Today…

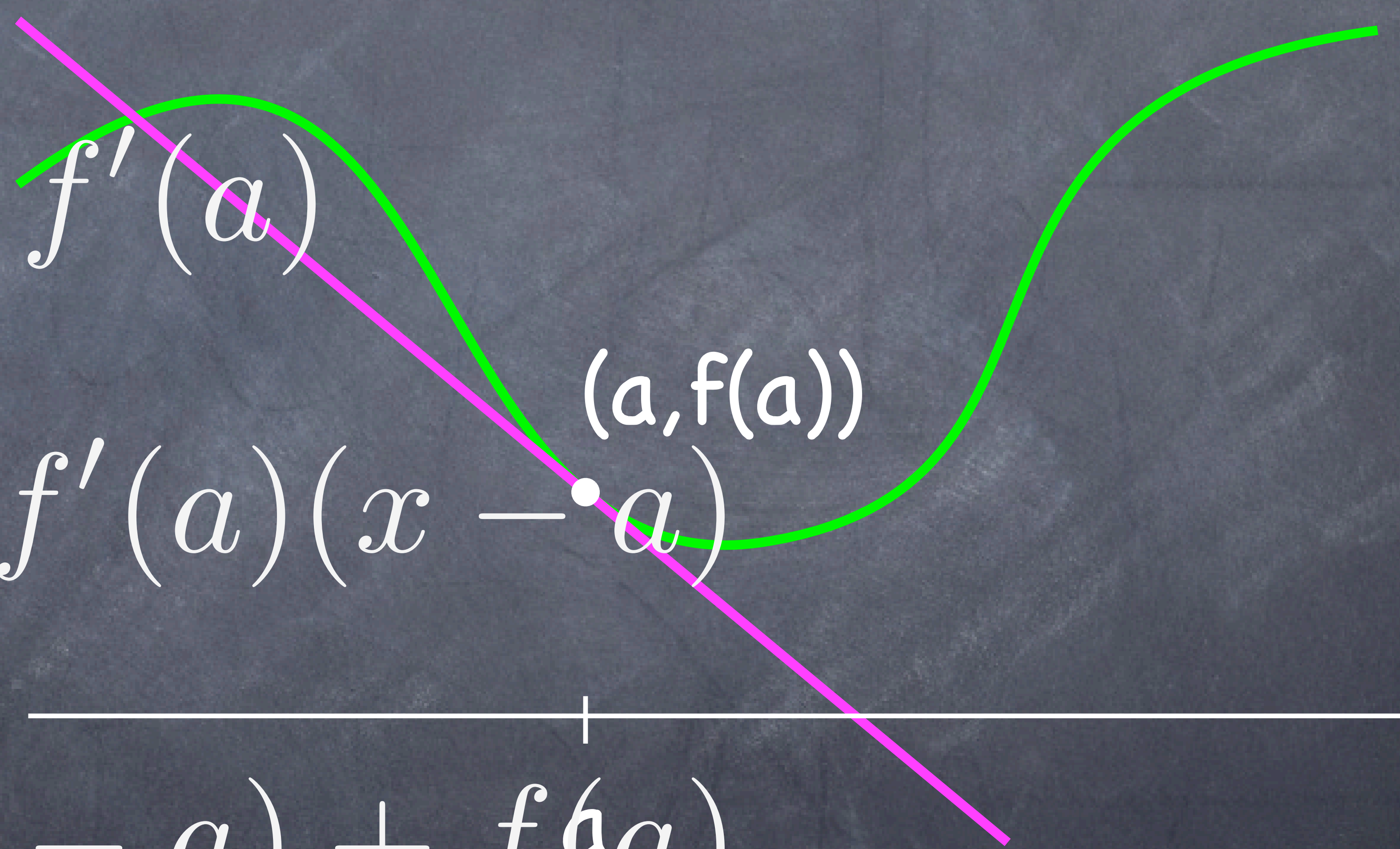
- Tangent lines
- Power rule

Find the tangent line to f(x) at (a,f(a)).

$$\frac{y - f(a)}{x - a} = f'(a)$$

$$y - f(a) = f'(a)(x - a)$$

$$y = f'(a)(x - a) + f(a)$$



Tangent line to sin(x) at x=0

- Slope of sin(x) at x=0 is 1 (spreadsheet last class).
- In general, tangent line: $y = f'(x_0)\,(x-x_0) + f(x_0)$.
- In this case, . . .

(A) $y = \cos(x)\,x + \sin(x)$

(B) $y = x$

(C) $y = x - \pi/2$

(D) $y = \cos(x_0)\,(x-x_0) + \sin(x_0)$

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General case

- Objects involved:

  (i) a function $f(x)$

  (ii) a point of tangency $(a, f(a))$

  (iii) slope at point of tangency $f'(a)$

  (iv) a tangent line $y = f'(a)(x-a) + f(a)$

- Some piece of information is missing - could be from any of these.

Example - simplest case

- Let $f(x) = x^3 + 2x^2 - x + 2$.
- Find tangent line at $x=3$.
- Need equation of line
  - slope is $f'(3)$, point on line is $(3,f(3))$
  - $y = f'(3)(x-3) + f(3) = 38(x-3) + 44$.

Example - slightly harder

- Let $f(x) = x^3 + 2x^2 - x + 2$.
- Find a tangent line parallel to $y=-x+3$.
- Need: a point of tangency, a slope --> line

We need to...

(i) a function $f(x)$

(ii) a point of tangency $(a,f(a))$

(iii) slope at point of tangency $f'(a)$

(iv) a tangent line $y = f'(a)(x-a) + f(a)$

(A) Find a such that $f(a) = -a+3$.

(B) Find a such that $f'(a)=-1$.

(C) Solve $x^3 + 2x^2 - x + 2 = -x + 3$.

Example - slightly harder

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(A) Find a such that $f(a) = -a+3$.

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(C) Solve $x^3 + 2x^2 - x + 2 = -x +3$.

Example - even harder

- Find tangent line to $f(x)=x^2$ that goes through (1,-1).

(i) check, (ii) nope, (iii) if we had (ii), (iv) nope.

- Name unknown point (a,f(a)). Pretend you know a. Means you also know f(a), f'(a).
- What can we now write down?

(i) a function f(x)

(ii) a point of tangency (a,f(a))

(iii) slope at point of tangency f'(a)

(iv) a tangent line $y = f'(a)(x-a) + f(a)$

- $y = f'(a)(x-a) + f(a) = 2a\,(x-a) + a^2$
- (1,-1) must be on this line so

$-1 = 2a\,(1-a) + a^2$. Solve for a.

Find tangent line to $f(x)=x^2$ that goes through (1,-1).

Point of tangency is at

(A) $(1+\sqrt{2}, 3-2\sqrt{2})$

(B) $(1+\sqrt{2}, 3+2\sqrt{2})$

(C) (1, -1)

(D) $(1-\sqrt{2}, 3-2\sqrt{2})$

Find tangent line to $f(x)=x^2$ that goes through (1,-1).

Point of tangency is at

(A) $(1+\sqrt{2}, 3-2\sqrt{2})$

(B) $(1+\sqrt{2}, 3+2\sqrt{2})$

(C) (1, -1)

(D) $(1-\sqrt{2}, 3-2\sqrt{2})$

Power rule

$$f(x) = x^2$$

Find f′ at x=2 (using the definition of the derivative).

$$f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h}$$

$$= \lim_{h \to 0} \frac{4\not{h} + h^{\not{2}}}{\not{h}} = 4$$

Power rule

$$f(x) = x^2$$

Find f' at all points x at the same time

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}x + h^{\cancel{2}}}{\cancel{h}} = 2x$$

Power rule

$$f(x) = x^3$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3hx^2 + 3h^2x + h^3 - x^3}{h}$$

$$= 3x^2$$

Power rule

$$f(x) = x^n$$

$$f'(x) = nx^{n-1}$$

Derivative properties

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df}{dx} + \frac{dg}{dx} \tag{4.1}$$

$$\frac{d}{dx}Cf(x) = C\frac{df}{dx} \tag{4.2}$$

- Notation:

$$y = f(x)$$

Leibniz → $$\frac{dy}{dx} = f'(x)$$ ← Newton