Today...

• Analyzing the studying experiment.
• Office hour poll - see email.
• Clicker questions on graphing simple polynomials.
• Hill functions.
• Introduction to the derivative.
• By now, you should have already done the Diagnostic, one or both of the PLs and started OSH1 (we’ll discuss how to write it up on Wed).
Interpreting the experiment

• Retrieval is critical for memory/learning.
• Reading notes or watching a lecture not as good as actively accessing.
• Interleaving versus blocking (did you notice the interleaving? see next point…).
• “Desirable difficulties” improve long term recall.
• If a method of learning feels easier - be skeptical that it’s better!
• Math is not a spectator sport!
Office hour poll

A. Monday 12 pm and Wednesday 1 pm
B. Monday 11 am and Wednesday 1 pm
C. Monday 1 pm and Wednesday 12 pm
D. Monday 11 am and Wednesday 12 pm
The following slides were not shown in class but are useful review...
Even and odd functions

• A function $f$ is called even if $f(-x) = f(x)$ for all values of $x$.

• A function $f$ is called odd if $f(-x) = -f(x)$ for all values of $x$.

• For power functions, even/odd-ness of the function is the same as even/odd-ness of the power.

• What about for polynomials?
Which function is odd?

A. \( f(x) = 2 \)
B. \( g(x) = x^2 - 3x^4 \)
C. \( h(x) = x + x^2 \)
D. \( k(x) = 3x + x^5 \)
Which function is odd?

A. $f(x) = 2$
B. $g(x) = x^2 - 3x^4$
C. $h(x) = x + x^2$
D. $k(x) = 3x + x^5$
Even or odd? \[ f(x) = \frac{x^n}{a^n + x^n} \].

A. \( f(x) \) is even when \( n \) is even and \( f(x) \) is odd when \( n \) is odd.

B. \( f(x) \) is even when \( n \) is odd and \( f(x) \) is odd when \( n \) is even.

C. \( f(x) \) is even when \( n \) is even and \( f(x) \) is neither even nor odd when \( n \) is odd.

D. \( f(x) \) is even for all values of \( n \).

E. \( f(x) \) is neither even nor odd for any value of \( n \).
Even or odd?  \[ f(x) = \frac{x^n}{a^n + x^n} \].

A. f(x) is even when \( n \) is even and f(x) is odd when \( n \) is odd.

B. f(x) is even when \( n \) is odd and f(x) is odd when \( n \) is even.

C. f(x) is even when \( n \) is even and f(x) is neither even nor odd when \( n \) is odd.

D. f(x) is even for all values of \( n \).

E. f(x) is neither even nor odd for any value of \( n \).
Which is the graph of the function

\[ g(x) = x^5 - x^2 \] ?

(A) 

(B) 

(C) 

(D)
Which is the graph of the function

\[ g(x) = x^5 - x^2 \]?
Which is the graph of the function 
\[ f(x) = x^5 - x^3 \]?
Which is the graph of the function 
\[ f(x) = x^5 - x^3 \]?

(A)  
(B)  
(C)  
(D)
Hill functions

\[ f(x) = \frac{ax^n}{bn + xn} \]

- A useful function for studying saturating phenomena.
- Important functions in biochemistry - Michaelis-Menten kinetics
- We will see these several times this semester.
If $|x| \ll b$, then

$$f(x) = \frac{ax^n}{b^n + x^n}$$

can be approximated by...

A. $a$

B. $\frac{a}{b^n}$

C. $a \left(\frac{x}{b}\right)^n$

(D) 0

(E) 1
If $|x| \ll b$, then

$$f(x) = \frac{ax^n}{bn + x^n}$$

can be approximated by...

A. $a$

B. $\frac{a}{b^n}$

C. $a \left( \frac{x}{b} \right)^n$

(D) 0

(E) 1
If $x \gg b$, then
\[ f(x) = \frac{ax^n}{bn + x^n} \]
can be approximated by...

A. $a$
B. $\frac{a}{b^n}$
C. $a \left(\frac{x}{b}\right)^n$

(D) 0
(E) 1

(assume $b>0$)
If $x \gg b$, then

$$f(x) = \frac{ax^n}{b^n + x^n}$$

can be approximated by...

A. $a$

B. $\frac{a}{b^n}$

C. $a \left(\frac{x}{b}\right)^n$

(D) 0

(E) 1

(assume $b>0$)
Implications for graphing

\[ f(x) = \frac{ax^n}{bn + x^n} \]

Why always below the asymptote?

We'll talk about filling in the rest later in the semester.
Comparing Hill functions with different $n$ values

(A) Green: $n=2$, yellow: $n=3$, red: $n=4$, blue: $n=5$.

(B) Green: $n=4$, yellow: $n=3$, red: $n=2$, blue: $n=1$.

(C) Green: $n=5$, yellow: $n=4$, red: $n=3$, blue: $n=2$.

(D) Either (B) or (C) (not enough info).

$$f(x) = \frac{ax^n}{b^n + x^n}$$
Comparing Hill functions with different n values

(A) Green: n=2, yellow: n=3, red: n=4, blue: n=5.

(B) Green: n=4, yellow: n=3, red: n=2, blue: n=1.

(C) Green: n=5, yellow: n=4, red: n=3, blue: n=2.

(D) Either (B) or (C) (not enough info).

\[ f(x) = \frac{ax^n}{b^n + x^n} \]
What is the slope of the line connecting the points?

(A) \( m = \frac{x_1 - x_2}{y_1 - y_2} \)

(B) \( m = \frac{x_2 - x_1}{y_1 - y_2} \)

(C) \( m = \frac{y_1 - y_2}{x_1 - x_2} \)

(D) \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
What is the slope of the line connecting the points?

(A) \( m = \frac{x_1 - x_2}{y_1 - y_2} \)

(B) \( m = \frac{x_2 - x_1}{y_1 - y_2} \)

(C) \( m = \frac{y_1 - y_2}{x_1 - x_2} \)

(D) \( m = \frac{y_2 - y_1}{x_2 - x_1} \)
What is the slope of the secant line to the graph of \( f(x) \)?

(A) \( m=\frac{f(x_1)-f(x_2)}{x_2-x_1} \)

(B) \( m=\frac{f(x_2)-f(x_1)}{x_2-x_1} \)

(C) \( m=\frac{x_1-x_2}{f(x_1)-f(x_2)} \)

(D) \( m=\frac{x_2-x_1}{f(x_1)-f(x_2)} \)

Slope of secant line = average rate of change from \( x_1 \) to \( x_2 \).
What is the slope of the secant line to the graph of f(x)?

(A) \( m = \frac{f(x_1) - f(x_2)}{x_2 - x_1} \)

(B) \( m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \)

(C) \( m = \frac{x_1 - x_2}{f(x_1) - f(x_2)} \)

(D) \( m = \frac{x_2 - x_1}{f(x_1) - f(x_2)} \)

Slope of secant line = average rate of change from \( x_1 \) to \( x_2 \).
What if you want the rate of change AT $x_1$?

Take a point $x_2$ so that the secant line is closer to the “secant line” AT $x_1$.

Alternate notation: let $x_2 = x_1 + h$ so that

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x_1 + h) - f(x_1)}{x_1 + h - x_1}$$
If we take $h$ values closer and closer to 0...

- The secant line approaches the **tangent line**.
- The slope of the secant line approaches the slope of the tangent line.
- We call the resulting slope **the derivative at** $x_1$.
- We now have to learn how to take **limits**!

\[
\text{slope at } x_1 = f'(x_1) = \lim_{h \to 0} \frac{f(x_1 + h) - f(x_1)}{h}
\]