Name: Solution Quiz Score:_____/16

Student Number:________________

Answer questions in the space provided. Show your work. No calculators or notes.

1.

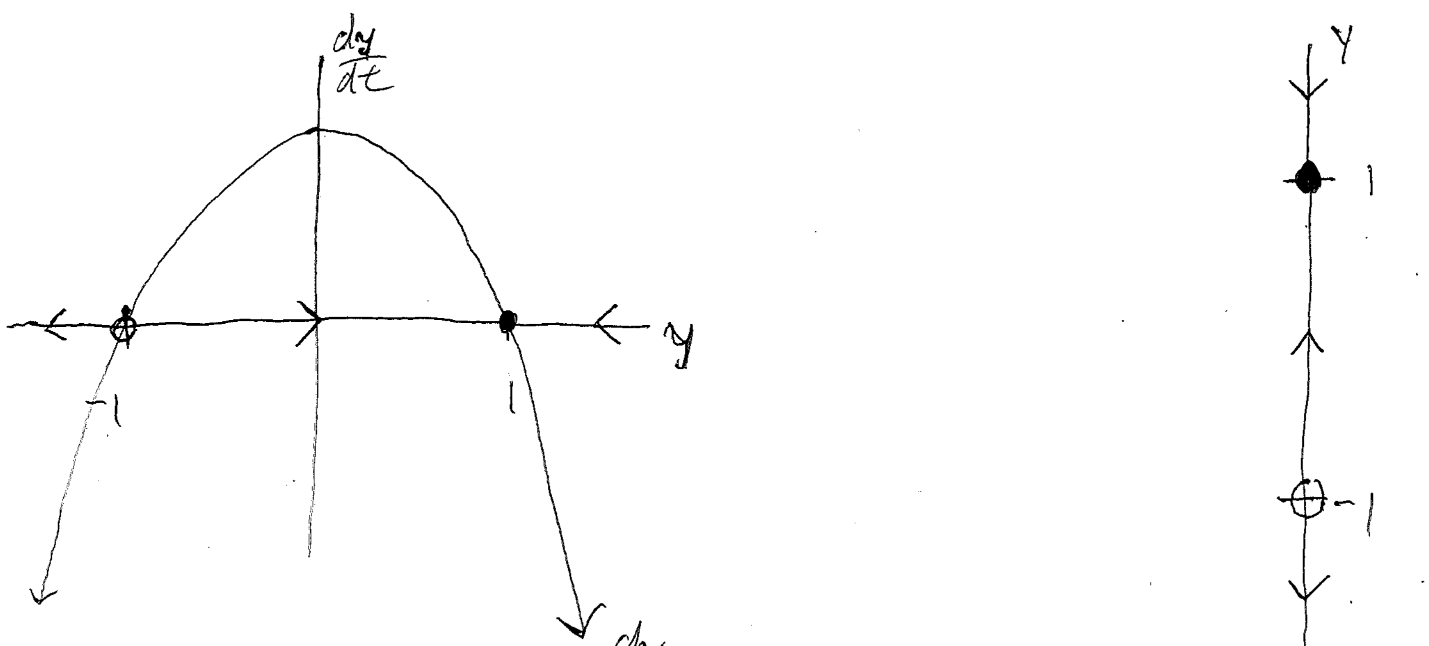
$$\frac{dy}{dt} = (1-y)(1+y)$$

(a) (2 points) Determine the steady state values of the differential equation.

Steady state values occur when $\frac{dy}{dt} = 0$, which occur when $y=1$, $y=-1$.

$$\boxed{y=1,\ y=-1}$$

(b) (5 points) Sketch the state space (phase line) diagram for the differential equation.



$\frac{dy}{dt} = (1-y)(1+y)$

(c) (2 points) Determine stability of steady state values [classify each steady state value as stable or unstable].

$y = -1$ is unstable

$y = 1$ is stable

(d) (1 point) For $y(0) = -1$, determine $\lim_{t\to\infty} y(t)$.

$$\lim_{t \to \infty} y(t) = -1$$

(e) (1 point) For $y(0) = 0$, determine $\lim_{t\to\infty} y(t)$.

$$\lim_{t \to \infty} y(t) = 1$$

(f) (1 point) For $y(0) = 2$, determine $\lim_{t\to\infty} y(t)$.

$$\lim_{t \to \infty} y(t) = 1$$

(g) (3 points) For initial condition $y(0) = 2$ and step size $\Delta t = 1/2$, use two steps of Euler's method to determine approximate solution values of the differential equation at $t = 1/2$ and $t = 1$.

$$y(1/2) \approx y_1 = y_0 + \Delta t \left.\frac{dy}{dt}\right|_{y=y_0}$$
$$= 2 + 1/2(1-2)(1+2)$$
$$= 2 - 3/2 = 1/2$$

$$y(1) \approx y_2 = y_1 + \Delta t \left.\frac{dy}{dt}\right|_{y=y_1}$$
$$= 1/2 + 1/2(1-1/2)(1+1/2)$$
$$= 1/2 + 3/8 = 7/8.$$

$$\boxed{y(1/2) \approx 1/2, \quad y(1) \approx 7/8}$$

(h) (1 point) Are the approximate solution values you found in part (g) underestimates or overestimates? Explain your reasoning.

From the state space diagram, $y(t) > 1$ for all $t \geq 0$ when $y(0) = 2$. Both solution estimates are less than one. Thus, the approximate solution values are underestimates.