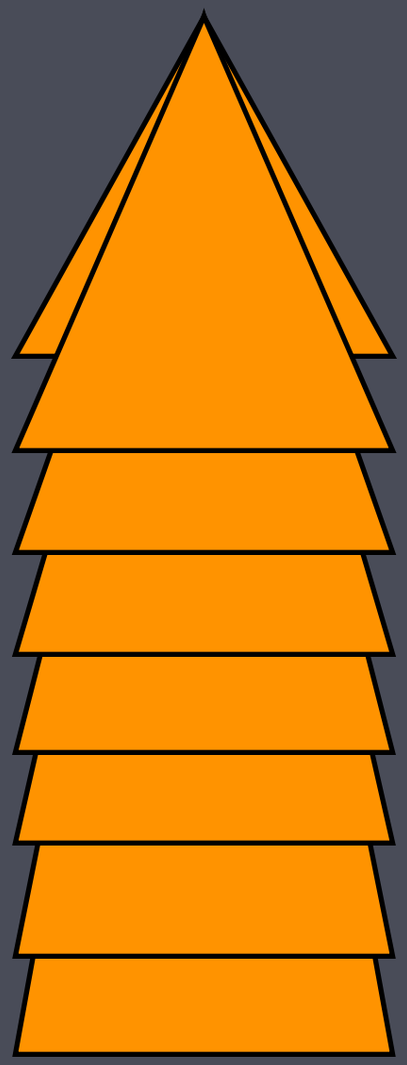
Today

- Related rates with trig
  - Zebra Danio
- Reminders:
  - Friday is the last day of classes...
  - ... and π pie day.
  - Exam: Dec 15 @ 3:30 pm - CIRS 1250

Trig-related rates

- These usually come down to a triangle that changes in time. For example...



If the height of an isosceles triangle with base 2m changes at a rate h′=3 m/s, how quickly is the angle opposite the base changing when h=sqrt(3) m?

Relate the two changing quantities (h and θ):

(A) $\sin(\theta) = 2/h$

(B) $\sin(\theta/2) = 1/h$

(C) $\sin(\theta/2) = 1/\mathrm{sqrt}(1+h^2)$

(D) $\tan(\theta) = 2/h$

(E) $\tan(\theta/2) = 1/h$

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This will get messy.

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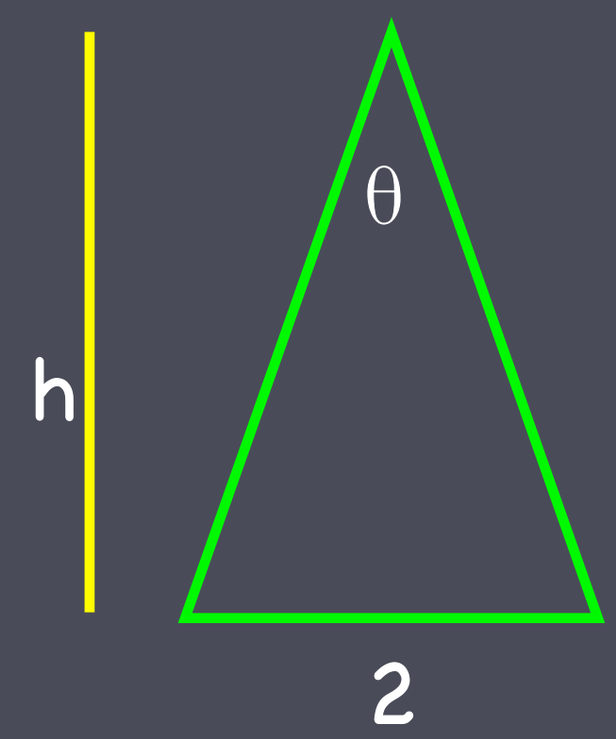
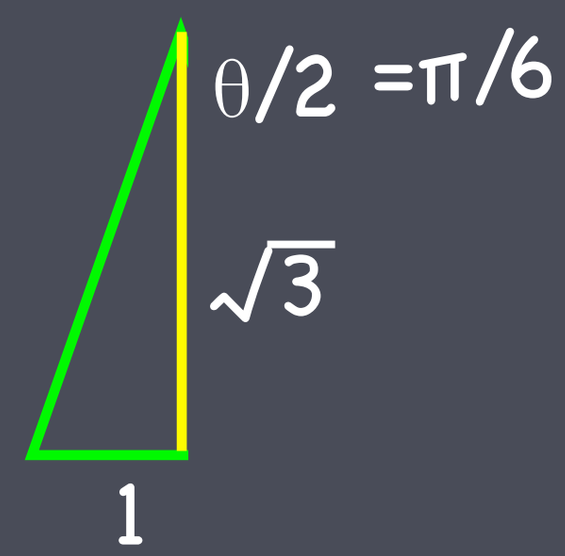


If the height of an isosceles triangle with base 2m changes at a rate $h'=3$ m/s, how quickly is the angle opposite the base changing when $h=\sqrt{3}$ m?

- Take derivatives to relate their rates of change (h' and θ'):



- $\tan(\theta/2) = 1/h$
- $\sec^2(\theta/2)\ \theta'/2 = -h'/h^2$
- $\theta' = -2\ h'/(h^2 \sec^2(\theta/2)) = -2\ h'\cos^2(\theta/2)\ /h^2$

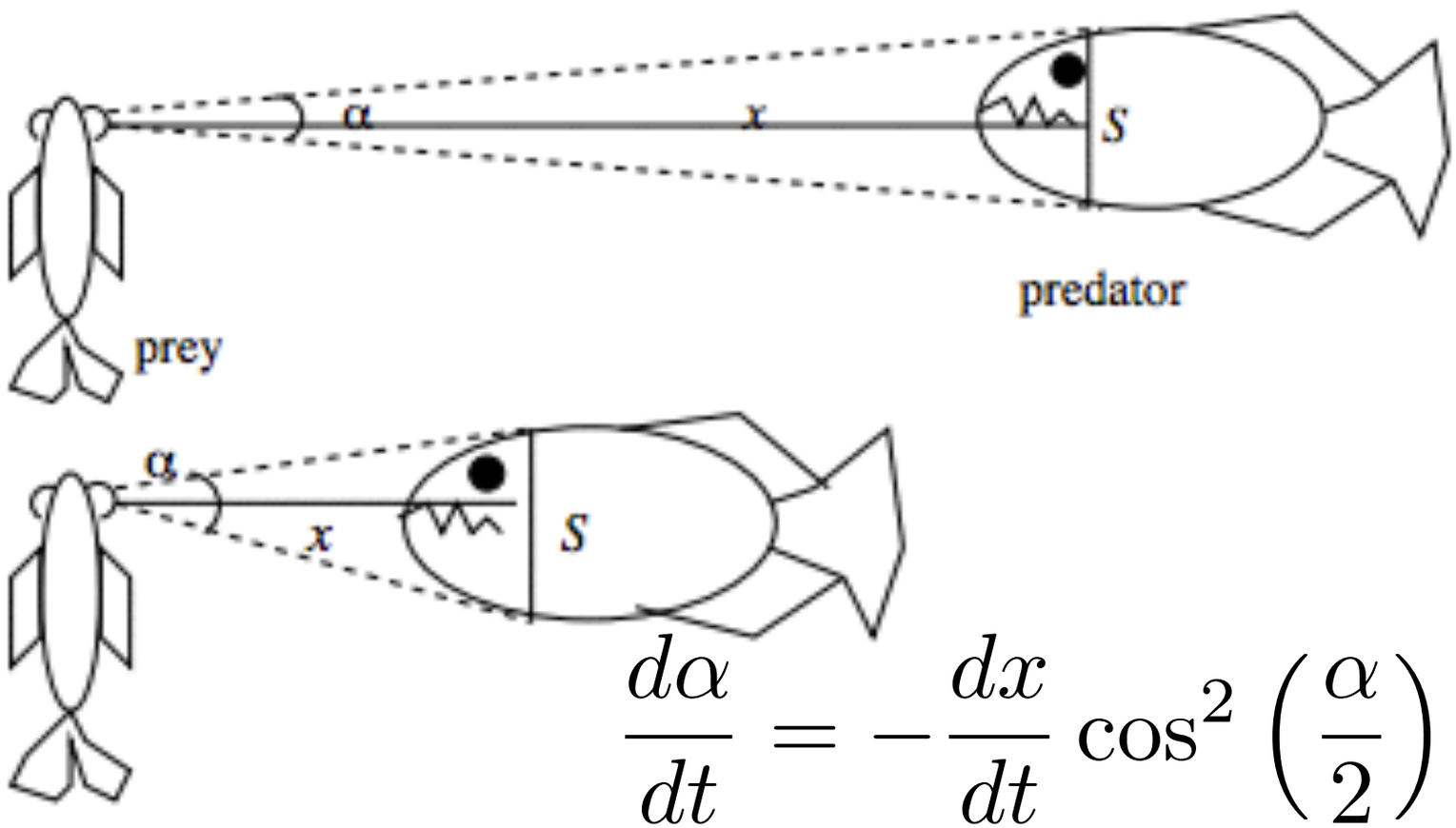
$= -2\cos^2(\theta/2) \quad = -3/2$ radians/s

$\theta = \ldots$ (A) $\pi/6$ (B) $\pi/4$ (C) $\pi/3$ (D) $2\pi/3$ (E) π.

Zebra Danio escape response

Zebra Danio escape response

10.9.1 The Zebra danio and its escape response



$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

ZD tries to escape when α' is above a threshold value.

What is $\cos^2(a)$ when $\tan(a)=p/q$?

(A) $(p^2+q^2) / q^2$

(B) $(p^2+q^2) / p^2$

(C) $p^2 / (p^2+q^2)$

(D) $q^2 / (p^2+q^2)$

(E) p^2/q^2

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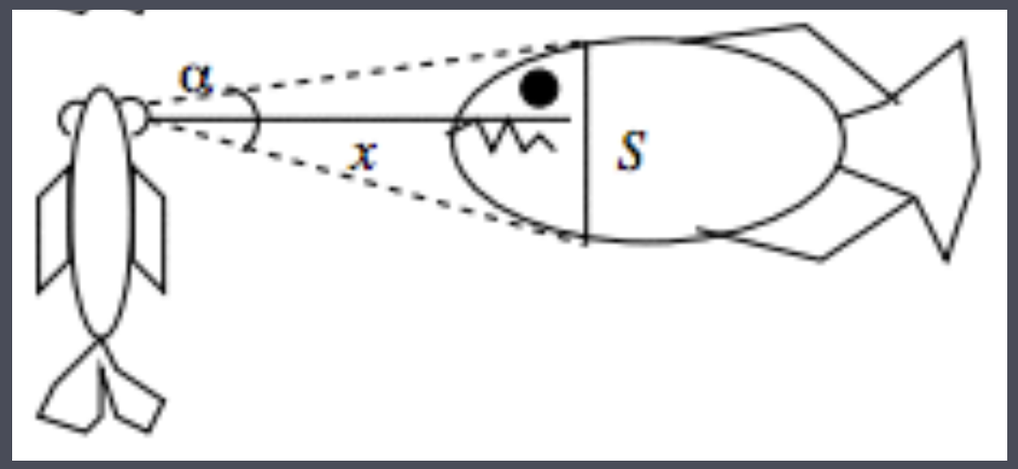

$$\frac{d\alpha}{dt} = -\frac{dx}{dt}\cos^2\left(\frac{\alpha}{2}\right)\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{x^2}{x^2+\frac{S^2}{4}}\frac{S}{x^2}$$

$$= -\frac{dx}{dt}\frac{S}{x^2+\frac{S^2}{4}} = v\frac{S}{x^2+\frac{S^2}{4}}$$

Assuming the Zebra Danio reacts to a rapidly changing optical angle α, it will try to escape from...

(A) ...a very large predator (large S).

(B) ...a very small predator (small S).

(C) ...a predator that is far away (large x).

(D) ...a slow-moving predator (small v).

(E) ...a fast-moving predator (large v).

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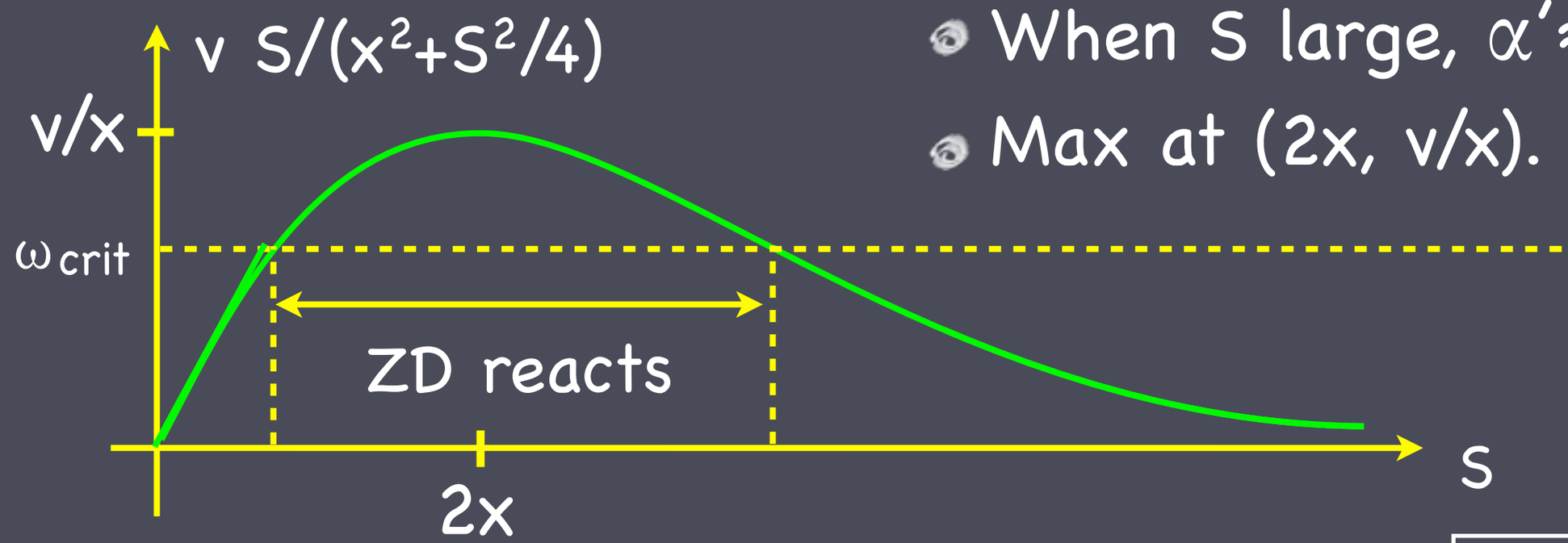
$$\frac{d\alpha}{dt} = v \frac{S}{x^2 + \frac{S^2}{4}}$$

If the ZD reacts when $\alpha' > \omega_{crit}$ then...

Hold predator distance x constant, plot

$\alpha' = v\ S/(x^2+S^2/4)$ as function of S.

- When S small, $\alpha' \approx S/x^2$.
- When S large, $\alpha' \approx 4v/S$.
- Max at (2x, v/x).



(A)	...a very large predator.
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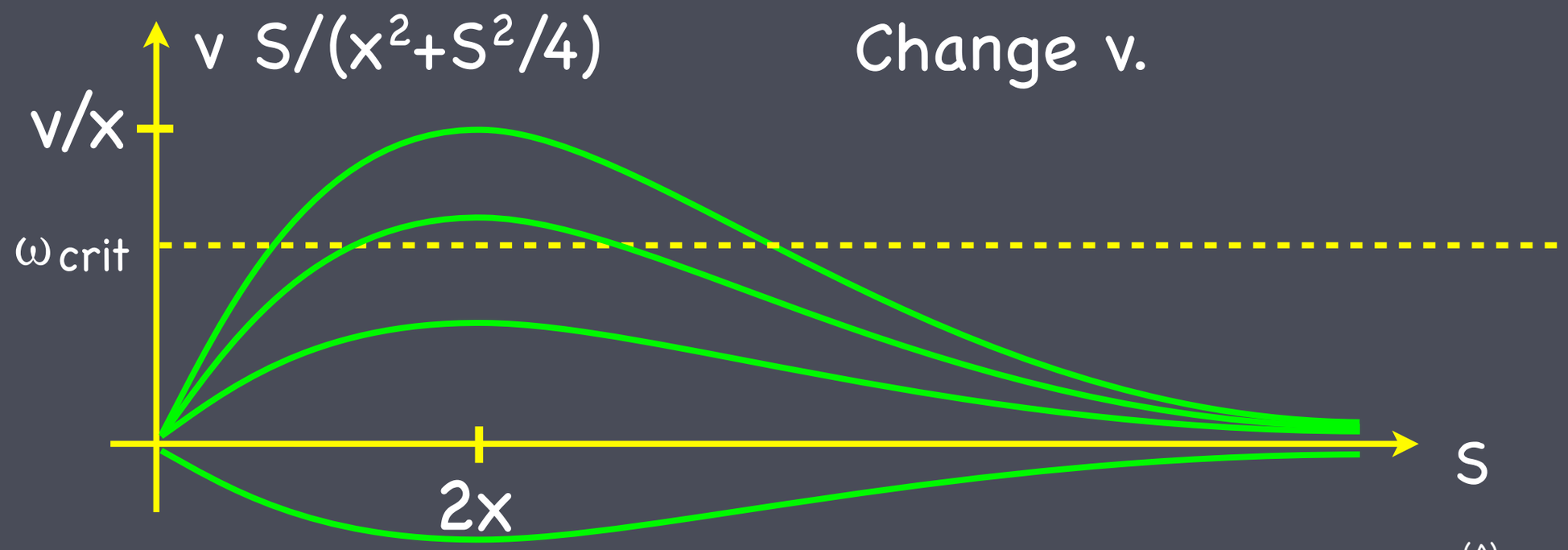

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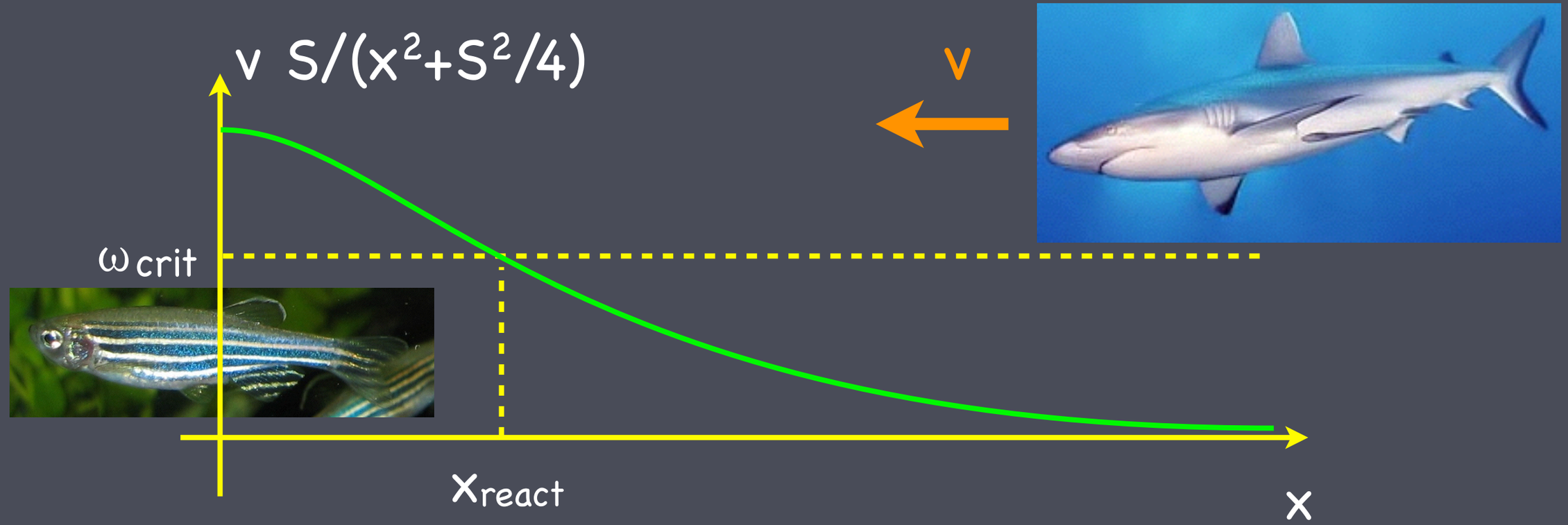

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(E) ...a fast-moving predator.

Shark image - http://en.wikipedia.org/wiki/File:Tibur%C3%B3n.jpg

Triangle with two sides of fixed length, angle between them changes.

Relate the two changing quantities:

(A) $a^2 = b^2+c^2$

(B) $a^2 = b^2+c^2 - 2bc \cos(\theta)$

(C) $a/\sin(A) = b/\sin(B)$

(D) $\sin(\theta) = a/b$