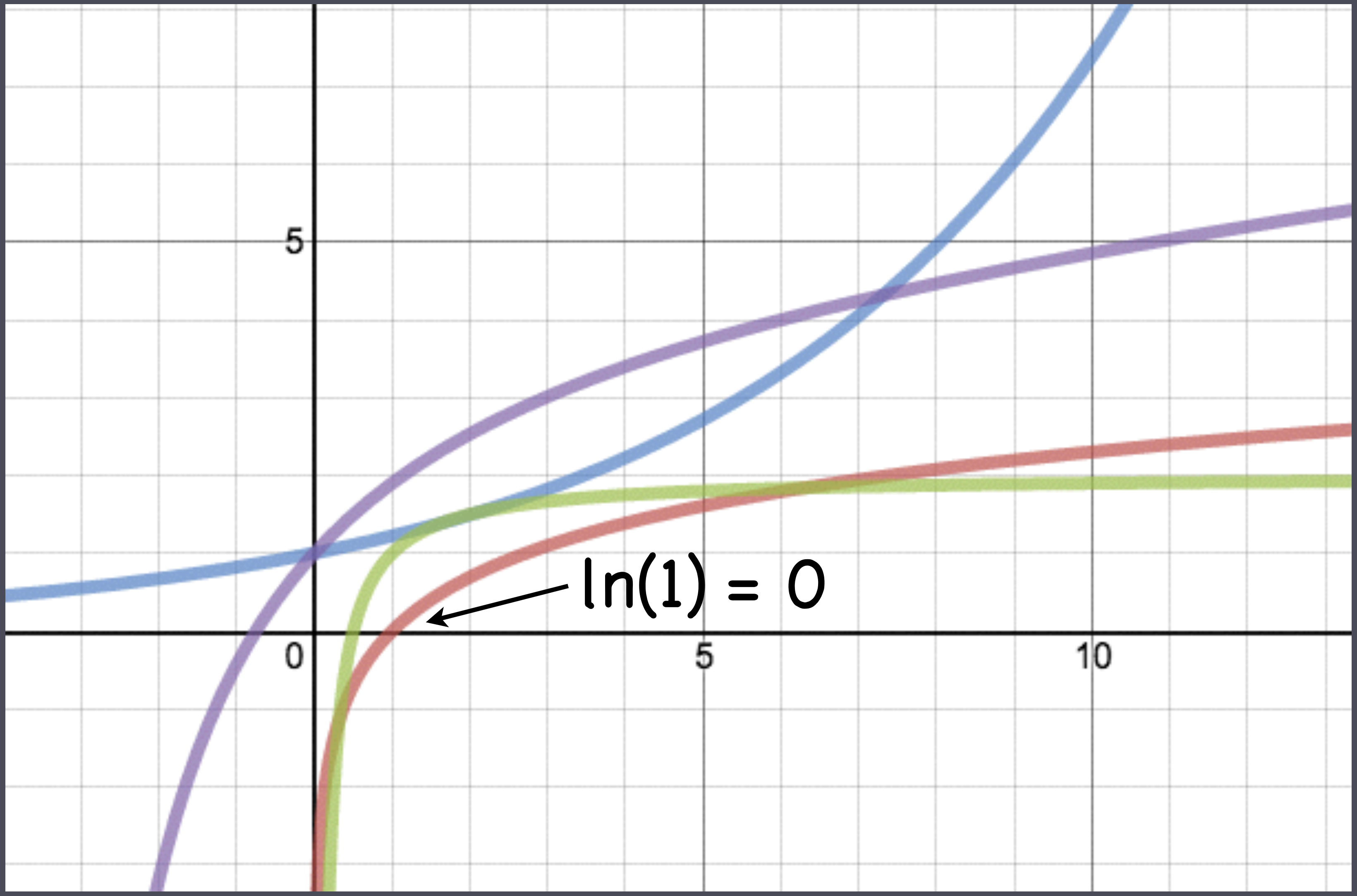

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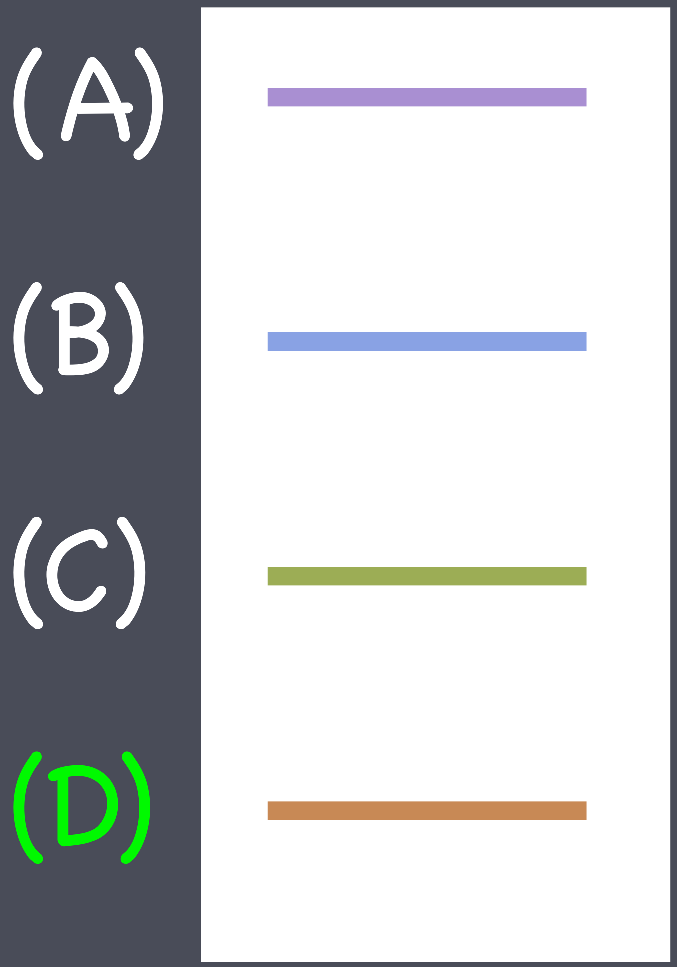

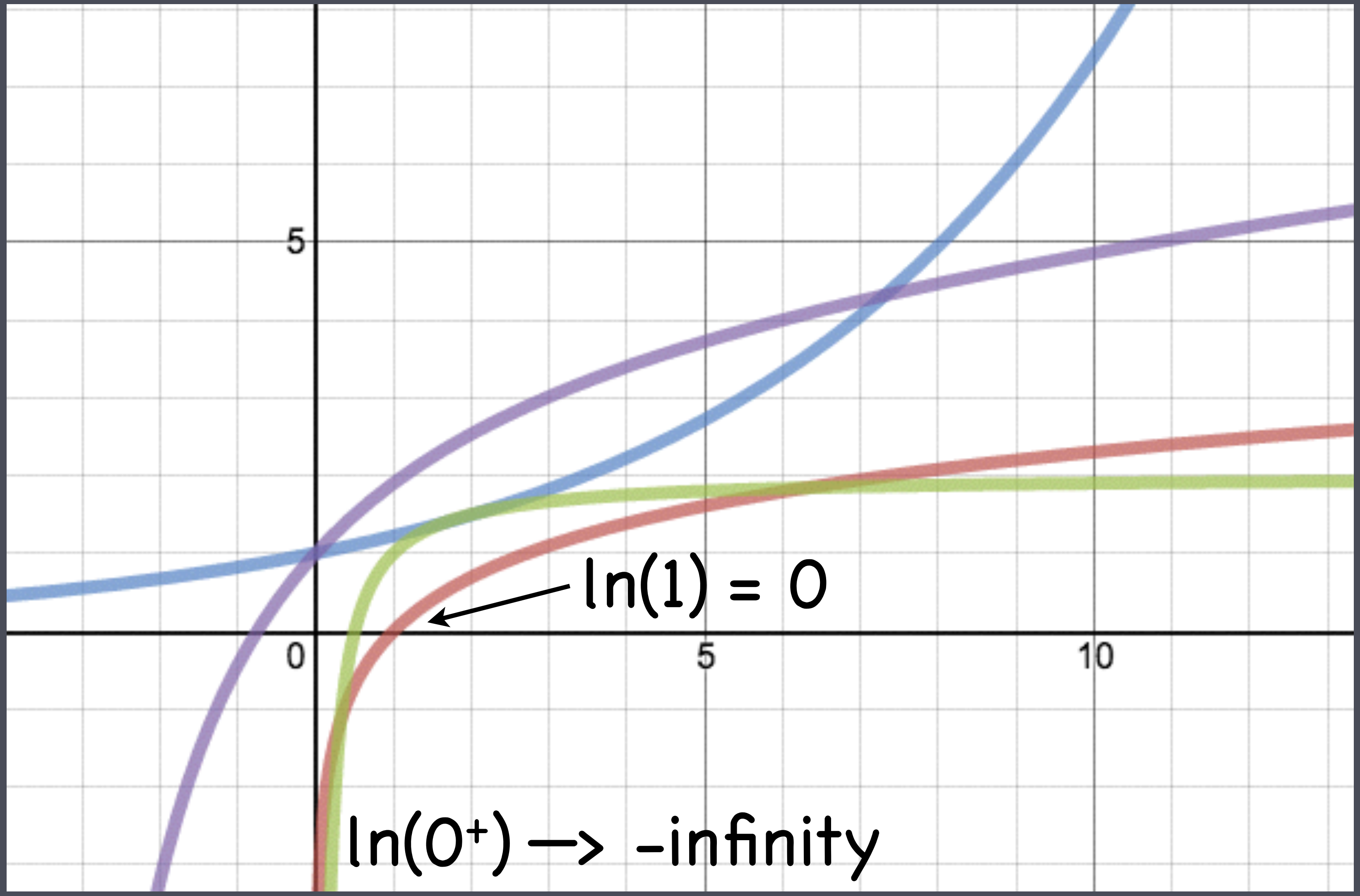

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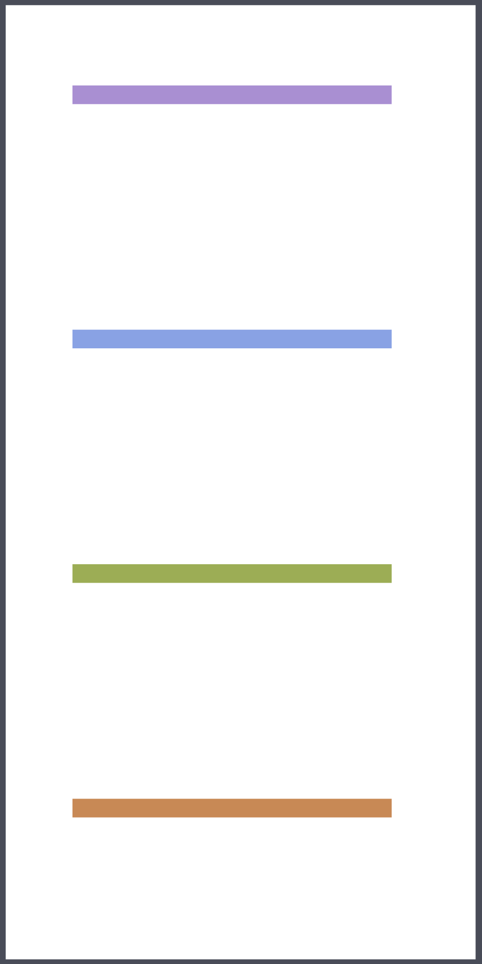

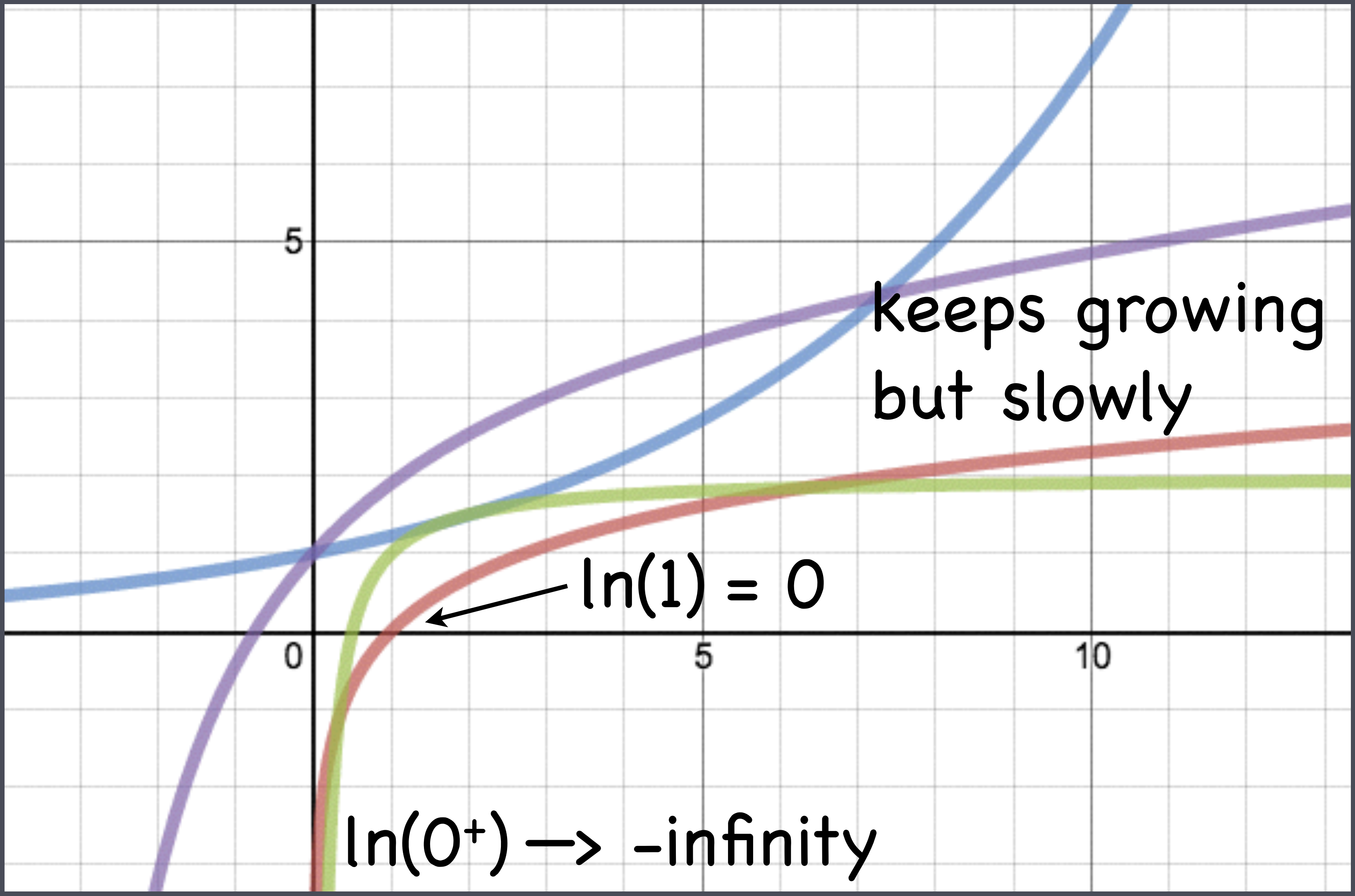

Log-log and semi-log plots

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- A semi-log plot is a plot on which you plot log(y) versus x instead of y versus x.

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<---this is what OSH 5 asks you to use

Semi-log plot of exponential function

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- $V = A + kx$ where $A=\ln(a)$.
- On a semi-log plot, $y= ae^{kx}$ looks linear.

Log-log plot of power function

Slide not shown in class but include for interest.

Log-log plot of power function

- Suppose $y = ax^p$.

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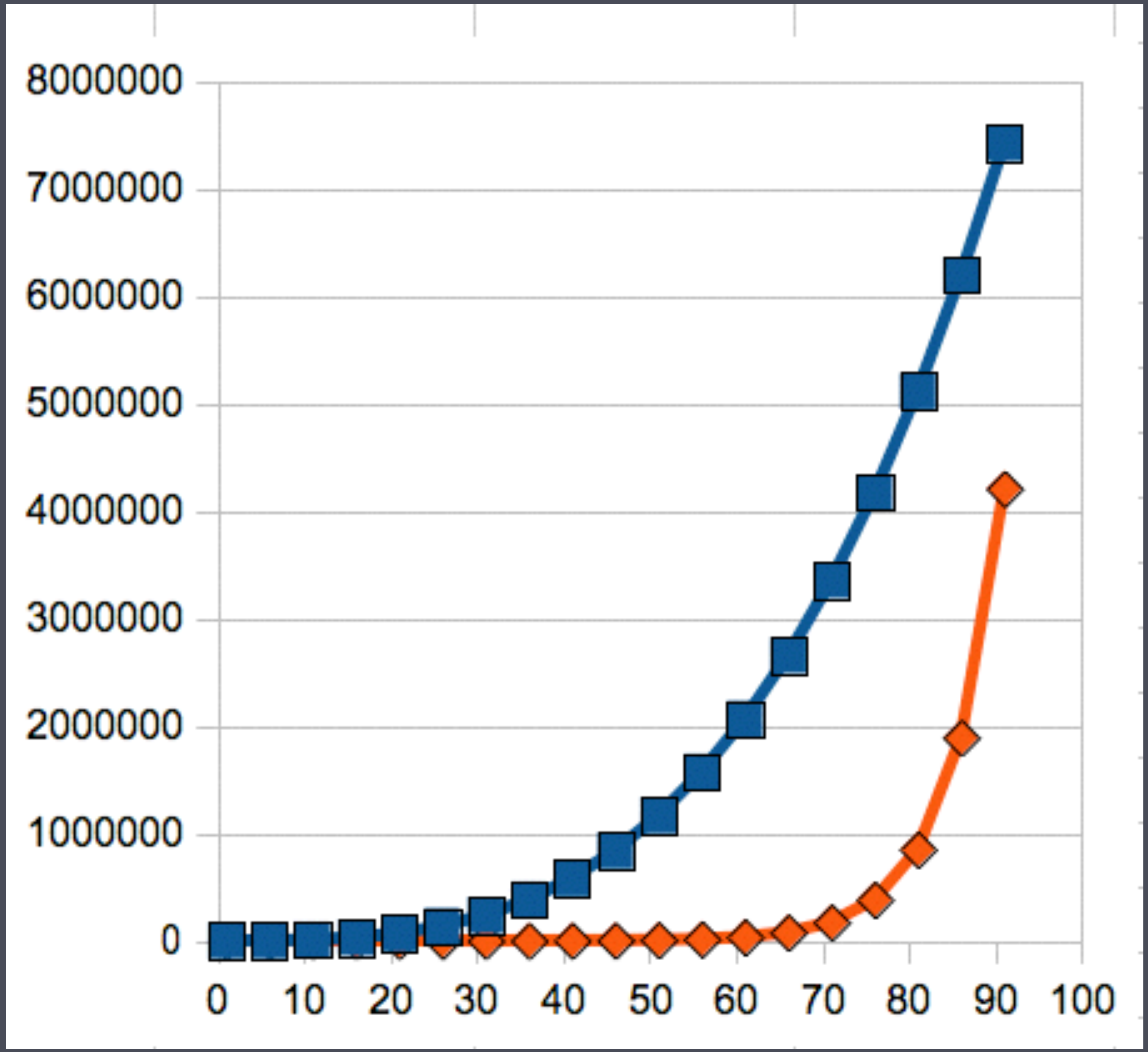
Regular, log-log and semi-log plots

Two data sets.

Power function?

Exponential function?



Regular x-y plot.

Slide not shown in class but include for interest.

Plot $Y_i = \ln(y_i)$ versus x_i.

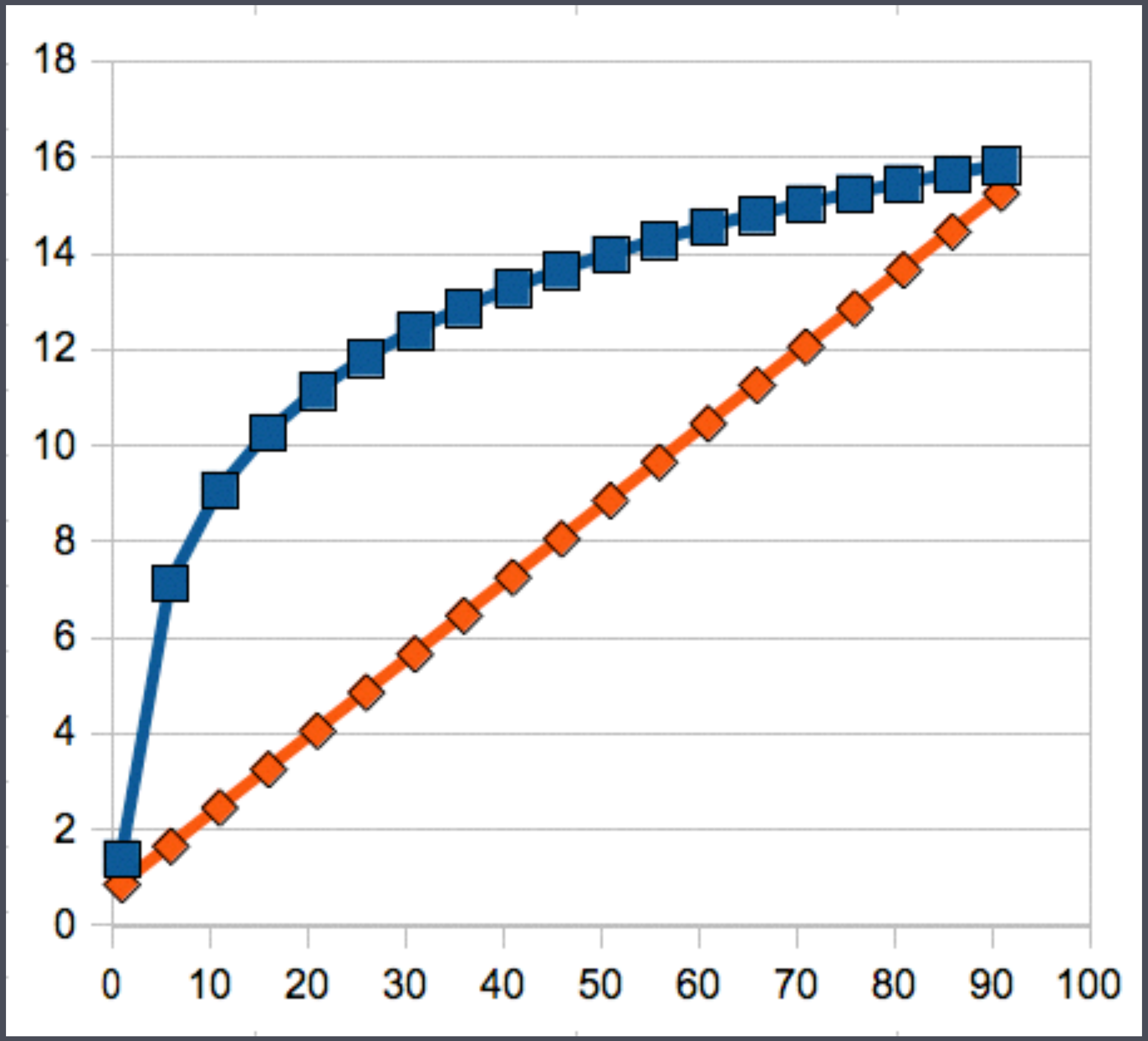
Conclude that:

(A) Blue is power function.

(B) Blue is exponential.

(C) Orange is power function.

(D) Orange is exponential.



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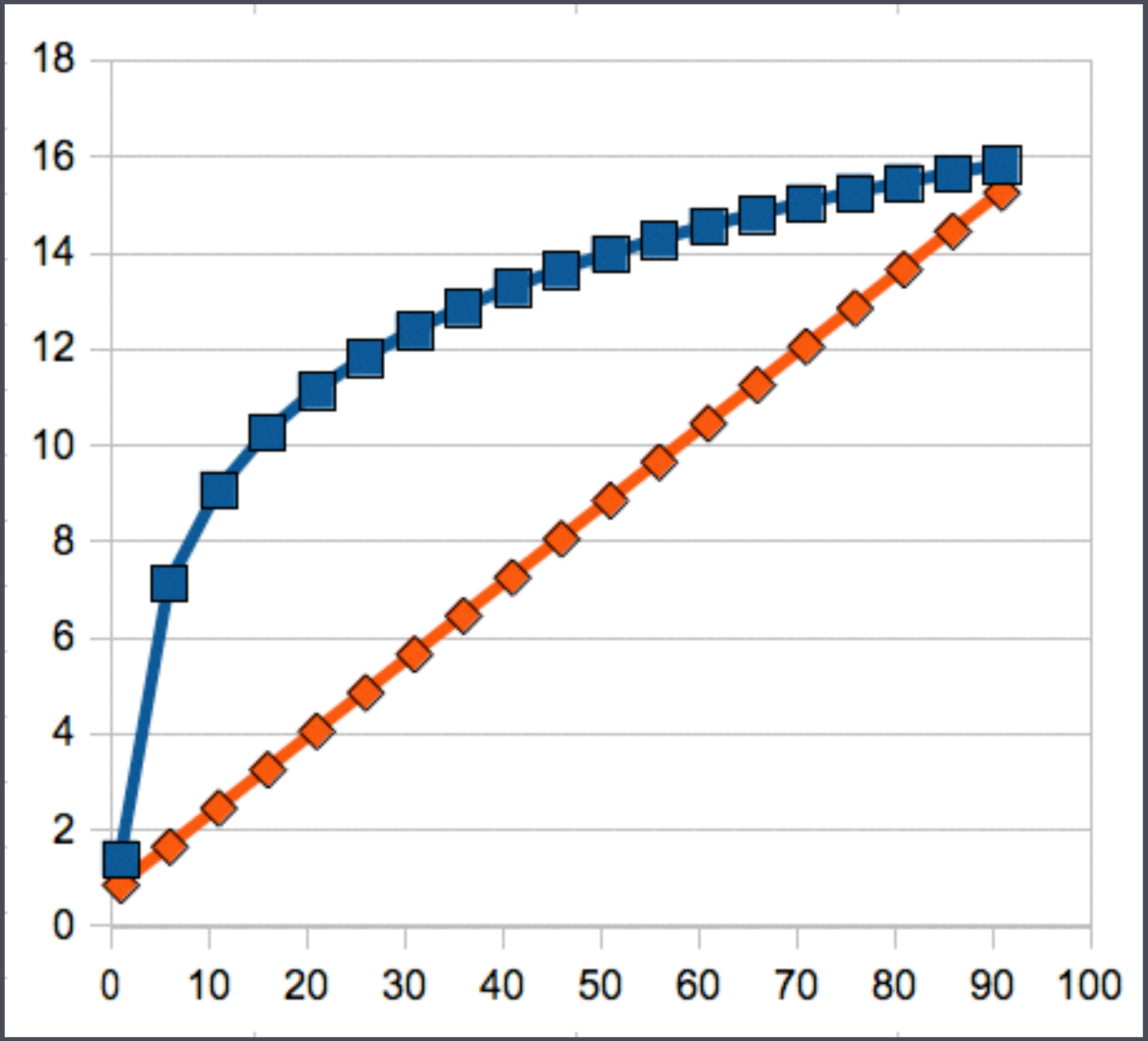

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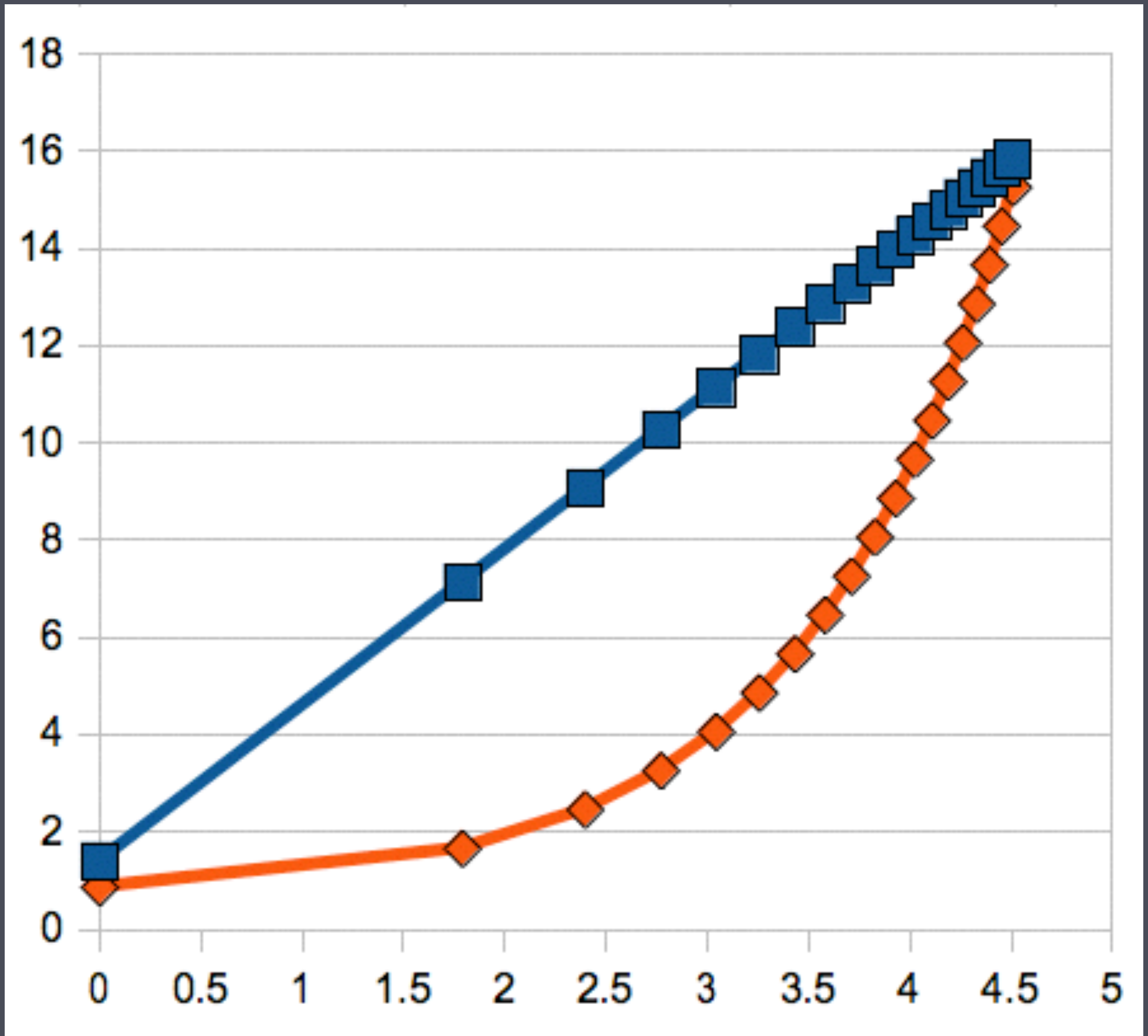

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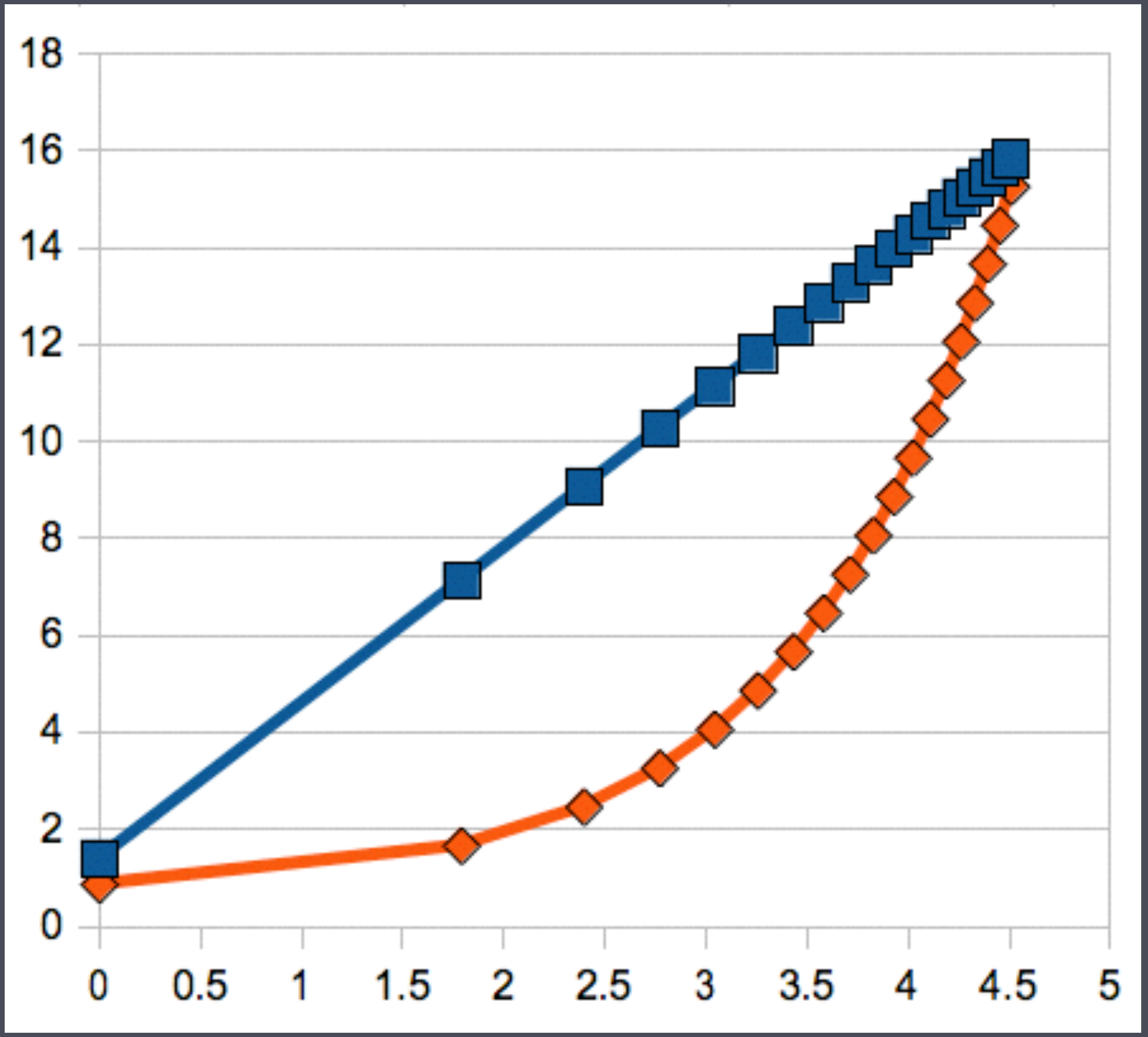

Log-log plot.

Slide not shown in class but include for interest.

$f(x)=a^x$. $f'(x)=C_a a^x$. $C_a=??$

- Recall that we got stuck on this derivative.
- Time to get unstuck...

$$f(x) = e^{\ln(2)x}.$$

(A) $f'(x) = e^{\ln(2)x}$.

(B) $f'(x) = \ln(2)e^{\ln(2)x}$.

(C) $f'(x) = \ln(2) \cdot 1/2 \cdot e^{\ln(2)x}$.

(D) $f'(x) = \ln(2)xe^{\ln(2)x-1}$.

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$$f(x) = e^{\ln(2)x}.$$

(A) $f(x) = 2x.$

(B) $f(x) = (e^{\ln(2)})^x = 2^x.$

(C) $f(x) = e^{\ln(2)}\, e^x = 2e^x.$

(D) $f(x) = e^{\ln(x^2)} = x^2.$

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- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.

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- So $f(x) = 2^x$ --> $f'(x) = 2^x \ln(2)$.
- In general, $f(x) = a^x$ --> $f'(x) = a^x \ln(a)$.

What value of k makes $a^x = e^{kx}$?

(A) $k=e^a$

(B) $k=e^{-a}$

(C) $k=\ln(a)$

(D) $k=-\ln(a)$

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$$a^x = (e^k)^x$$
$$a = e^k$$
$$\ln(a) = \ln(e^k)$$
$$\ln(a) = k\ \ln(e)$$
$$\ln(a) = k$$

$f(x) = a^x = e^{\ln(a)x}$

--> $f'(x) = a^x \ln(a)$.

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

Rough estimate (no calcs, just intuition):

(A) ~1 week.

(B) ~2 weeks.

(C) ~1 month.

(D) ~1 year.

(E) ~10^4 days ≈ 27 years.

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A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $p(t) = e^{t/24}$.

(B) $p(t) = 100{,}000\ 2^{t/24}$.

(C) $p(t) = e^{\ln(2)t}$.

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A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

(A) $t=\ln(10^5)/\ln(2)$

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$$p(t) = 2^t$$

$$10^5 = 2^t$$

$$\ln(10^5) = \ln(2^t)$$

$$\ln(10^5) = t \ln(2)$$

$$t \approx 16.6 \text{ days}$$

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- If written $c(t) = c_0e^{-kt}$ with $k>0$ then $t=\ln(2)/k$ (same as doubling time).

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- If $k<0$, $c(t)$ is decreasing and halves when $c_0e^{kt} = c_0/2$.
- That is when $t=-\ln(2)/k$.
- If written $c(t) = c_0e^{-kt}$ with $k>0$ then $t=\ln(2)/k$ (same as doubling time).
- This is called the half-life.

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- Let $c(t) = c_0e^{-kt}$. At $t=0$, $c(0) = c_0e^0 = c_0$.
- If $k>0$, $c(t)$ is decreasing and reaches 1/e its original value when $c_0e^{kt} = c_0/e$.
- That is when $t=1/k$.
- This is called the characteristic time or mean life. Just like half-life but replace 2 with e (could be called 1/e-life).