Today

- Midterms - come to my office
  - Today 2:30 - 5 pm
  - Tues. 12 - 2 pm
  - Wed. 11 am - 12 pm
- Multiple choice.
- Optimization examples

Midterm stats

- Class average: 64%
- 18% got below 50%
- 8% got above 90%
- Max score 98%
- MC: 53%, SAP: 79%,

  LAP1: 90%, LAP2: 70%, LAP3 47%.

The average rate of change of this function over the interval [a,b] is 1. If a=0, which of the following is possible?

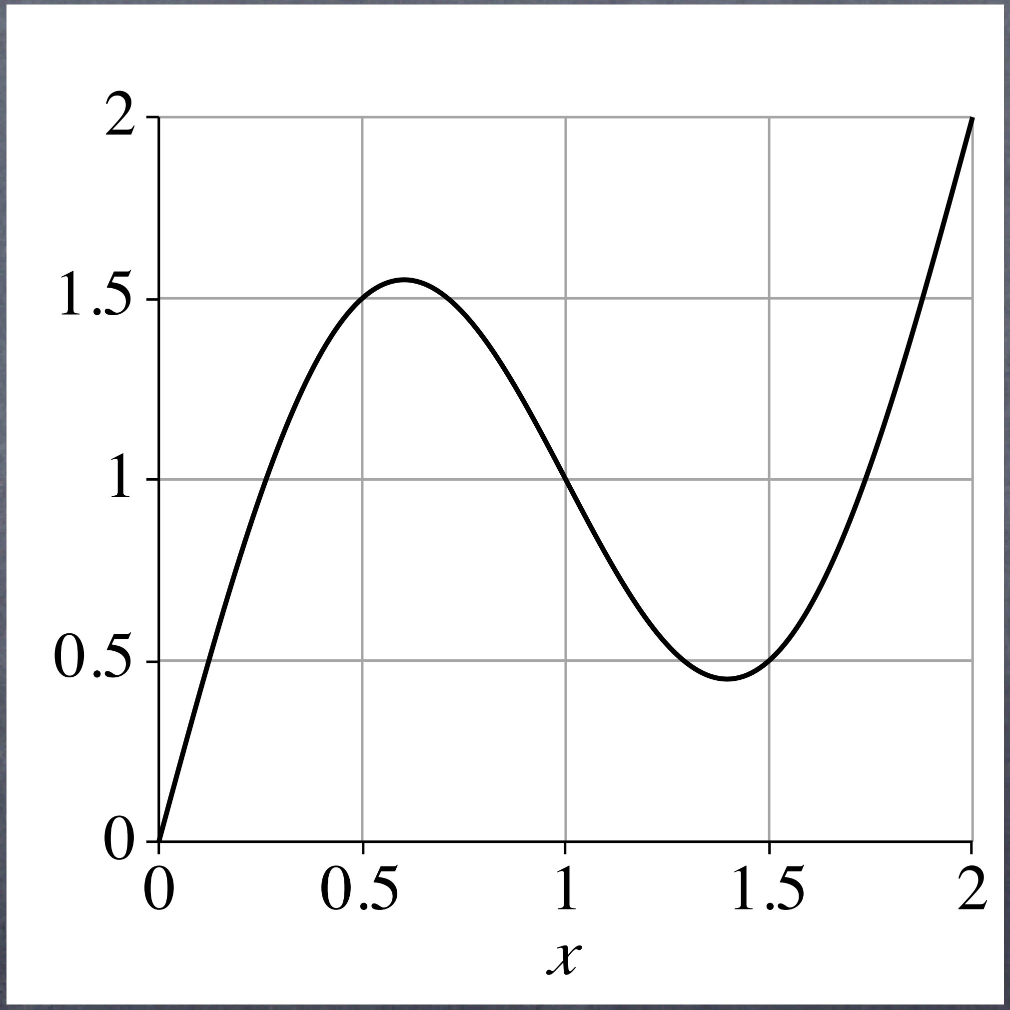
(A) b=0.5

(B) b=1

(C) b=1.25

(D) b=1.5

(E) b=1.75



Which of the following describes the derivative of a function f(x)?

(A) It is defined as $\dfrac{f(x+h)-f(x)}{h}$.

(B) The line we see when we zoom into the graph of f(x).

(C) The average rate of change of f(x) over the interval $0 < x < h$.

(D) More than one of the above.

(E) None of the above.

Which ONE of the following statements is always true for all differentiable functions f that satisfy the stated condition.

(A) When $f''(a) = 0$, the function $f(x)$ has an inflection point at $x=a$.

(B) If $f(x)$ has a local maximum at a then $f'(a) < 0$ and $f''(a) = 0$.

(C) If $f(x)$ has a local minimum at a then $f'(a) = 0$ and $f''(a) < 0$.

(D) If $f(x)$ is increasing and concave up at a then $f'(a) > 0$ and $f''(a) > 0$.

(E) Both the functions $f(x) = x^3$ and $f(x) = x^6$ have inflection points at $x=0$.

As shown in the figure below, the tangent line to the graph of $f(x)$ at $x=a$ intersects the x-axis at $x=b$. Which of the following expressions gives the value of b?

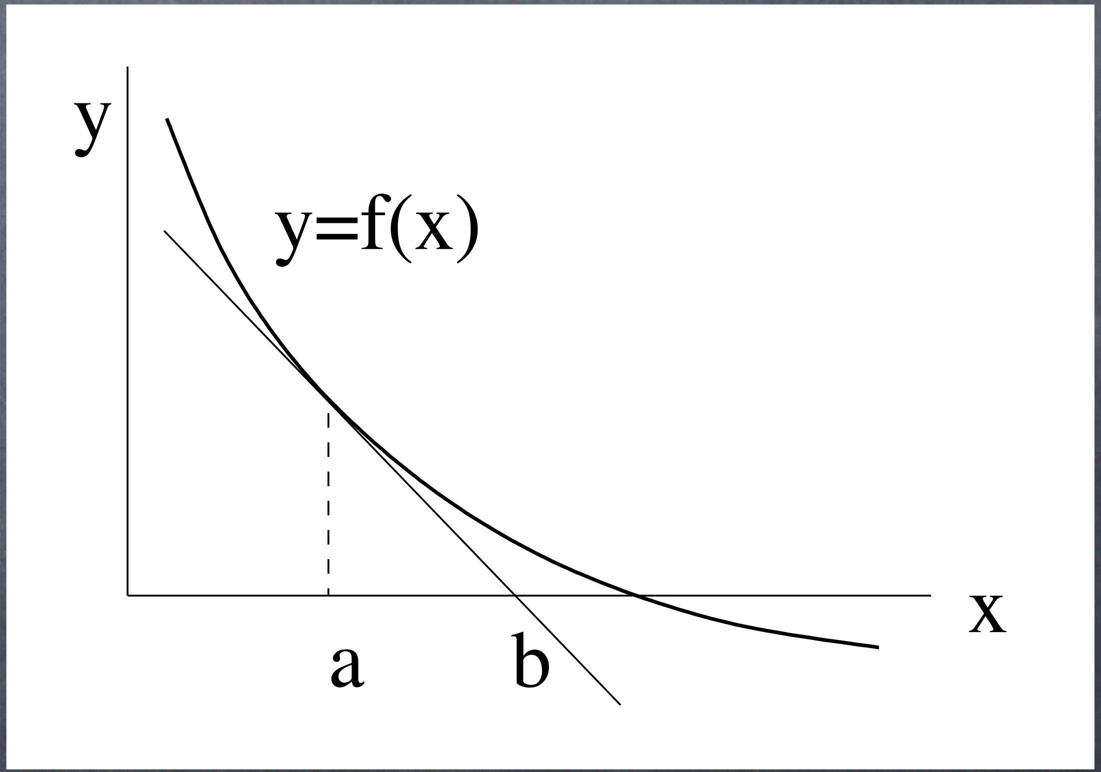
(A) $b=a-f(a)/f'(a)$.

(B) $b=a+f(a)/f'(a)$.

(C) $b=a+f'(b)/f(b)$.

(D) $b=f(a)-f'(a)a$.

(E) $b=f(a)+f'(a)(x-a)$.



In order for the function $f(x) = 1/3\ x^3 + 2x^2 + qx + 2$ to have any critical points, we require that the constant q satisfy which of the following statements?

(A) $0 \leq q \leq 16$

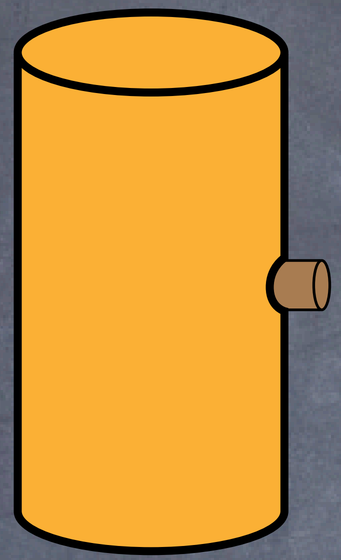
(B) $q \geq 2$

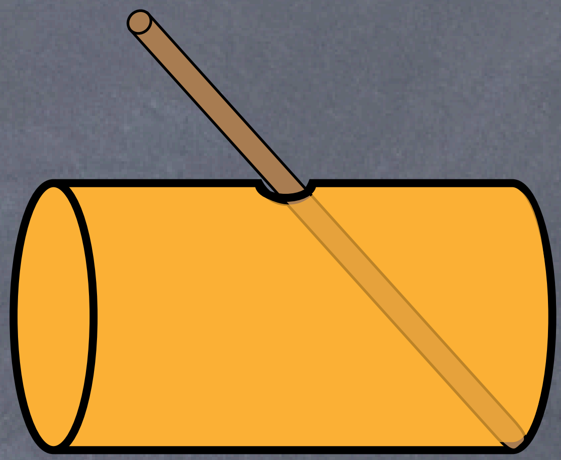
(C) $4 \leq q \leq 16$

(D) $q \leq -4$ or $q \geq 4$

(E) $q \leq 4$

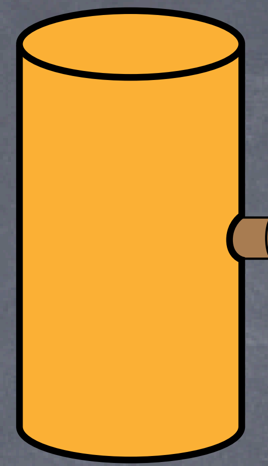
Wine for Kepler's wedding

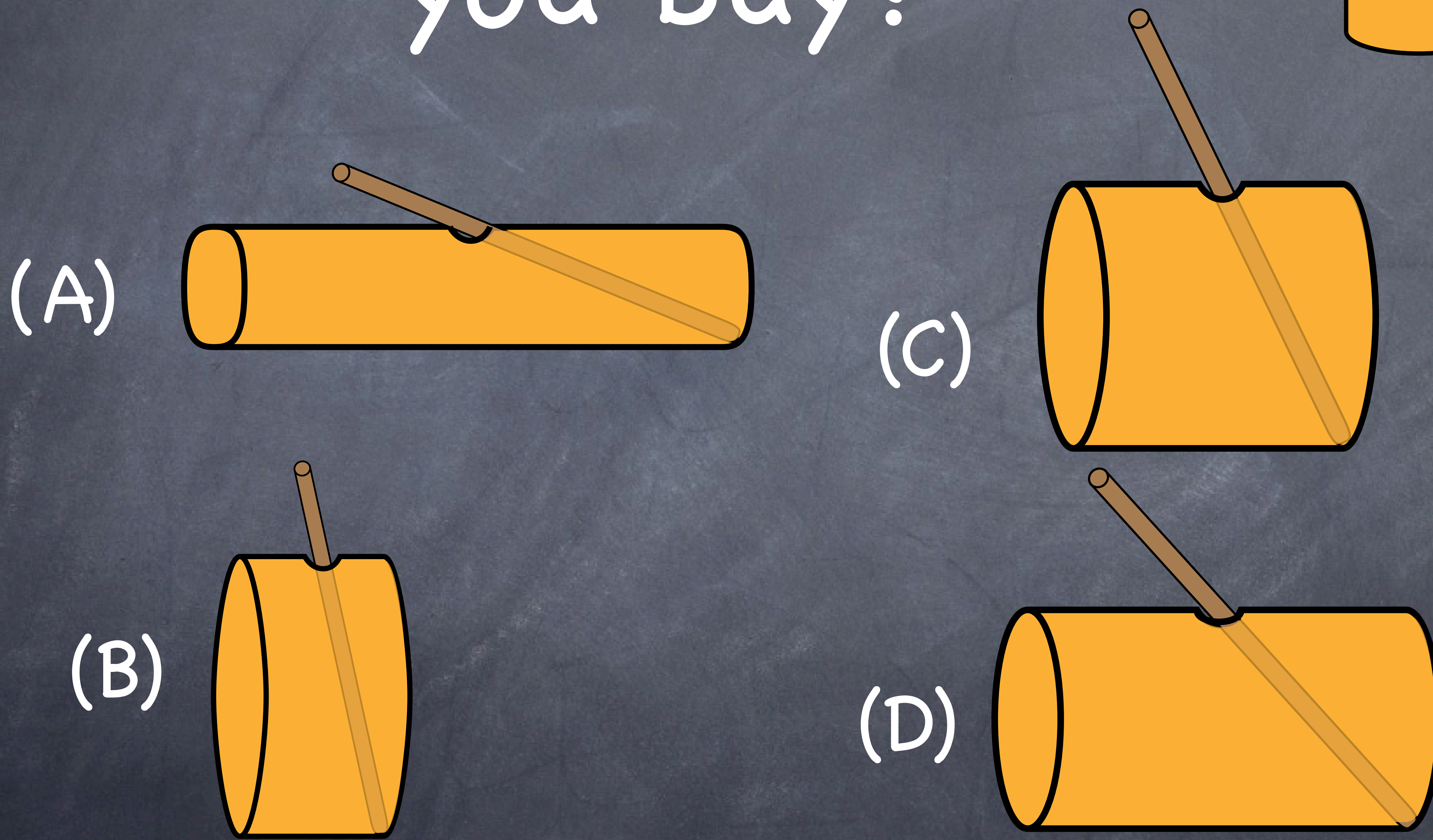




- Wine was sold by "the length of the submerged part of the rod"
- Same length of wet rod = same volume of wine?

Which barrel would you buy?





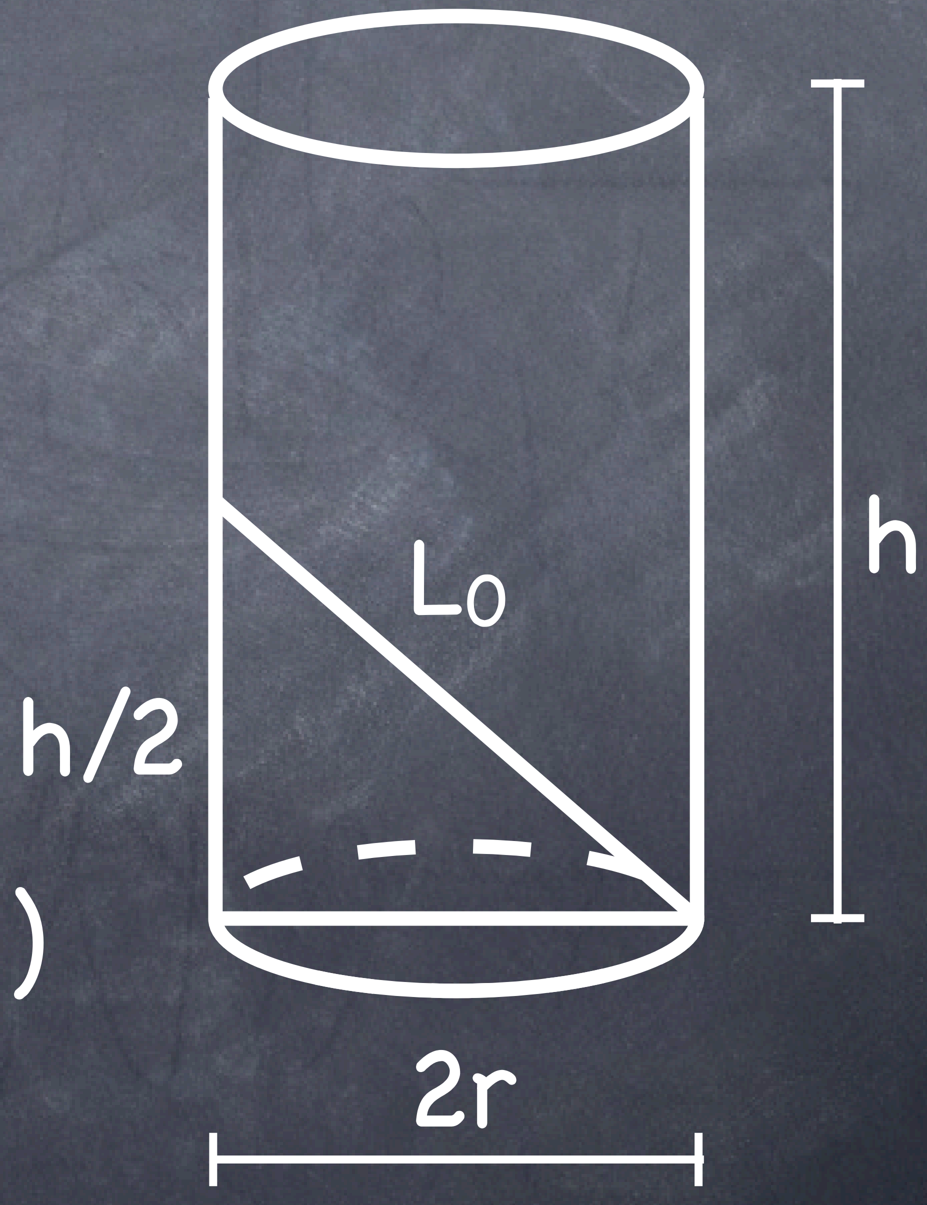
Objective function? (to be maximized)

(A) $V = 2\pi rh$

(B) $r^2 = L_0^2/4 - h^2/16$

(C) $V = \pi r^2 h$

(D) $L_0 = \text{sqrt}((2r)^2 + (h/2)^2)$



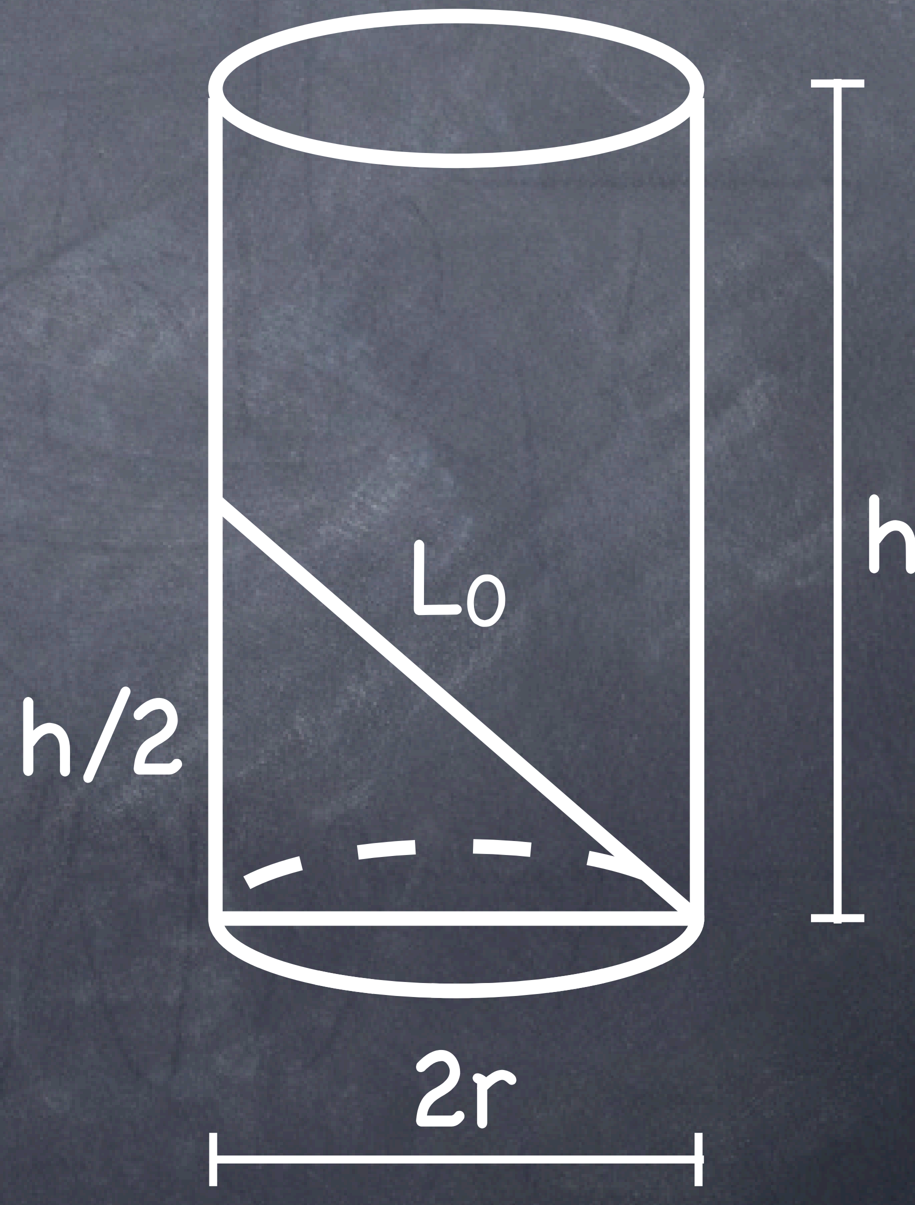
Constraint?
(used to simplify OF)

(A) $L_0^2 = (2r)^2 + (h/2)^2$

(B) $L_0^2 = (2r)^2 + h^2$

(C) $V = 2\pi rh$

(D) $L_0 = \tan(h/4r)$



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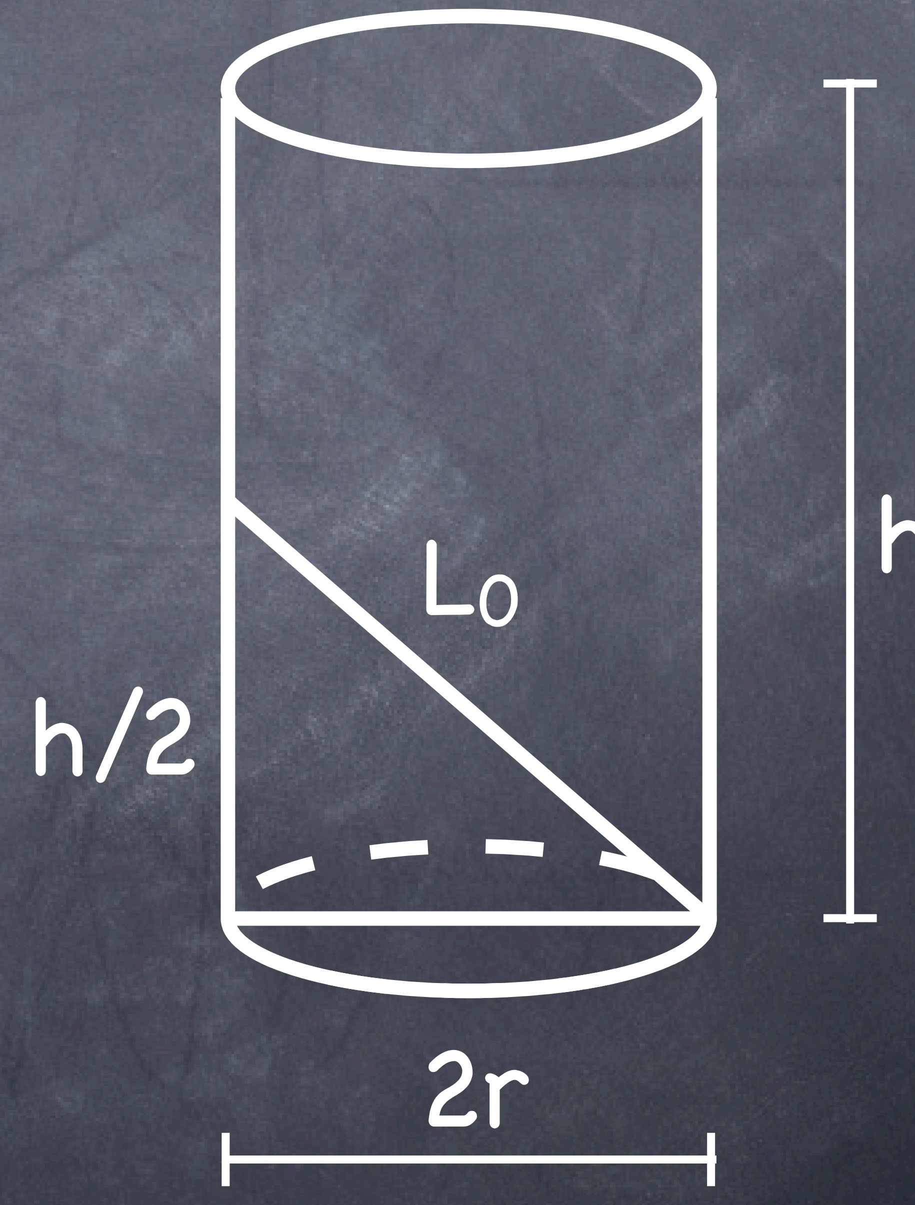

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Solve for:

(A) r

(B) r^2

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$V = \pi h(4L_0^2 - h^2)/16$
What is the best h?

(A) $h = 0$

(B) $h = 2L_0$

(C) $h = \sqrt{3}\, L_0$

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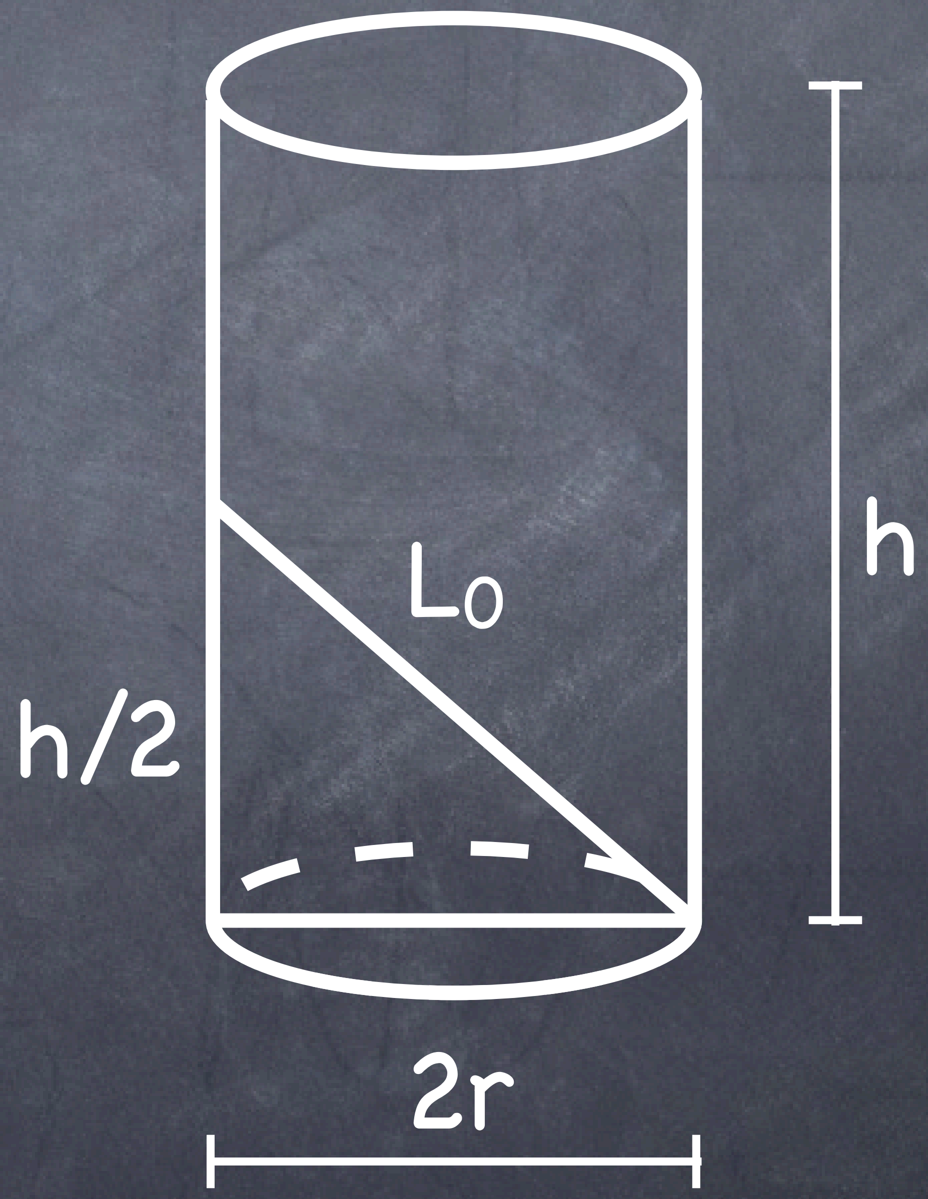

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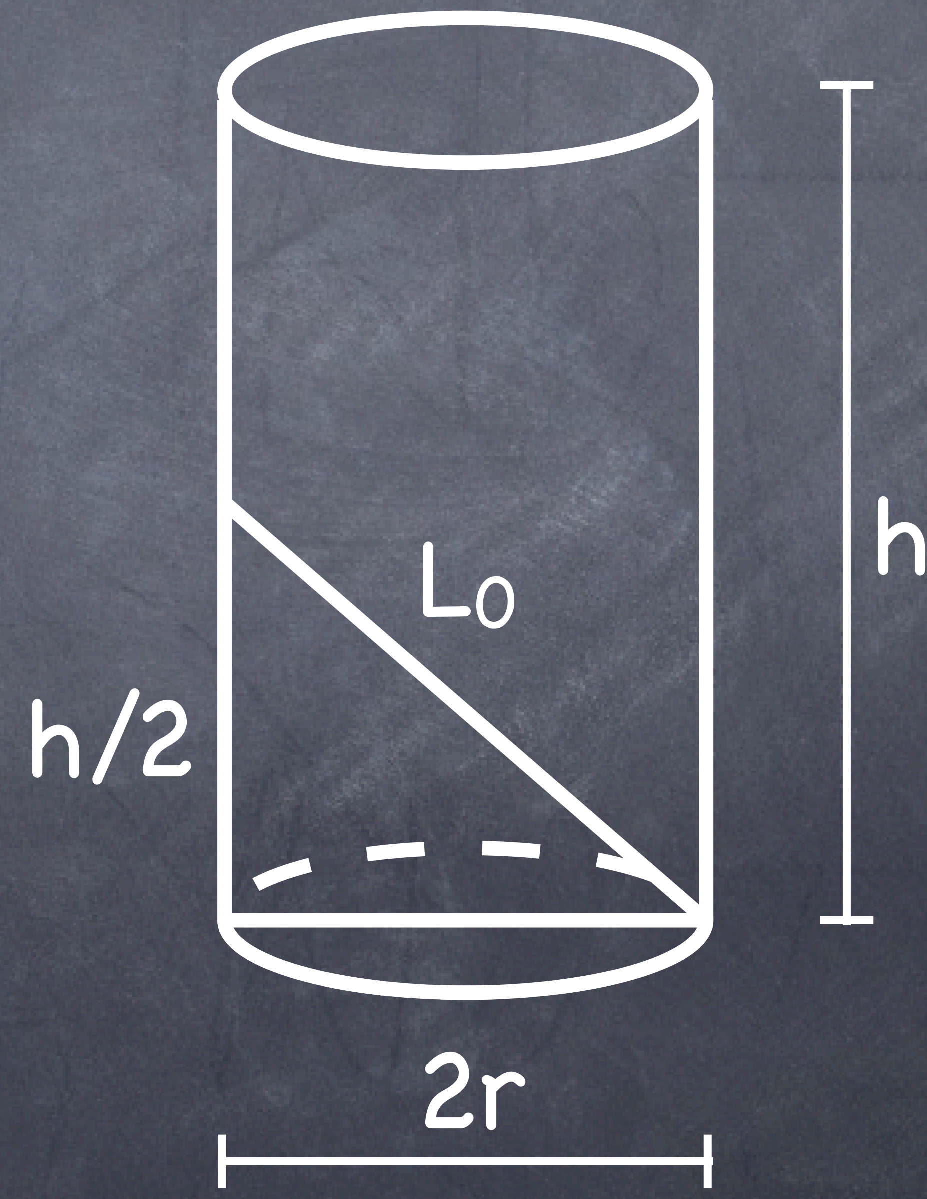

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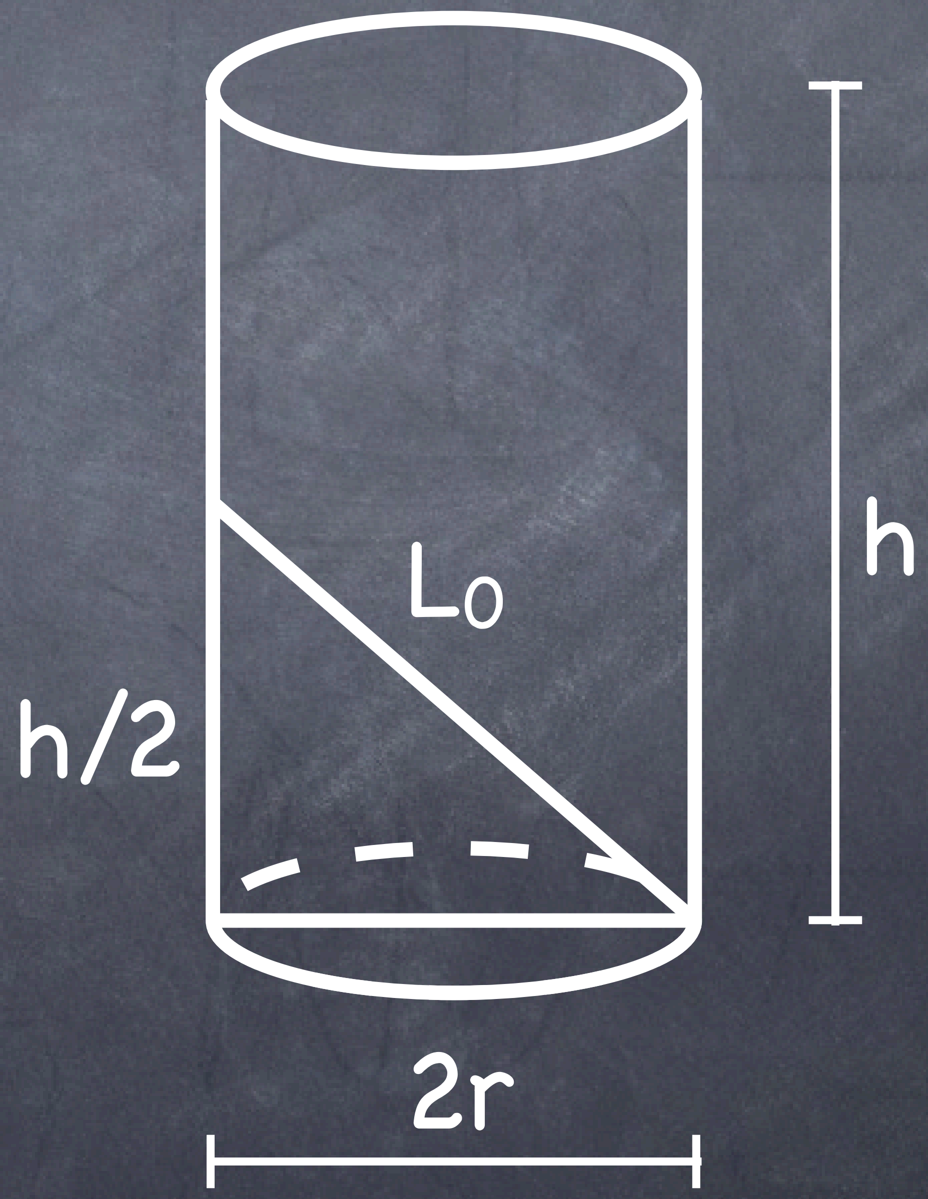

http://www.matematicasvisuales.com/english/html/history/kepler/doliometry.html

Overall procedure

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6. Substitute it into the objective function.
7. Find the absolute extremum (check concavity!).