# Today

- Note about Hill functions: $\frac{x^n}{k^n + x^n} = 1 - \frac{k^n}{k^n + x^n}$
- Shape of graphs
- Intro to optimization

$g(x) = 12x^3 - 12x^2$ has...

(A) a maximum at x=0 and a minimum at x=1/3.

(B) a minimum at x=0 and a maximum at x=1/3.

(C) a maximum at x=0 and an inflection pt at x=1/3.

(D) an inflection pt at x=0 and a minimum at x=1/3.

# $g(x) = 12x^3 - 12x^2$ has...

(A) a maximum at $x=0$ and a minimum at $x=1/3$.

(B) a minimum at $x=0$ and a maximum at $x=1/3$.

(C) a maximum at $x=0$ and an inflection pt at $x=1/3$.

(D) an inflection pt at $x=0$ and a minimum at $x=1/3$.

$g(x) = 12x^2(x-1)$, also has a zero at $x=1$.

$f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at x=0 and a minimum at x=1.

(B) a minimum at x=0 and a maximum at x=1.

(C) a maximum at x=0 and an inflection pt at x=1.

(D) an inflection pt at x=0 and a minimum at x=1.

# $f(x) = 3x^4 - 4x^3$ has...

(A) a maximum at x=0 and a minimum at x=1.

(B) a minimum at x=0 and a maximum at x=1.

(C) a maximum at x=0 and an inflection pt at x=1.

(D) an inflection pt at x=0 and a minimum at x=1.

$f'(x) = g(x) = 12x^2(x-1)$ has a max at x=0 and a zero at x=1 with $f'(1^-) < 0$ and $f'(1^+) > 0$.

# $f(x) = 3x^4 - 4x^3$

- $f'(x) = 12\,(x^3 - x^2) = 0$ --> $x=0$, $x=1$.
- $f''(x) = 12\,(3x^2 - 2x)$.
- SDT: $f''(1) = 1 > 0$

  --> $f'(x)$ is increasing near $x=1$.

  --> $f'(x)$ goes from $-$ to 0 to $+$ near $x=1$.

  --> $f(x)$ has a minimum at $x=1$.
- SDT: $f''(0) = 0$ --> Min/max? Inflection point?

Could also do FDT:
$f'(0^{+/-})$

# Is $x=0$ an inflection point of $f(x) = 3x^4-4x^3$?

(A) Yes because $f''(0) = 0$.

(B) Yes because $f''(0) = 0$ and $f'''(0) < 0$.

(C) No because $f''(-1) = 60$ and $f''(1) = 12$.

(D) Yes because $f''(-1) = 60$ and $f''(1/2) = -3$.

Note: $f'(x) = g(x) = 12x^3-12x^2$ from earlier and we agreed that $g(x)$ had a max at $x=0$!

# Is x=0 an inflection point of $f(x) = 3x^4-4x^3$?

(A) Yes because $f''(0) = 0$.

"SDT" applied to $f'(x)$.

(B) Yes because $f''(0) = 0$ and $f'''(0) < 0$.

Jumped over a zero of $f''(x)$

(C) No because $f''(-1) = 60$ and $f''(1) = 12$.

"FDT" applied to $f'(x)$.

(D) Yes because $f''(-1) = 60$ and $f''(1/2) = -3$.

| x | $(-\infty,0)$ | 0 | $(0,2/3)$ | 2/3 | $(2/3,\infty)$ |
|---|---|---|---|---|---|
| $f''(x)$ | + | 0 | − | 0 | + |

$f'''(0)<0$

# A table for sketching the graph of

$$f(x) = 3x^4-4x^3 = x^3(3x-4)$$

$$f'(x) = 12(x^3-x^2) = 12x^2(x-1)$$

$$f''(x) = 12(3x^2-2x) = 12x(3x-2)$$

| x | $(-\infty,0)$ | 0 | $(0,2/3)$ | 2/3 | $(2/3,\infty)$ |
|---|---|---|---|---|---|
| $f''(x)$ | + | 0 | − | 0 | + |

# The whole table

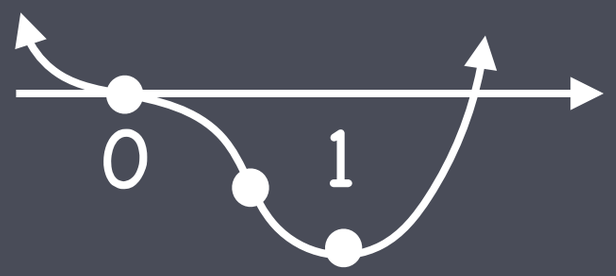
| x | (−∞,0) | 0 IP | (0,2/3) | 2/3 IP | (2/3,1) | 1 min | (1,4/3) | 4/3 | (4/3,∞) |
|---|---|---|---|---|---|---|---|---|---|
| f(x) | + | 0 | − | − | − | − | − | 0 | + |
| f′(x) | − | 0 | − | − | − | 0 | + | + | + |
| f″(x) | + | 0 | − | 0 | + | + | + | + | + |

Not a min/max

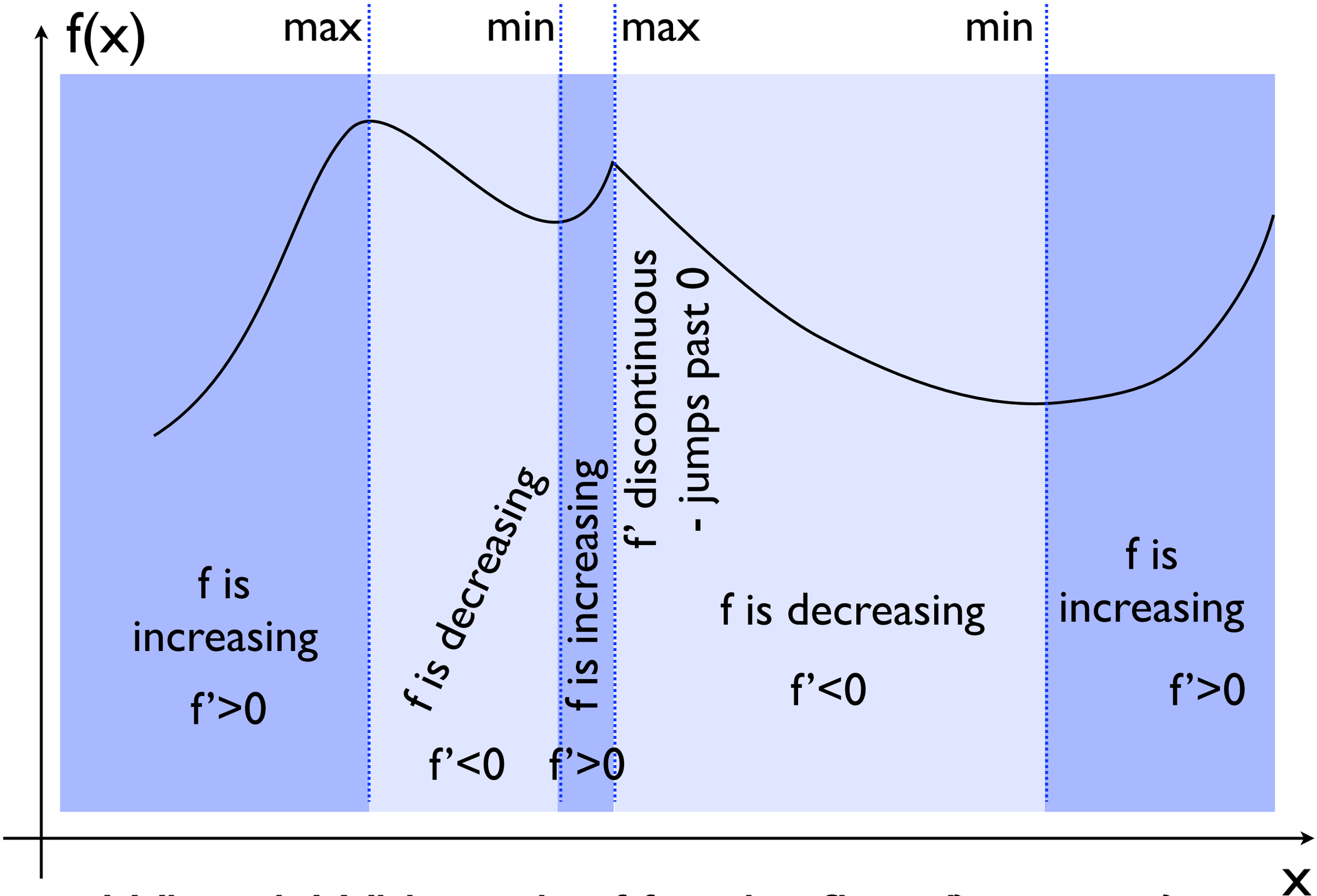
minimum

inflection inflection point

$f(x) = 2 - 4x^3$



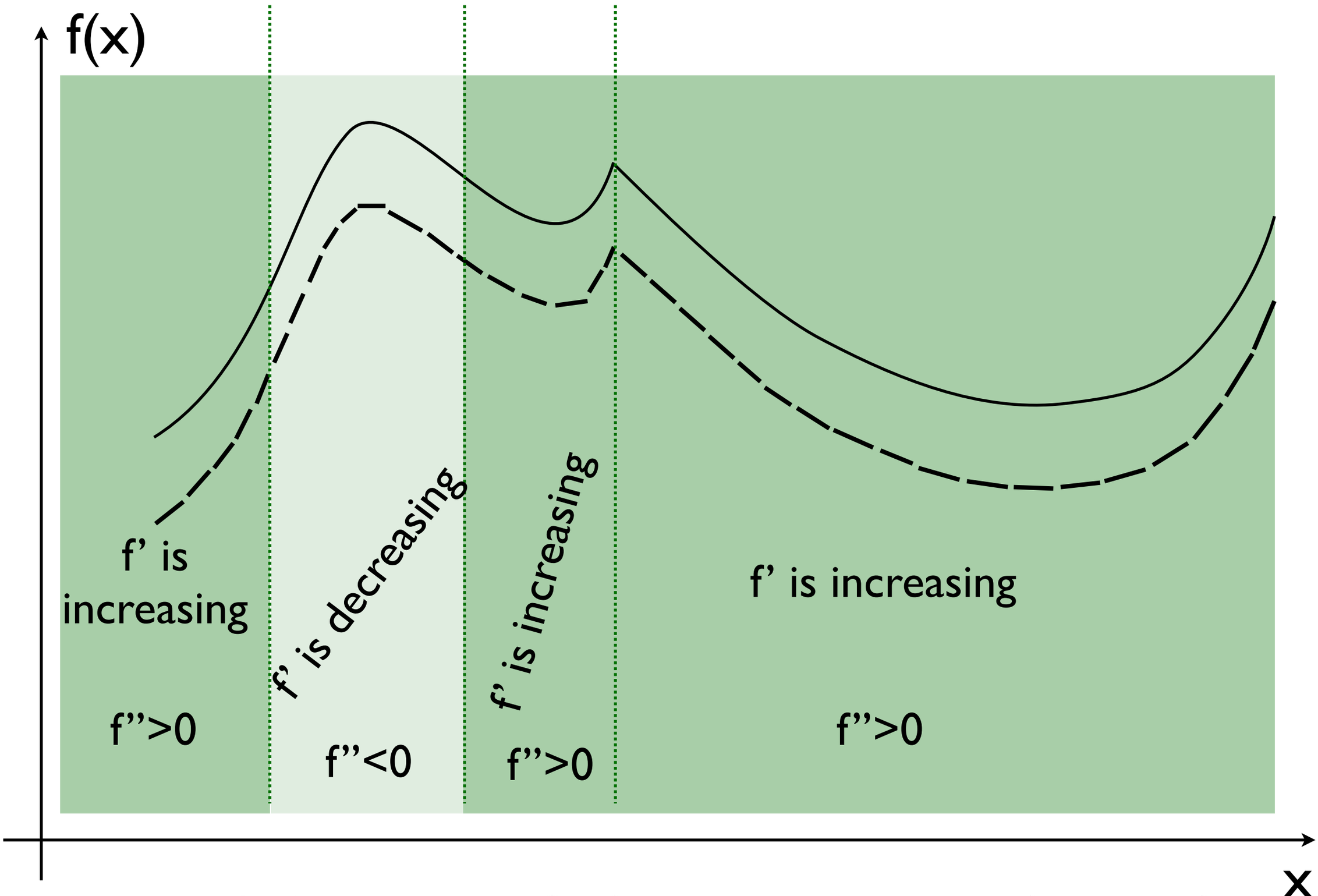
# Annotating the graph of f(x) with f'(x) info




What does that mean for the original function?

What does that mean for the first derivative?

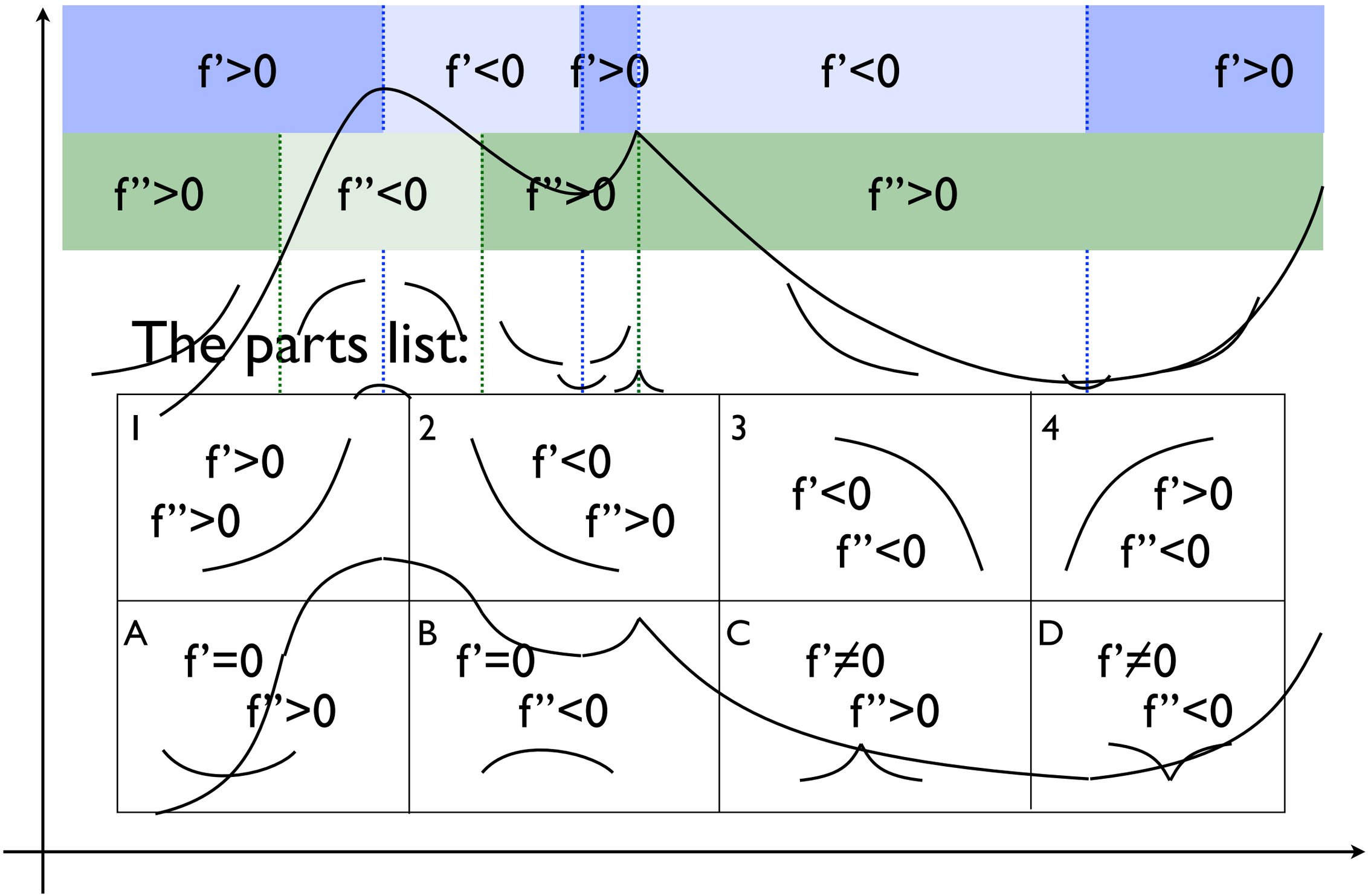
# Annotating the graph of f(x) with f''(x) info




What does this mean for the second derivative?

What is the meaning of the second derivative?

# From table to graph



# From table to graph