Today

- Exponential growth and decay
- Differential equations
- Initial value problems
- Model of population growth

Cell growth

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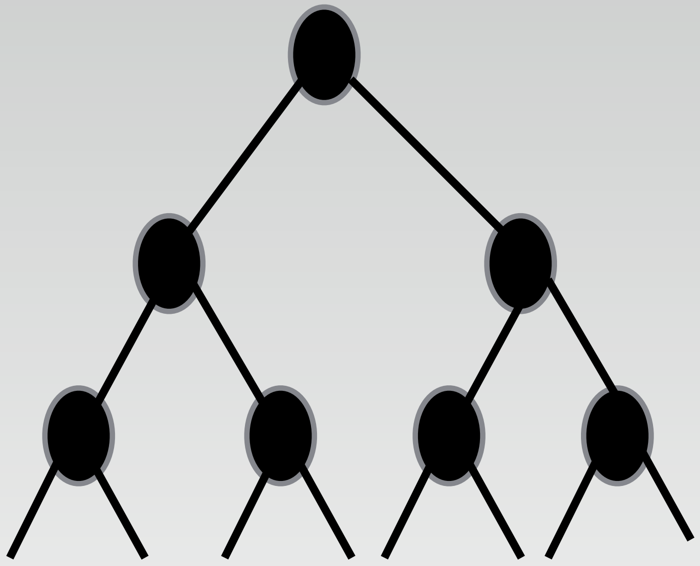

Question 1

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. Each cell divides once every 24 hours. What is the formula for the number of cells in the dish at time t?

A) $c(t) = 2^{24t}$

B) $c(t) = e^{t/24}$

C) $c(t) = 2^{t/24}$

D) $c(t) = e^{\ln(2)t}$

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Question 2

A single cell is placed in a dish containing a growth medium. The dish can support 100,000 cells. If each cell divides once every 24 hours, how long before the dish is full?

A) $t = \ln(10^5)/24$

B) $t = 100{,}000 \cdot \ln(24)$

C) $t = 100{,}000/\ln(24)$

D) $t = \ln(10^5)/\ln(2)$

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C) $t = 100{,}000/\ln(24)$

D) $t = \ln(10^5)/\ln(2)$ ~16.6 days -> $c(t) = 2^t$

Question 3

In 2001, 100 deer were put on a desert island with plenty of resources. By 2011 the population of deer had reached 1000. How many deer will there be in 2015?

A) $d(14) = \ln(10)/10$

B) $d(14) = \ln(1000)/\ln(100)$

C) $d(14) = 100e^{1.4\ln(10)}$

D) $d(14) = 1000e^{14\ln(100)}$

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The rate of decay of a radioactive isotope is ***k***. At what time will the amount of the isotope have decreased by half?

A) $t = (1/2)e^k$

B) $t = k \cdot \ln(2)$

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D) $t = -\ln(2)/k$ -> if $k<0$

Half-life

From the previous clicker question:

- Let $y(t) = y_0 e^{kt}$. At $t=0$, $y(0) = y_0 e^0 = y_0$.

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- If $k<0$, $y(t)$ is decreasing and halves when: $y_0/2 = y_0e^{kt}$.
- That is when $t=-\ln(2)/k$.
- This is called the **half-life**.

Doubling time

Similarly:

- Let $y(t) = y_0e^{kt}$.
- If $k>0$, $y(t)$ is increasing and doubles when $2y_0 = y_0e^{kt}$.
- That is when $t=\ln(2)/k$.
- This is called the **doubling time.**

Question 4

Which of the following functions does not satisfy the equation $y'(t) = ky(t)$?

A) $y(t) = e^{kt}$

B) $y(t) = 2e^{kt}$

C) $y(t) = e^{kt+5}$

D) $y(t) = Ce^{kt}$

Question 4

Which of the following functions does not satisfy the equation $y'(t) = ky(t)$?

They all satisfy the differential equation!

A) $y(t) = e^{kt}$

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Differential equations

The function $y'(t) = ky(t)$ is a type of differential equation.

- **Differential equations** relate one (or more) derivative of a function to the function itself.
- A solution to a differential equation is a **function**.

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  - We know a general solution is $y(t) = Ce^{kt}$.
  - From the initial condition: $y(0) = Ce^{k0}=C=2$
  - Therefore: $y(t) = 2e^{kt}$

a) $y(t) = 2k$

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c) $y(t)= e^{kt+2}$

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Question 5

N is the number of Icelandic people. The average birth rate in Iceland is r=0.0145 births per capita per year, the average mortality rate is m=0.006 deaths per capita per year and the average emigration rate is b=0.003 people per capita per year. Which of the following equations would describe the rate of change of Icelanders?

A) $dN/dt = -rN-mN+bN$

B) $dN/dt = rN-mN +bN$

C) $dN/dt = rN-mN-bN$

D) $dN/dt = -rN+(m-b)N$

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Question 6

Which of the following equations solves our model for number of Icelanders from the previous question? ($dN/dt = rN-mN-bN$)

A) $N(t) = (r/2)N^2 - (m/2)N^2 - (b/2)N^2$

B) $N(t) = Ce^{rt} + Ce^{-mt} + Ce^{-bt}$

C) $N(t) = Ce^{r-m-b+t}$

D) $N(t) = Ce^{(r-m-b)t}$

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Question 7

Because of the Bárðabunga eruption in Iceland the emigration rate has increased to b=0.009 per year. Recall the average mortality rate in Iceland is m=0.006 deaths per person per year. What does the birth rate need to be for the Icelandic population to be growing?

A) >0.009

B) >0.0015

C) >0.015

D) >0.006

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C) >0.015

D) >0.006