Which of the following four graphs is the graph of the function 

\[ f(x) = 7x^5 - 3x^2 \]?
Let $f$ be the function 

$$f(x) = \frac{4x^4 - 5.5x^2 + x}{3x^7 - 5x^2}.$$ 

Then $f(0.02)$ is approximately:

(A) 3 (B) $-10$ (C) $-0.0008$ (D) $\frac{5.5}{5}$
Which of the following limits do not exist?

(A) \( \lim_{x \to -2} \frac{x^2-4}{x-2} \)

(B) \( \lim_{x \to 2} \frac{x^2-4}{x-2} \)

(C) \( \lim_{x \to 2} \frac{x+2}{x-2} \)

(D) More than one.

(E) All three exist.
For which number $a$ is the function
\[ f(x) = \begin{cases} \ ax + 1 & , x > 1 \\ -3 & , x \leq 1 \end{cases} \]
continuous?

(A) $-4$
(B) $-3$
(C) 3
(D) For some other value of $a$.
(E) For no number $a$. 
Let \( f(x) = \sqrt{x} \). Calculate \( f'(x) \) from the definition of the derivative at \( x \). Hint:

\[
\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} (\sqrt{a} + \sqrt{b}) = \frac{a - b}{\sqrt{a} + \sqrt{b}}
\]
For any function $g$, answer the following questions with (A) true or (B) false.

a) If $g$ is continuous at $a$, then $g$ has a tangent at $a$. 
For any function $g$, answer the following questions with (A) true or (B) false.

a) If $g$ is continuous at $a$, then $g$ has a tangent at $a$.

b) If $g$ has a tangent at $a$, then $g$ is continuous at $a$. 
For any function $g$, answer the following questions with (A) true or (B) false.

a) A tangent of $g$ can intersect the graph of $g$ in several points.
For any function $g$, answer the following questions with (A) true or (B) false.

a) A tangent of $g$ can intersect the graph of $g$ in several points.

b) Suppose that $g$ is increasing. A tangent of $g$ has to stay on the same side of the graph of $g$. 
Lecture 4: Related exam problem

The following three graphs represent position $x(t)$, velocity $v(t)$ and acceleration $a(t)$. Label them.
Suppose that the energy expenditure of a fish swimming at velocity $v$ is $E(v) = \alpha\left(\frac{1}{v} + \beta v\right)$, where $\alpha, \beta > 0$ are constant. The derivative of $E$ with respect to $v$ is

(A) $\alpha(v + \beta)$
(B) $\alpha\left(\frac{1}{\sqrt{v}} + \beta\right)$
(C) $\alpha\left(-\frac{1}{\sqrt{v}} + \beta v\right)$
(D) $\alpha\left(-\frac{1}{\sqrt{v}} + \beta\right)$
(E) $\alpha(1 + \beta)$
Lecture 5: Related exam problem

Suppose that

\[ f(x) = (3 + x^2)g(x) \]

and that

\[ g(2) = 2, \quad g'(2) = -1. \]

Compute \( f'(2) \).
Let \( f(x) = 3x - 2 \), \( g(x) = -2x + 1 \) and \( h(x) = g(f(x)) \). Which of the following statements is true?

(A) \( h(x) = -6x + 5 \)
(B) \( h(x) = -6x^2 + 7x - 2 \)
(C) \( h(x) \) is a quadratic polynomial
(D) More than one of the above.
(E) None of the above.
Lying on a 10 meter platform, you are throwing down a ball. The ball leaves your hand with a downwards speed of $3\text{m/s}$.

Which function $y(t)$ describes the height of the ball in meters at time $t$ in seconds?

(A) $y(t) = -\frac{9.81}{2} \frac{m}{s^2} t^2 + 10m$

(B) $y(t) = -\frac{9.81}{2} \frac{m}{s^2} t^2 - 3\frac{m}{s} t + 10m$

(C) $y(t) = -\frac{9.81}{2} \frac{m}{s^2} t^2 + 3\frac{m}{s} t + 10m$

(D) $y(t) = \frac{9.81}{2} \frac{m}{s^2} t^2 + 3\frac{m}{s} t + 10m$
5. (2 pts) Consider the function \( h(x) = f(g(x)) \) where the graphs of \( f \) and \( g \) are shown in the figure below.

Circle the letter next to the true statement.

(a) \( h'(0.5) > 0 \).
(b) \( h'(1.5) > 0 \).
(c) \( h'(2.5) = 0 \).
(d) None of the above.
Let $f(x) = x^3$ and $x_0 \neq 0$. We use linear approximation to estimate $f(x)$ for $x$ close to $x_0$. Which of the following statements is true?

(A) Our approximation overestimates $f(x)$.
(B) Our approximation is an overestimate if $x_0 > 0$.
(C) Our approximation is an underestimate if $x_0 < 0$.
(D) More than one is true.
(E) None is true.
Using linear approximation to estimate $\sqrt{99}$, you would calculate the tangent line at:

(A) 100
(B) 99
(C) 10
(D) 1
(E) 0
Using linear approximation to estimate $\sqrt{99}$, the equation of the tangent line at 100 yields

(A) an overestimate.

(B) an underestimate.

(C) I don’t know.
Consider the function $f(x) = \frac{3}{x-2}$. At which points $(a, f(a))$ does the graph of this function have a tangent line parallel to the line $y = -x$? What is the equation of the tangent line at each of these points?
Which of the following statements is true?

(A) Newton’s method can fail because the derivative at some $x_n$ can be zero.

(B) Newton’s method never fails.

(C) If Newton’s method converges, then it finds the zero closest to the initial guess $x_0$.

(D) Newton’s method can fail because the calculated approximations $x_n$ can start repeating themselves in a loop.

(E) More than one of the above.
[4 pts] Consider each of the labeled points (solid dot) on the graph of \( f(x) \) as a starting point for Newton’s method. To which zero of the function \( f(x) \) (empty dots \( Z_1, Z_2 \), or neither) will Newton’s method converge for each one? You may assume that the graph of the function continues off the edges of the graph with no significant change in direction.

<table>
<thead>
<tr>
<th>Initial point</th>
<th>zero ( (Z_1, Z_2, \text{ or neither}) )</th>
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<tbody>
<tr>
<td>A</td>
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<tr>
<td>B</td>
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<td>D</td>
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Suppose that the second derivative of a certain function $f$ can be written in the factored form $f''(x) = x^2(1 - x)(x - 2)^2(x^2 - 9)$. Then $f$ has inflection points at

(A) $x = -3, 0, 1, 2, 3$
(B) $x = 0, 1, 2, 3$ only
(C) $x = 1$ only
(D) $x = 1, 3$ only
(E) $x = -3, 1, 3$ only
Let $f(x)$ be a continuous function at $x = 0$. Which of the following statements is correct? Recall that if there exists any function for which a statement is false then the statement is false.

(A) If $f'(0)$ exists and $f'(0) = 0$ then $x = 0$ is an extremum.

(B) if $x = 0$ is an extremum then $f'(0)$ exists and $f'(0) = 0$.

(C) If $f'(x)$ exists for all $x$ and $f'(0) = 0$, $f'(1) > 0$, and $f'(-1) < 0$ then $x = 0$ is a minimum.

(D) If $f'(0)$ and $f''(0)$ exist and $f'(0) = f''(0) = 0$ then $x = 0$ is not an extremum.

(E) None of the above are true.
Suppose that the energy expenditure of a fish swimming at velocity \( v \) is \( E(v) = \alpha \left( \frac{1}{v} + \beta v \right) \), where \( \alpha, \beta > 0 \) are constant. At which velocity does the fish minimize its energy expenditure?

(A) \( v = \frac{1}{\sqrt{\beta}} \)
(B) \( v = \sqrt{\beta} \)
(C) \( v = \alpha \sqrt{\beta} \)
(D) \( v = \alpha \beta \)
(E) \( v = \beta \)
C.4 [6 pts.] Shown in the figure below is the view from above of the path taken by a penguin from point A to a feeding area on the shore at point C. The penguin must choose the point B toward which it starts walking. It takes twice as much energy per unit distance for a penguin to walk over land (AB) as to swim through water (BC). The distance AD is 300 m and the distance DC is 400 m. Calculate the value of the distance $x$ (and hence the location of the point B - see figure) that minimizes the energy spent on the entire trip.
Kepler wants to find out for which shape of the barrel he can buy the most wine. Assume that the cost is proportional to the length $L$ measuring the wet portion of the rod. What are possible strategies?

(A) Maximize the volume for fixed $L$ with the constraint $V = hr^2\pi$.

(B) Maximize the volume for fixed $L$ with the constraint $L^2 = \left(\frac{h}{2}\right)^2 + (2r)^2$.

(C) Minimize $L$ for fixed volume $V$ with the constraint $V = hr^2\pi$.

(D) Minimize $L$ for fixed volume $V$ with the constraint $L^2 = \left(\frac{h}{2}\right)^2 + (2r)^2$.

(E) More than one of the above.
An architect is designing a house in the form of a cylinder covered by a roof in the shape of half a sphere (extending above the cylinder). Suppose the material used to build the cylindrical wall is half the price of the material that is used to build the roof per unit area. If the total volume of the house is fixed, what ratio between the height of the wall and the radius of the roof will minimize the cost?