Lecture 8 (Sept. 23, 2013)

Learning Goals: ① Product Rule; Quotient Rule

② Sketch the curve of the derivative based on a given function.

• Product Rule: If $f(x)$ and $g(x)$ are differentiable in the domain of interest

then $[f(x)\cdot g(x)]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$$\frac{d}{dx}[f(x)\cdot g(x)] = \frac{d}{dx}f(x)\cdot g(x) + f(x)\cdot\frac{d}{dx}g(x)$$

Example 1: Find the derivative of $h(x) = (4x^3+5x^2)(x^2-1)$

Assume $f(x) = 4x^3+5x^2$, $g(x) = x^2-1$

then $f'(x) = 12x^2+10x$, $g'(x) = 2x$

$$\Rightarrow h'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$
$$= (12x^2+10x)\cdot(x^2-1) + (4x^3+5x^2)\cdot 2x$$
$$= 12x^4+10x^3-12x^2-10x+8x^4+10x^3$$
$$= 20x^4+20x^3-12x^2-10x$$

Check the result by expanding $h(x)$ first and taking the derivative

$h(x) = (4x^3+5x^2)(x^2-1) = 4x^5+5x^4-4x^3-5x^2$

$\Rightarrow h'(x) = 20x^4+20x^3-12x^2-10x$

* ~~Simply~~ Simplify your final answer as much as possible
* Compute the derivative by the rule that you are most comfortable with, unless the problem explicitly requires the particular method to use.

• Quotient Rule: If $f(x)$ and $g(x)$ are differentiable in the domain of interest and $g(x)\neq 0$

then $$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Proof: $h(x) = \frac{f(x)}{g(x)} \Rightarrow f(x) = \cancel{f}\, g(x)\cdot h(x)$

$$\Rightarrow \underset{\text{know}}{f'(x)} = \underset{\text{know}}{g'(x)}\cdot \underset{\text{know}}{h(x)} + \underset{\text{unknown}}{h'(x)}\cdot \underset{\text{know}}{g(x)}$$

$$\Rightarrow h'(x) = \frac{f'(x) - h(x)\cdot g'(x)}{g(x)} = \frac{f'(x) - \frac{f(x)}{g(x)}\cdot g'(x)}{g(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Example 2: Find the derivative of $h(x) = x^{-n}$, n- positive integer

$h(x) = x^{-n} = \frac{1}{x^n}$, then assume $f(x)=1$, $g(x) = x^n$

$\Rightarrow f'(x) = 0$, $g'(x) = n \cdot x^{n-1}$

By Quotient Rule $h'(x) = \frac{0 \cdot x^n - 1 \cdot nx^{n-1}}{x^{2n}} = -nx^{-n-1} = (-n) \cdot x^{(-n)-1}$

* Power Rule holds for negative integer powers

Example 3: Find the derivative of $f(x) = \frac{4-3x}{3x^2+x}$

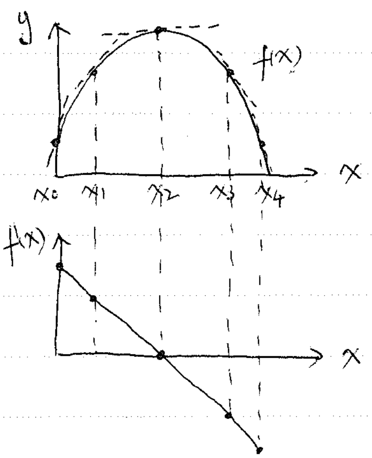
$$f'(x) = \frac{(-3)\cdot(3x^2+x) - (4-3x)\cdot(6x+1)}{(3x^2+x)^2} = \frac{9x^2-24x-4}{(3x^2+x)^2}$$

Example 4: Suppose $h(x) = \frac{8x^4}{g(x)}$, and we know $h(1)=4$ and $h'(1)=16$. Find $g(1)$ and $g'(1)$

$h(1) = \frac{8 \cdot 1^4}{g(1)} = 4 \Rightarrow g(1) = 2$

$$g(x) = \frac{8x^4}{h(x)} \Rightarrow g'(1) = \left.\frac{32x^3 \cdot h(x) - 8x^4 \cdot h'(x)}{[h(x)]^2}\right|_{x=1} = \frac{32\times4 - 8\times16}{16} = 0$$

- Sketch the derivative based on a given function (curve).



* start from $x=x_0$, the slope of at $f(x_0)$ is positive
* Compare the slope at $x=x_0$ and $x=x_1$, notice $f'(x_1) < f'(x_0)$ but also positive
* Notice at $x=x_2$, tangent line is a horizontal line $\Rightarrow f'(x_2)=0$
* At $x=x_3$, $f'(x_3)<0$
* Tangent line at $x=x_4$ is steeper than the one at $x=x_3$, $\Rightarrow f'(x_4) < f'(x_3) < 0$
* Connect all the points by smooth line
* Given $f(x) = -\frac{1}{2}gx^2 + v_0x + y_0$

  we have $f'(x) = -gx + v_0$ as a straight line