Today

- Midterm - end of class or online.
- Chain rule reminder for...
- Related rates examples

Definitions

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- The best fit model is the model with parameter value(s) (a, a&b, C&k) that gives the smallest SSR.

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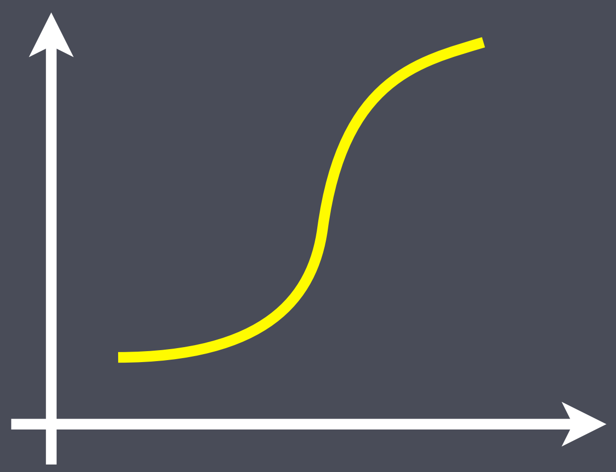

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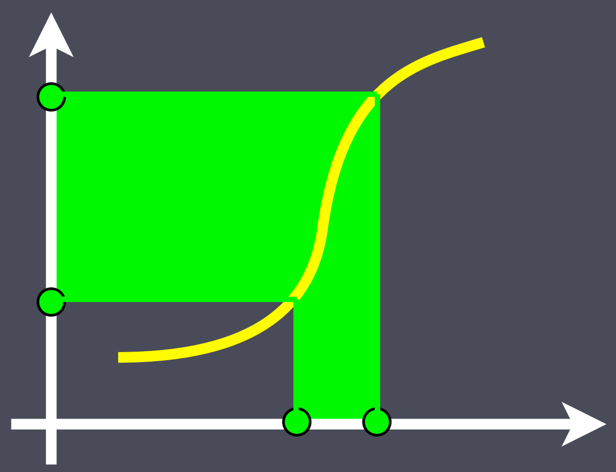

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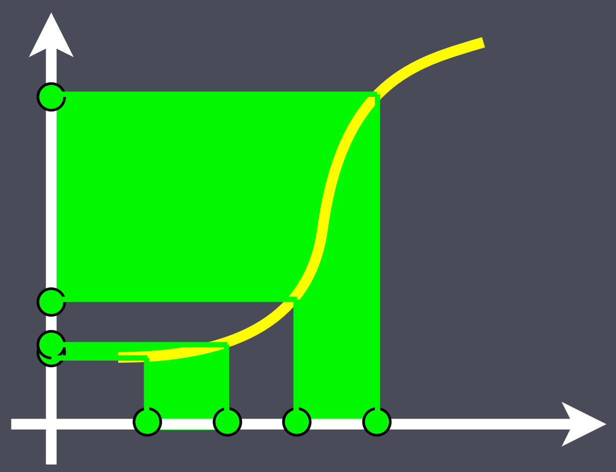

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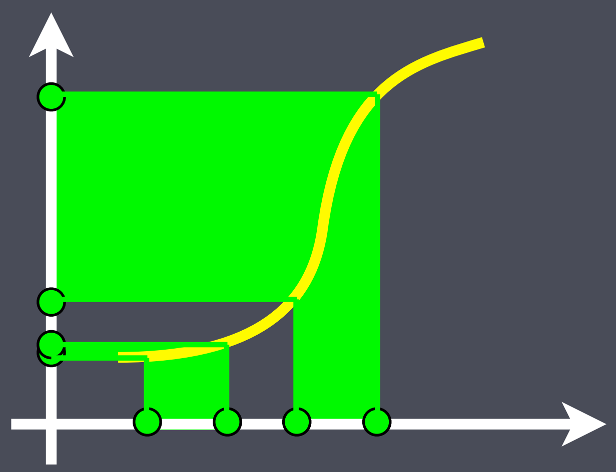

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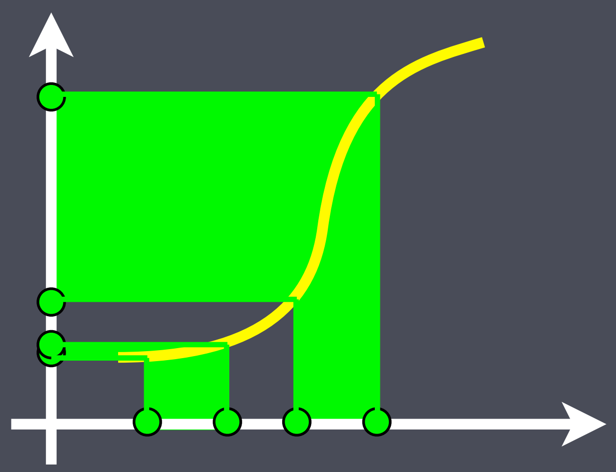
- Where you are matters!



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- When you do one and then the other, you multiply their effects.
- Where you are on the function matters - multiply the stretch factor of g near x: g’(x), by the stretch factor of f near g(x): f’(x).

Gas costs \$1.25/litre. Your car consumes 7 litres/100 km. You've driven 130 km. How much does it cost to drive one more km?

(A) $1.25 \cdot (7/100) \cdot 130$

(B) $1.25 \cdot (7/100)$

(C) $125/7$

(D) $7/125$

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- These are straight lines so the slopes are independent of L, x.

Related rates

- When two quantities (e.g. Q_1 and Q_2) are related to each other, if one changes in time so will the other.
- Knowing the relationship between Q_1 and Q_2 gives you the relationship between Q_1' and Q_2'.

The radius of a spherical tumor grows at a constant rate, k. Determine the rate of growth of the volume of the tumor when the radius is one centimeter.

Which is the relevant equation relating the quantities (not rates of change yet)?

(A) $V = 4/3\ \pi\ r^3$

(B) $V' = 4\ \pi\ r^2 k$

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Now we can plug in r=1.