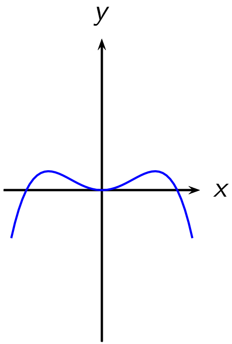
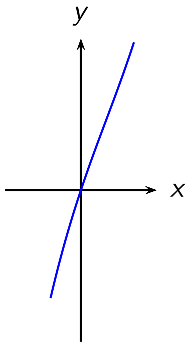
Lecture 2: Ch. 1, Question 8

Which of the following four graphs is the graph of the function

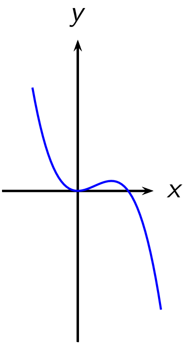
$$f(x) = 7x^5 - 3x^2?$$



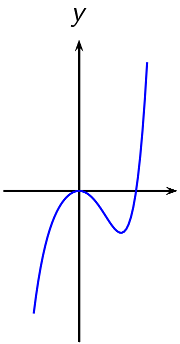
(A)



(B)



(C)



(D)

Lecture 2: Ch. 1, Question 11

Let f be the function

$$f(x) = \frac{4x^4 - 5.5x^2 + x}{3x^7 - 5x^2}.$$

Then $f(0.02)$ is approximately:

(A) 3 (B) -10 (C) -0.0008 (D) $\frac{5.5}{5}$

Lecture 3: Ch. 2, Question 7

Which of the following limits do not exist?

(A) $\lim_{x \to -2} \frac{x^2-4}{x-2}$

(B) $\lim_{x \to 2} \frac{x^2-4}{x-2}$

(C) $\lim_{x \to 2} \frac{x+2}{x-2}$

(D) More than one.

(E) All three exist.

Lecture 3: Ch. 2, Question 10

For which number a is the function

$$f(x) = \begin{cases} ax + 1 & , x > 1 \\ -3 & , x \leq 1 \end{cases}$$

continuous?

(A) -4

(B) -3

(C) 3

(D) For some other value of a.

(E) For no number a.

Lecture 3: Related exam problem

Let $f(x) = \sqrt{x}$. Calculate $f'(x)$ from the definition of the derivative at x. Hint:

$$\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}(\sqrt{a}+\sqrt{b}) = \frac{a-b}{\sqrt{a}+\sqrt{b}}$$

Lecture 4: Ch. 3, Question 4

For any function g, answer the following questions with (A) true or (B) false.

a) If g is continuous at a, then g has a tangent at a.

Lecture 4: Ch. 3, Question 4

For any function g, answer the following questions with (A) true or (B) false.

a) If g is continuous at a, then g has a tangent at a.

b) If g has a tangent at a, then g is continuous at a.

Lecture 4: Ch. 3, Question 6

For any function g, answer the following questions with (A) true or (B) false.

a) A tangent of g can intersect the graph of g in several points.

Lecture 4: Ch. 3, Question 6

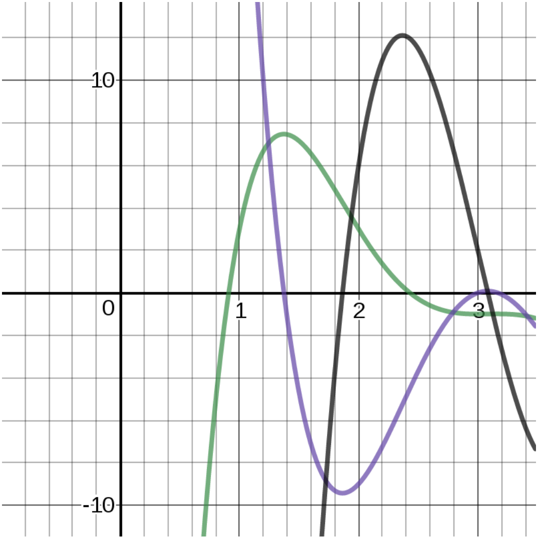
For any function g, answer the following questions with (A) true or (B) false.

a) A tangent of g can intersect the graph of g in several points.

b) Suppose that g is increasing. A tangent of g has to stay on the same side of the graph of g.

Lecture 4: Related exam problem

The following three graphs represent position $x(t)$, velocity $v(t)$ and acceleration $a(t)$. Label them.



Lecture 5: Ch. 4, Question 5

Suppose that the energy expenditure of a fish swimming at velocity v is $E(v) = \alpha(\frac{1}{v} + \beta v)$, where $\alpha, \beta > 0$ are constant. The derivative of E with respect to v is

(A) $\alpha(v + \beta)$

(B) $\alpha(\frac{1}{v^2} + \beta)$

(C) $\alpha(-\frac{1}{v^2} + \beta v)$

(D) $\alpha(-\frac{1}{v^2} + \beta)$

(E) $\alpha(1 + \beta)$

Lecture 5: Related exam problem

Suppose that

$$f(x) = (3 + x^2)g(x)$$

and that

$$g(2) = 2, \quad g'(2) = -1.$$

Compute $f'(2)$.

Lecture 6: Ch. 4, Question 7

Let $f(x) = 3x - 2$, $g(x) = -2x + 1$ and $h(x) = g(f(x))$. Which of the following statements is true?

(A) $h(x) = -6x + 5$

(B) $h(x) = -6x^2 + 7x - 2$

(C) $h(x)$ is a quadratic polynomial

(D) More than one of the above.

(E) None of the above.

Lecture 6: Ch. 4, Question 15

Lying on a 10 meter platform, you are throwing down a ball. The ball leaves your hand with a downwards speed of $3m/s$.
Which function $y(t)$ describes the height of the ball in meters at time t in seconds?

(A) $y(t) = -\frac{9.81}{2}\frac{m}{s^2}t^2 + 10m$

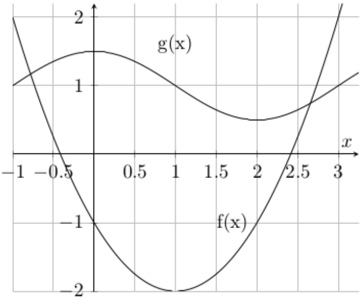
(B) $y(t) = -\frac{9.81}{2}\frac{m}{s^2}t^2 - 3\frac{m}{s}t + 10m$

(C) $y(t) = -\frac{9.81}{2}\frac{m}{s^2}t^2 + 3\frac{m}{s}t + 10m$

(D) $y(t) = \frac{9.81}{2}\frac{m}{s^2}t^2 + 3\frac{m}{s}t + 10m$

Lecture 6: Related exam problem

5. (2 pts) Consider the function $h(x) = f(g(x))$ where the graphs of f and g are shown in the figure below.



Circle the letter next to the true statement.

(a) $h'(0.5) > 0$.

(b) $h'(1.5) > 0$.

(c) $h'(2.5) = 0$.

(d) None of the above.

Lecture 7: Ch. 5, Question 3

Let $f(x) = x^3$ and $x_0 \neq 0$. We use linear approximation to estimate $f(x)$ for x close to x_0. Which of the following statements is true?

(A) Our approximation overestimates $f(x)$.

(B) Our approximation is an overestimate if $x_0 > 0$.

(C) Our approximation is an underestimate if $x_0 < 0$.

(D) More than one is true.

(E) None is true.

Lecture 7: Ch. 5, Question 4

Using linear approximation to estimate $\sqrt{99}$, you would calculate the tangent line at:

(A) 100

(B) 99

(C) 10

(D) 1

(E) 0

Lecture 7: Ch. 5, Question 5

Using linear approximation to estimate $\sqrt{99}$, the equation of the tangent line at 100 yields

(A) an overestimate.

(B) an underestimate.

(C) I don't know.

Lecture 7: Related exam problem

Consider the function $f(x) = \frac{3}{x-2}$. At which points $(a, f(a))$ does the graph of this function have a tangent line parallel to the line $y = -x$? What is the equation of the tangent line at each of these points?

Lecture 8: Ch. 5, Question 8

Which of the following statements is true?

(A) Newton's method can fail because the derivative at some x_n can be zero.

(B) Newton's method never fails.

(C) If Newton's method converges, then it finds the zero closest to the initial guess x_0.

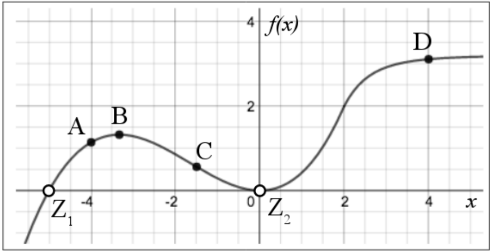
(D) Newton's method can fail because the calculated approximations x_n can start repeating themselves in a loop.

(E) More than one of the above.

Lecture 8: Related exam problem

[4 pts] Consider each of the labeled points (solid dot) on the graph of $f(x)$ as a starting point for Newton's method. To which zero of the function $f(x)$ (empty dots Z_1, Z_2, or neither) will Newton's method converge for each one? You may assume that the graph of the function continues off the edges of the graph with no significant change in direction.

Initial point	zero (Z_1, Z_2, or neither)
A	
B	
C	
D	



Lecture 9: Ch. 6, Question 13

Suppose that the second derivative of a certain function f can be written in the factored form $f''(x) = x^2(1-x)(x-2)^2(x^2-9)$. Then f has inflection points at

(A) $x = -3, 0, 1, 2, 3$

(B) $x = 0, 1, 2, 3$ only

(C) $x = 1$ only

(D) $x = 1, 3$ only

(E) $x = -3, 1, 3$ only

Lecture 9: Related exam problem

Let $f(x)$ be a continuous function at $x = 0$. Which of the following statements is correct? Recall that if there exists any function for which a statement is false then the statement is false.

(A) If $f'(0)$ exists and $f'(0) = 0$ then $x = 0$ is an extremum.

(B) if $x = 0$ is an extremum then $f'(0)$ exists and $f'(0) = 0$.

(C) If $f'(x)$ exists for all x and $f'(0) = 0$, $f'(1) > 0$, and $f'(-1) < 0$ then $x = 0$ is a minimum.

(D) If $f'(0)$ and $f''(0)$ exist and $f'(0) = f''(0) = 0$ then $x = 0$ is not an extremum.

(E) None of the above are true.

Lecture 10: Ch. 7, Question 1

Suppose that the energy expenditure of a fish swimming at velocity v is $E(v) = \alpha(\frac{1}{v} + \beta v)$, where $\alpha, \beta > 0$ are constant. At which velocity does the fish minimize its energy expenditure?

(A) $v = \frac{1}{\sqrt{\beta}}$

(B) $v = \sqrt{\beta}$

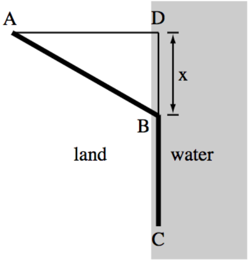
(C) $v = \alpha\sqrt{\beta}$

(D) $v = \alpha\beta$

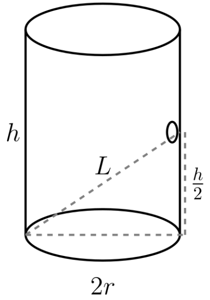
(E) $v = \beta$

Lecture 10: Related exam problem

C.4 [6 pts.] Shown in the figure below is the view from above of the path taken by a penguin from point A to a feeding area on the shore at point C. The penguin must choose the point B toward which it starts walking. It takes twice as much energy per unit distance for a penguin to walk over land (AB) as to swim through water (BC). The distance AD is 300 m and the distance DC is 400 m. Calculate the value of the distance x (and hence the location of the point B - see figure) that minimizes the energy spent on the entire trip.



Lecture 11: Ch. 7, Question 5 (wine for Kepler's wedding)



Kepler wants to find out for which shape of the barrel he can buy the most wine. Assume that the cost is proportional to the length L measuring the wet portion of the rod. What are possible strategies?

(A) Maximize the volume for fixed L with the constraint $V = hr^2\pi$.

(B) Maximize the volume for fixed L with the constraint $L^2 = (\frac{h}{2})^2 + (2r)^2$.

(C) Minimize L for fixed volume V with the constraint $V = hr^2\pi$.

(D) Minimize L for fixed volume V with the constraint $L^2 = (\frac{h}{2})^2 + (2r)^2$.

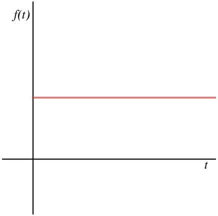
(E) More than one of the above.

Lecture 11: Related exam problem

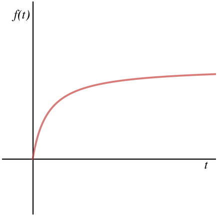
An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? The energy gained is proportional to the time in the food patch.



(A)



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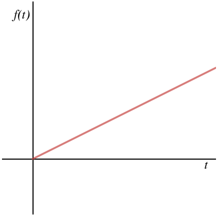

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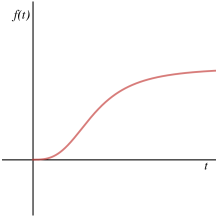

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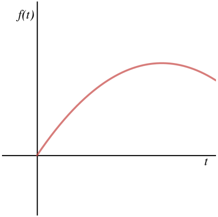
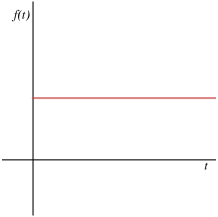


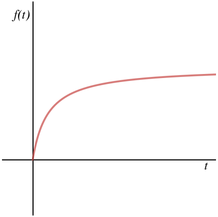
(E)

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? The energy gained is proportional to the time in the food patch. e.g. bear salmon fishing



(A)



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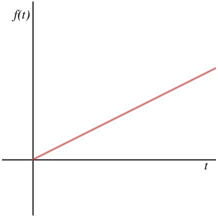

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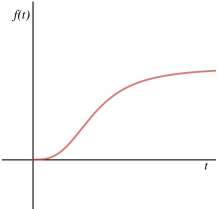

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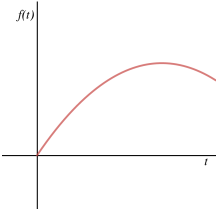
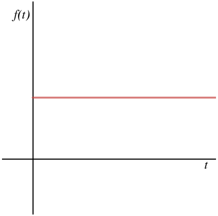


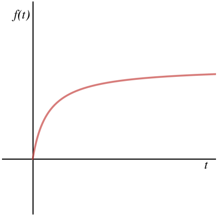
(E)

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? At first I gain energy rapidly, but then I slow down.



(A)



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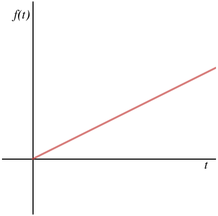

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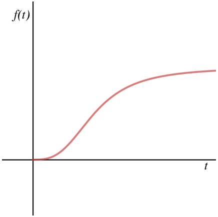

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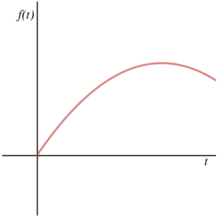
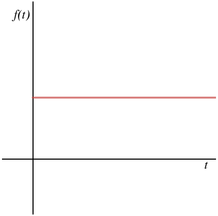


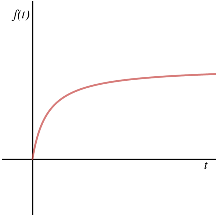
(E)

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? At first I gain energy rapidly, but then I slow down. e.g. bear eating berries



(A)



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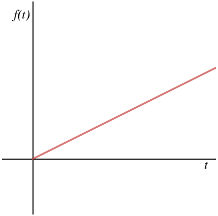

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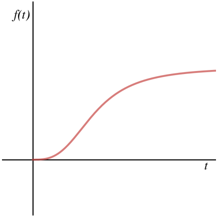
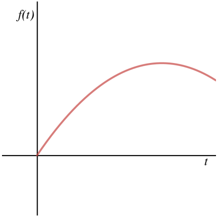


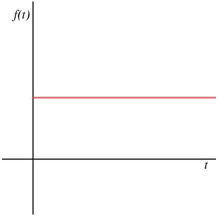
(D)



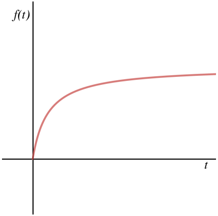
(E)

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? All the energy is gained right at the beginning.



(A)



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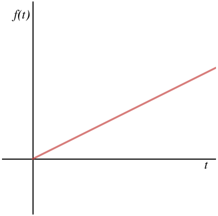

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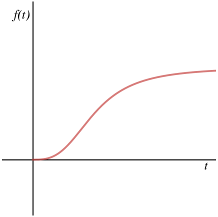

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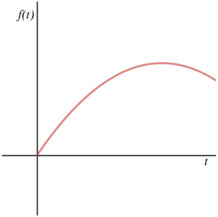
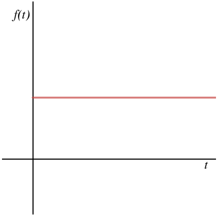


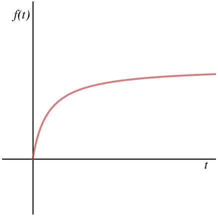
(E)

Lecture 12: Ch. 7, Question 6

Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? All the energy is gained right at the beginning. e.g. fox eating eggs



(A)



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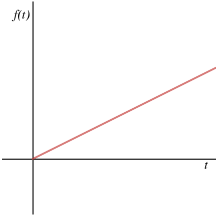

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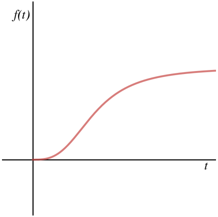

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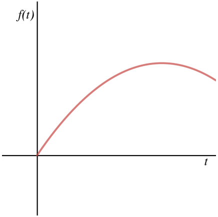
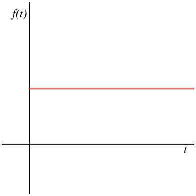


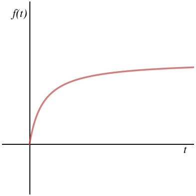
(E)

Lecture 12: Ch. 7, Question 6

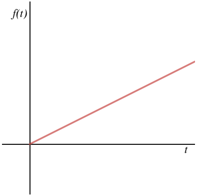
Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? At first it is difficult to reach the food, then I gain energy rapidly and afterwards I slow down.



(A)



(B)



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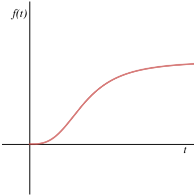

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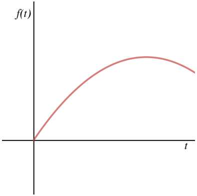
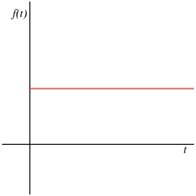


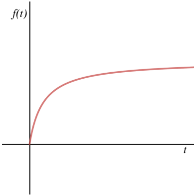
(E)

Lecture 12: Ch. 7, Question 6

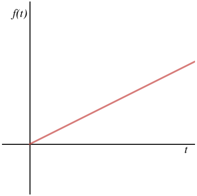
Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? At first it is difficult to reach the food, then I gain energy rapidly and afterwards I slow down. e.g. bear eating ant larvae from colonies in logs



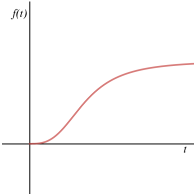
(A)



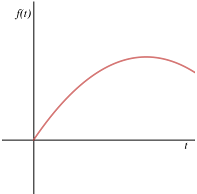
(B)



(C)



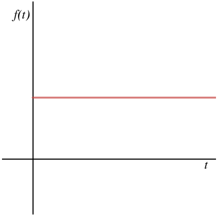
(D)



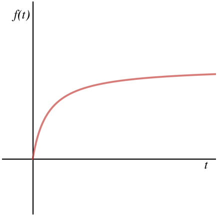
(E)

Lecture 12: Ch. 7, Question 6

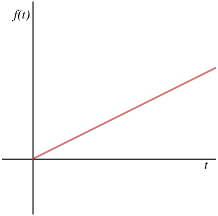
Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? After some gain, I lose energy.



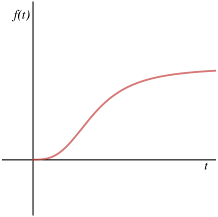
(A)



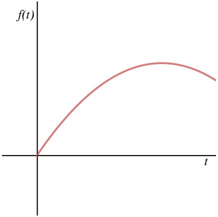
(B)



(C)



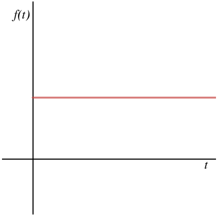
(D)



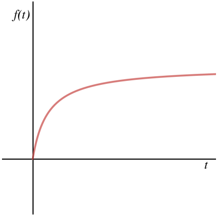
(E)

Lecture 12: Ch. 7, Question 6

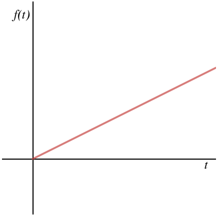
Let $f(t)$ denote the energy gained up to time t in a food patch. Which graph represents the following food patch? After some gain, I lose energy. e.g. booby gets its fish stolen by a frigatebird



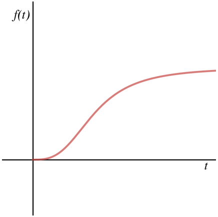
(A)



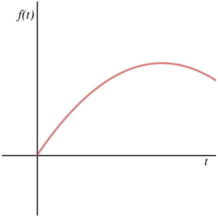
(B)



(C)



(D)



(E)

Lecture 12: Related exam problem

OSH 4, part 1: Optimal length of shifts for medical residents.

Lecture 13: Ch. 8, Question 1

Suppose that f is a differentiable function. What is the derivative of $(f(x))^2 - 7f(x)$?

(A) $2f(x) - 7$

(B) $2f'(x) - 7f'(x)$

(C) $(f'(x))^2 - 7f'(x)$

(D) $2f(x)f'(x) - 7f'(x)$

(E) $2f(x) - 7f'(x)$

Lecture 13: Ch. 8, Question 2

Suppose that f is a differentiable function. What is the derivative of $f(x^2 - 7x)$?

(A) $f'(x^2 - 7x)$

(B) $f'(x)(2x - 7)$

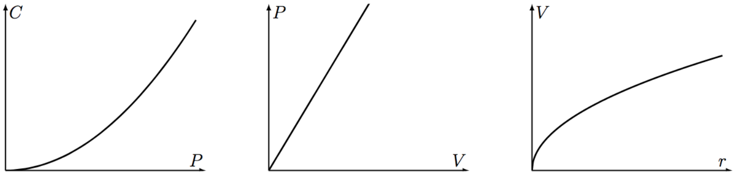
(C) $f(x^2 - 7x)(2x - 7)$

(D) $f'(2x - 7)$

(E) $f'(x^2 - 7x)(2x - 7)$

Lecture 13: Ch. 8, Question 3

Population of carnivores C, prey P, and vegetation $V(r)$ are given by



,

where r denotes the rainfall. What is the rate of change $\frac{\mathrm{d}C}{\mathrm{d}r}$ of the carnivore population with respect to the rain?

(A) $C'(P(V(r)))P'(V(r))V'(r)$

(B) $C'(P(V(r)))P'(V(r))$

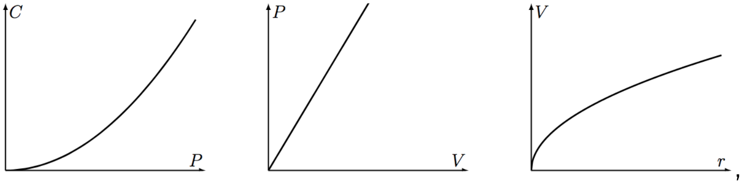
(C) $\frac{\mathrm{d}C}{\mathrm{d}P}\frac{\mathrm{d}P}{\mathrm{d}V}\frac{\mathrm{d}V}{\mathrm{d}r}$

(D) $\frac{\mathrm{d}C}{\mathrm{d}P}\frac{\mathrm{d}P}{\mathrm{d}V}$

(E) More than one of the above.

Lecture 13: Ch. 8, Question 4

Population of carnivores C, prey P, and vegetation $V(r)$ are given by



where r denotes the rainfall. What's $\left.\frac{\mathrm{d}C}{\mathrm{d}r}\right|_{r=9}$ if $V = \sqrt{r}$, $P = 2V$ and $\left.\frac{\mathrm{d}C}{\mathrm{d}P}\right|_{P=6} = 12$?

(A) 12

(B) 4

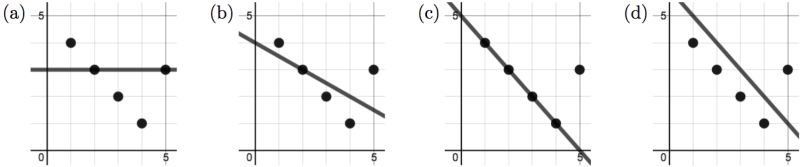
(C) 6

(D) 3

(E) Not enough information.

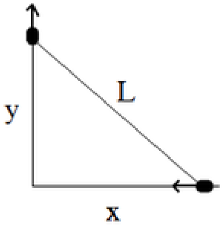
Lecture 13: Related exam problem

5. Which of the following lines $y = ax + b$ provides the best fit to the data in the least squares sense?



Lecture 14: Ch. 9, Question 2

In the figure shown here, there are two people walking away or towards the street corner. The distances of the individuals from the corner at time t are $x(t)$ and $y(t)$. The distance L is then



(A) $L = x + y$

(B) $L = y/x$

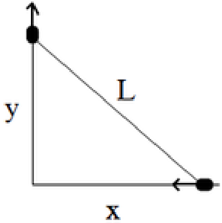
(C) $L = x/y$

(D) $L = \sqrt{x + y}$

(E) $L = \sqrt{x^2 + y^2}$

Lecture 14: Ch. 9, Question 2

If in this diagram one person walks towards the corner at the rate $1\mathrm{m/s}$ and the other walks away at rate $2\mathrm{m/s}$ then in $\mathrm{m/s}$:



(A) $\frac{dx}{dt} = 1, \frac{dy}{dt} = 2$

(B) $\frac{dx}{dt} = -1, \frac{dy}{dt} = 2$

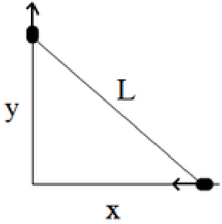
(C) $\frac{dx}{dt} = 1, \frac{dy}{dt} = -2$

(D) $\frac{dx}{dt} = 2, \frac{dy}{dt} = 1$

(E) $\frac{dx}{dt} = 2, \frac{dy}{dt} = -1$

Lecture 14: Ch. 9, Question 2

If one person walks towards the corner at the rate $1\mathrm{m/s}$ and the other walks away at rate $2\mathrm{m/s}$ then how fast (in $\mathrm{m/s}$) is L changing at the instant when $x = y = 10\mathrm{m}$?



(A) 2

(B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{3}{\sqrt{2}}$

(E) 30

Lecture 14: Related exam problem

A squirrel sitting $6\mathrm{m}$ up in a tree is watching a coyote walk past the tree. The squirrel measures the angle formed between a vertical line directly below her and the line connecting her and the coyote and finds that it is changing at a rate of $1/12$ radians per second when the coyote is $8\mathrm{m}$ away from the base of the tree. How fast is the coyote walking?

Lecture 15: Ch. 9, Question 5

Consider the hyperbola with equation

$$y^2 - x^2 = 1.$$

Implicit differentiation with respect to x yields the equation

(A) $2y - 2x = 0$

(B) $-2x = 0$

(C) $2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2x = 1$

(D) $2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2x = 0$

(E) None of the above.

Lecture 15: Ch. 9, Question 5

Consider the hyperbola with equation

$$y^2 - x^2 = 1.$$

What is the best approximation to the slope of the tangent at (x, y) with $x = 1010010001$ and $y < 0$?

(A) -2

(B) -1

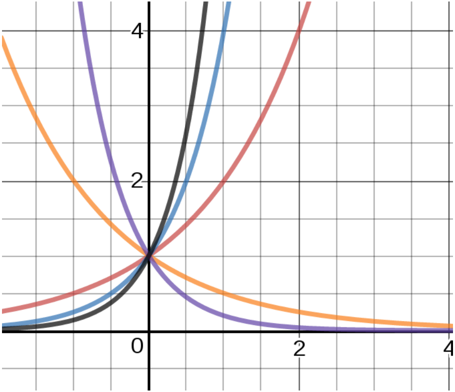
(C) 0

(D) 1

(E) 2.

Lecture 15: Ch. 10, Question 2

The derivative of the exponential function $f(x) = a^x$ is $f'(x) = C_a \cdot a^x$. One of the following exponential functions has $C_a \approx 1.386$. Which one?



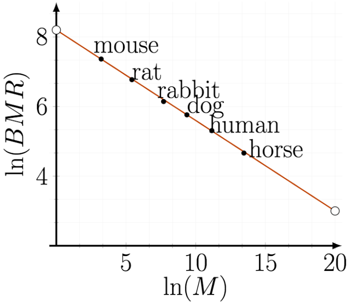
(A) red (B) blue (C) orange (D) purple (E) black

Lecture 15: Related exam problem

Find the tangent line of $y^2 = e^{(x^2)} + 5x$ at the point $(0, 1)$.

Lecture 16: Ch. 10, Question 8

animal	**body weight** M (gm)	**basal metabolic rate** (BMR)
mouse	25	1580
rat	226	873
rabbit	2200	466
dog	11700	318
man	70000	202
horse	700000	106



The red line passes through the points $(0, 8.2)$, $(20, 3)$. Which equation provides the best description of how BMR relates to M?

(A) $BMR = \frac{20}{-5.2} M + 8.2$

(B) $\ln(BMR) = \frac{20}{-5.2} \ln(M) + 8.2$

(C) $BMR = \frac{-5.2}{20} M + 8.2$

(D) $\ln(BMR) = \frac{-5.2}{20} \ln(M) + 8.2$

(E) $\ln(BMR) = \frac{-5.2}{20} M + 8.2$

Lecture 16: Ch. 10, Question 8

animal	body weight M (gm)	basal metabolic rate (BMR)
mouse	25	1580
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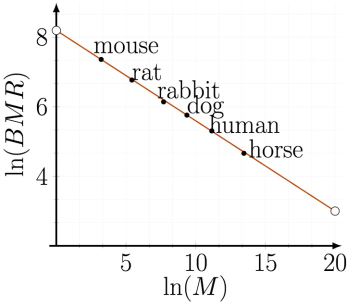


The red line passes through the points $(0, 8.2)$, $(20, 3)$. As a function of M, the basal betabolic rate BMR is best described as

(A) $M^{-5.2} - 20$

(B) $\frac{-5.2}{20} \ln(M)$

(C) a power function with exponent $\frac{-5.2}{20}$

(D) an exponential function with base $\frac{-5.2}{20}$

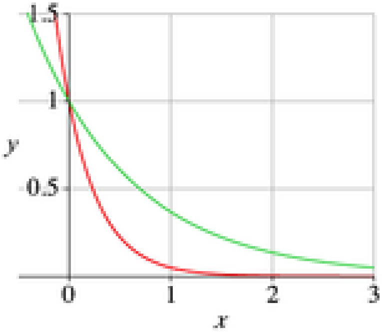
(E) $M \cdot e^{\frac{-5.2}{20}}$

Lecture 16: Related exam problem

Suppose that $f(x) = \ln(e^{g(x)} + x)$ and that $g(2) = 3$, $g'(2) = 5$. Compute $f'(2)$.

Lecture 17: Ch. 11, Question 2

If the green curve is the function $y = e^{-x}$ then the red curve could



be

(A) $y = 3e^{-x}$

(B) $y = e^{-3x}$

(C) $y = e^{-x/3}$

(D) $y = \frac{1}{3}e^{-x}$

(E) $y = e^{-10x}$

Lecture 17: Ch. 11, Question 4

Suppose a radioactive atom has probability k per unit time of emitting a particle and decaying. Suppose there are $N(t_0)$ atoms at time t_0. Then the number of remaining radioactive atoms that have not yet decayed at time $t_0 + h$ is approximately

(A) $N(t_0 + h) = khN(t_0)$

(B) $N(t_0 + h) = N(t_0) - khN(t_0)$

(C) $N(t_0 + h) = N(t_0) + khN(t_0)$

(D) $N(t_0 + h) = -k(t_0 + h)$

(E) $N(t_0 + h) = kh$

Lecture 17: Ch. 11, Question 5

Given the equation for the number of radioactive atoms remaining at time $t_0 + h$ that we found in the previous problem, $N(t_0 + h) = N(t_0) - khN(t_0)$, we arrive at the differential equation for decay by

(A) Taking a derivative of both sides with respect to t

(B) Taking a derivative of both sides with respect to N

(C) Subtracting $N(t_0)$ from both sides, dividing by h, and letting $h \to 0$

(D) Factoring out and then dividing both sides by $N(t_0)$

(E) None of the above

Lecture 17: Related exam problem

In the Chernobyl reactor explosion, which occurred on April 26, 1986, substantial amounts of the isotope strontium-90 ($^{90}\mathrm{Sr}$) contaminated the area around the nuclear plant. $^{90}\mathrm{Sr}$ decays at a rate proportional to its quantity. $^{90}\mathrm{Sr}$ has a half-life of 29 years; that is, it takes 29 years for a quantity of $^{90}\mathrm{Sr}$ to decrease by half. What is the proportion of $^{90}\mathrm{Sr}$ originally released which remains on April 26, 2012?

Lecture 18: Ch. 13, Question 1

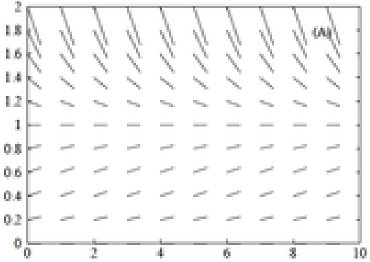
Which of the following functions is a solution to the differential equation

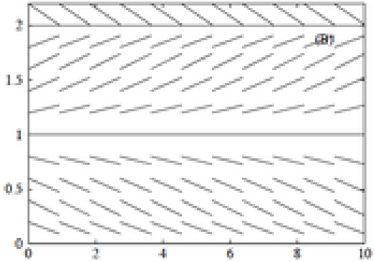
$$y'(t) = (y(t))^2?$$

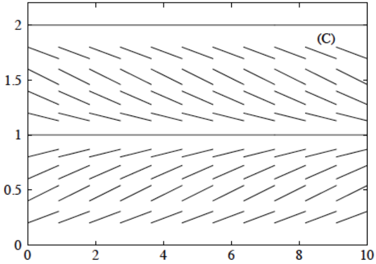
(A) $y(t) = 2^t$
(B) $y(t) = e^t$
(C) $y(t) = t^{-1}$
(D) $y(t) = -\frac{1}{t}$
(E) $y(t) = t^2$

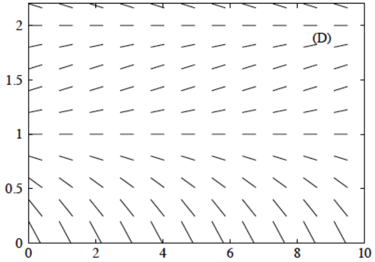
Lecture 18: Ch. 13, Question 2

Which slope field corresponds to the DE $y' = ry(1-y)(y-2)$, $r > 0$?



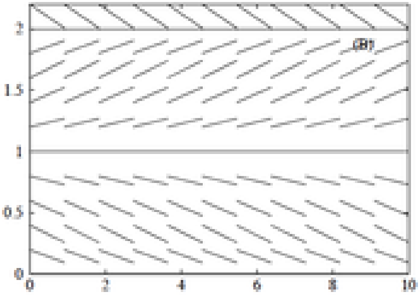






Lecture 18: Ch. 13, Question 3

Which steady states of the differential equation
$y' = ry(1-y)(y-2), r > 0$, are stable and which are unstable?



(A) Stable: 0,1; Unstable: 2
(B) Stable: 1,2; Unstable: 0
(C) Stable: 2; Unstable: 0,1
(D) Stable: 1; Unstable: 0,2
(E) Stable: 0,2; Unstable: 1

Lecture 18: Related exam problem

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{y^2 - 1}{y^2 + 1}.$$

a) Find all stable and unstable steady states, if any, for this differential equation.

b) What value will the solution of this differential equation with initial condition $y(0) = 0$ approach as t increases?

Lecture 19: Ch. 13, Question 5

Let $y(x)$ be the solution of the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2(y - a), \quad y(0) = 2a,$$

where $a > 0$ is a constant. Then

(A) $\lim_{x\to\infty} y(x) = 0$

(B) $\lim_{x\to\infty} y(x) = \infty$

(C) $\lim_{x\to\infty} y(x) = a$

(D) $\lim_{x\to\infty} y(x) = 2a$

(E) $\lim_{x\to\infty} y(x) = -\infty$

Lecture 19: Related exam problem

MC 2 A rumour started by one student spreads through UBC according to the model

$$I' = 10I - 0.5I^2,$$

in which $I(t)$ denotes the number of students who have heard the rumour by time t. Which of the following is the best estimate for the number of students who eventually will hear the rumour?

☐ 10 ☐ 200 ☐ 0.5 ☐ 20

Lecture 20: Ch. 12, Question 2

Consider the initial value problem

$$y' = 7 - by, \quad y(0) = y_0.$$

For what values of b and y_0 is $\lim_{t\to\infty} y(t) = \infty$?

(A) $b < 0$, $y_0 > 7/b$

(B) $b < 0$, $y_0 < 7/b$

(C) $b > 0$, $y_0 > 7/b$

(D) $b > 0$, $y_0 < 7/b$

(E) None of the above.

Lecture 20: Ch. 12, Question 3

Which of the following differential equations expresses Newton's law of cooling with constant $k > 0$?

(A) $E = k(T' - T)$

(B) $E = k(T' - T)$

(C) $T' = kT - E$

(D) $T' = k(T - E)$

(E) $T' = k(E - T)$

Lecture 20: Related exam problem

7. Rainbow Trout in Deer Lake can no longer reproduce due to habitat destruction. On November 1st when the population is estimated at 800 fish, locals start stocking the lake with fish at a constant rate of 100 fish per day and fish them out at a rate proportional to their population size. In the options below, $r > 0$. Which of the following equations describes the size of the fish population?

   (a) $F(t) = 100t - 800e^{-rt}$

   (b) $\dfrac{dF}{dt} = (100 - r)F$

   (c) $F(t) = 100/r + (800 - 100/r)e^{-rt}$

   (d) $\dfrac{dF}{dt} = r(100 - F)$

   (e) $\dfrac{dF}{dt} = 100t - rF$

Lecture 22: Ch. 13, Question 6

What are the steady states of

$$I' = \beta(N - I)I - \mu I?$$

(A) 0 and $N - \mu$

(B) 0 and $\beta N - \mu$

(C) 0 and $N - \mu/\beta$

(D) 0 and N

(E) None of the above.

Lecture 22: Related exam problem

5. The model given below on the left has been suggested for the spread of HIV within the immune system of an infected person. $C(t)$ is the density of healthy immune cells, $I(t)$ is the density of HIV-infected immune cells and $V(t)$ is the density of virus in the blood of a patient. Which of the options on the right gives a correct interpretation of some part of the model?

$$\frac{dC}{dt} = P - \alpha CV - \gamma_1 C$$

$$\frac{dI}{dt} = \alpha CV - \gamma_2 I$$

$$\frac{dV}{dt} = \beta I - \gamma_3 V$$

(a) Healthy cells can become infected when they encounter infected cells.

(b) Healthy cells can become infected when they encounter virus.

(c) Virus is produced at a rate proportional to the current viral density.

(d) Infected cells die at a rate proportional to the viral density.

(e) Virus is killed/removed at a rate proportional to the density of healthy immune cells.

Lecture 23: Ch. 14, Question 6

Which of the following functions approximates the annual variation in daylight per day in Vancouver with $t = 0$ at March 21?

(A) $L(t) = 12 + 4\cos(2\pi t/365)$

(B) $L(t) = 12 - 4\cos(\pi t/365)$

(C) $L(t) = 12 + 4\sin(2\pi t/365)$

(D) $L(t) = 12 + 4\sin(\pi t/365)$

Lecture 23: Related exam problem

The daily temperature in Vancouver during the month of March varies from an average low of $2^\circ C$ to an average high of $10^\circ C$. It is coolest just before sunrise which is roughly at 7 AM. Construct a function (using cosine) that provides a good description of the temperature $T(t)$ throughout a day in March where t is measured in hours from 12 AM.

Lecture 24: Ch. 15, Question 1

What is the derivative of $(x^2e^{-x} + \tan(3x))^2$?

(A) $2(x^2e^{-x} + \tan(3x))$

(B) $2(2xe^{-x} - x^2e^{-x} + 3\sec^2(3x))$

(C) $2(2xe^{-x} - x^2e^{-x} + 3\sec^2(3x))(x^2e^{-x} + \tan(3x))$

(D) $2(2xe^{-x} + 3\sec^2(3x))(x^2e^{-x} + \tan(3x))$

(E) $2(-2xe^{-x} + 3\sec^2(3x))(x^2e^{-x} + \tan(3x))$

Lecture 24: Ch. 15, Question 2

For which of the following differential equations is $\sin(t)$ not a solution?

(A) $y''(t) = -\sin(t)$

(B) $y''(t) = -y(t)$

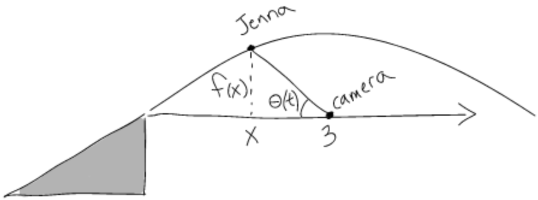
(C) $y'(t) = \cos(t)$

(D) $y'(t) = y(t + \pi/2)$

(E) $y'''(t) = \cos(t)$

Lecture 24: Related exam problem

Jenna is riding off a snowboarding ramp while her father takes a video of her flight. The ramp tilts up with a slope of $1/2$. Her father stands 3 meters in front of the ramp with his camera pointed at the ramp and held at the same level as the top of the ramp. The angular elevation of the camera, $\theta(t)$, starts at 0 and increases with time so that the camera follows Jenna. When Jenna is a horizontal distance x from the ramp, she is a vertical distance $f(x)$ above the top end of the ramp. She is moving horizontally at a speed of 2 m/s as she leaves the ramp. How rapidly must Jenna's father be rotating the camera so as to follow her flight path just as she leaves the ramp ($x = 0$)? Note that $f(0) = 0$ and $f'(0) = 1/2$.



Lecture 25: Ch. 15, Question 4

Let
$$f(x) = \arccos(\ln(3x)).$$

Which of the following equations does not hold?

(A) $f'(x) = \frac{1}{-x\sqrt{1-\ln(3x)^2}}$

(B) $f'(1) = \frac{1}{-\sqrt{1-\ln(3)^2}}$

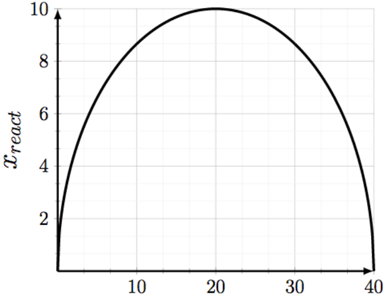
(C) $f'(1/3) = -3$

(D) $f'(2/3) = \frac{3}{-2\sqrt{1-\ln(2)^2}}$

(E) All of the above hold.

Lecture 25: Ch. 15, Question 5

Which of the following is the independent variable?



(A) S

(B) K_{crit}

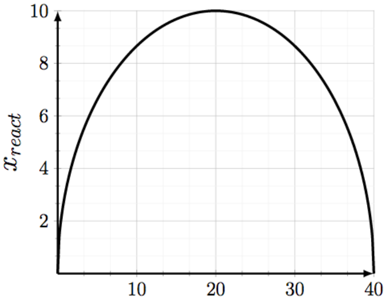
(C) α

(D) v

(E) α'

Lecture 25: Ch. 15, Question 6

Reaction distance x_{react} versus predator size S



How would you interpret this graph?

(A) The larger the predator, the faster the reaction of the prey.

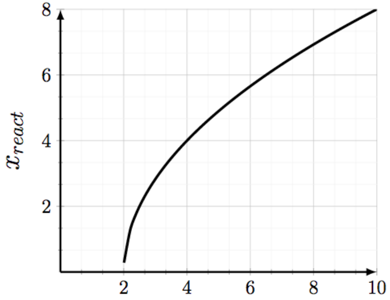
(B) The larger the predator, the larger the reaction distance.

(C) Predators that are very large may not be noticed.

(D) The predator has an optimal size if it is to catch the prey.

Lecture 25: Ch. 15, Question 7

Which of the following is the independent variable?



(A) S

(B) K_{crit}

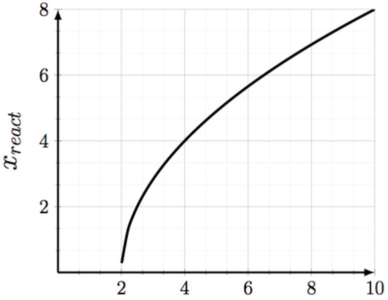
(C) α

(D) v

(E) α'

Lecture 25: Ch. 15, Question 8

Reaction distance x_{react} versus speed v



How would you interpret this graph?

(A) If the prey has a large velocity, it can get away faster.

(B) The faster the predator, the further away it is detected.

(C) If the predator is very slow, the prey can always get away.

(D) If the prey has small velocity, it can not react.